Contents

Foundation 10th Mathematics

Booklet 1

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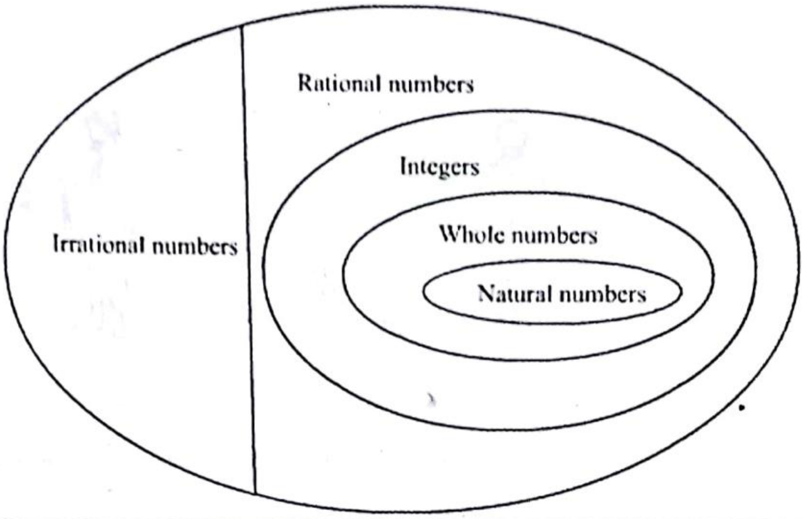
chapter 1

The Real Number System



REAL NUMBERS

Introduction

REAL NUMBERS (R)

Rational Numbers (Q)

Numbers in the form $\frac{p}{q}$, where $q \neq 0$ and p and q are coprime (i.e. the integers which have no common factor except 1), are rational numbers.

The decimal representation of rational numbers are either terminating or non terminating repeating.

Examples: $-3, 0, 1, 25, \frac{3}{5}, \frac{-4}{5}, -\frac{5}{8}, 4.25, -4.5,$ $0.534, 0.024, 8.3232, \ldots\ldots, 5.48\overline{62}, 0.\overline{531}$

Irrational Numbers (I)

Numbers which are not in the form $\frac{p}{q}$, where $q \neq 0$ and p and q are coprime (i.e. the integers which have no common factor except 1), are irrational numbers.

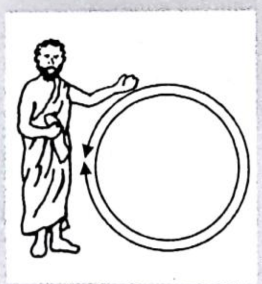
The decimal representation of irrational numbers are non-terminating non repeating.

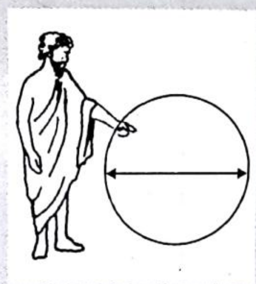
Examples: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt[3]{11}, 5.373773\ldots,$ $-\sqrt[5]{29}, 0.1010010001\ldots, \pi, \frac{\sqrt[2]{3}}{5}$

Note: $\frac{22}{7}$ or 3.14 are not the exact value of π

A short History of π

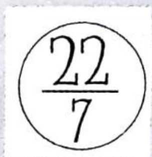
(i) From ancient times, people have known that the circumference of a circle is more than 3 times its diameter.





(ii) Through the centuries different civilizations have tried to find a fraction which would be exactly equal to the ratio : $\frac{\text{circumference}}{\text{diameter}}$. About 1700 B.C., the Egyptians used the fraction $\frac{256}{81}$ to approximate this ratio.

(iii) About 220 B.C. the Greeks used the Greek letter π to represent this ratio.

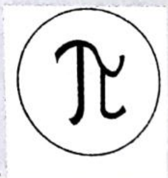


And they used the fraction $\frac{22}{7}$ as its approximate value.

(iv) In their search for the elusive rational number equal to π, various other civilizations used these approximations.

Civilization	Date	Value for π
Chinese	470	$\frac{355}{113}$
Hindu	530	$\frac{3927}{1250}$
European	1220	$\frac{864}{275}$

(vi) Finally, in 1761, Johann Lambert proved that there is no rational number equal to π.
That is, the decimal form of πdoes not repeat. π= 3.141 592 653 589...
In 1987, mathematicians at the University of Tokyo used a super computer to determine π to 201 326 000 decimal places. As expected, the decimal representation did not repeat.



NOTE:(i) The union of the set of rational numbers and the set of irrational numbers is the Real Numbers.

(ii) Sum, difference, product or quotient of any two rational numbers is always a rational number.

(iii) Sum, difference, product or quotient of any two irrational numbers can either be rational or irrational.

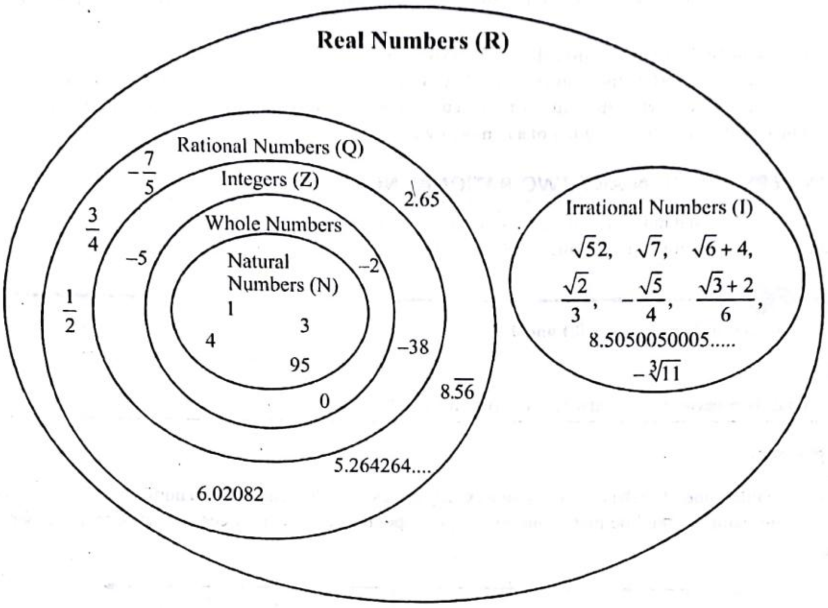
(iv) Sum, difference, product or quotient of a rational and an irrational number is always an irrational number.

All type of numbers which we will study up to class X are real numbers. In class XI, we shall study new type of numbers: Imaginary numbers and complex numbers.

We know $\sqrt{4} = \pm 2$, because $2 \times 2 = 4 = (-2) \times (-2)$

Can you find the value of $\sqrt{-4}$ (think) ?

DIFFERENT TYPES OF REAL NUMBERS :



Symbols of some sets of specific type numbers:

R : set of all real numbers
R^+ : set of all positive real numbers
Z : set of all integers
Z^+ : set of all positive integers
Q : set of all rational numbers
Q^+ : set of all positive rational numbers
N : set of all natural numbers

NOTE: (i) 0 is called the identity element of addition for any real number 'a' because $a + 0 = 0 + a = a$, for all real number a.

(ii) 1 is called the identity element of multiplication for any real number 'a', because $1.a = a.\ 1 = a$, for all real number a

(iii) $-a$ is called the additive inverse of any real number a, because $a + (-a) = (-a) + a = 0$, $\forall$ real number a

(iv) $\frac{1}{a}$ is the multiplicative inverse of any non zero real number a, because $a \times \frac{1}{a} = 1$, for all real number a.

TEXT FOR DIVISIBILITY :

(i) A number is divisible by 2 when its units digit is even or zero e.g., 350, 7916, etc.

(ii) A number is divisible by 3 when the sum of its digits is divisible by 3 e.g., 342, 13791, etc.

(iii) A number is divisible by 4 when the number formed by last two right hand digits is divisible by 4 or if the last two digits are zero. e.g., 1264, 1500, etc.

(iv) A number is divisible by 5 when its unit's digit is either 5 or 0 e.g., 50, 245, etc.

(v) A number is divisible by 6 when it is divisible by 2 and 3 both e.g., 354.

(vi) A number is divisible by 8 when the number formed by the last three right hand digits is divisible by 8 or when the last three digits are zero e.g., 1000, 87895432 etc.

(vii) A number is divisible by 9 when the sum of its digits is divisible by 9 e.g., 39537.

(viii) A number is divisible by 10 when its unit's digit is 0 e.g., 510, 1350, etc.

(ix) A number is divisible by 11 when the difference between the sum of the digits in the odd and even places is either zero or a multiple of 11.

(x) A number is divisible by 12 when it is divisible by 3 and 4 both e.g., 624, etc.

(xi) A number is divisible by 25 when the number formed by last two right hand digits is divisible by 25 e.g., 123475, etc.

(xii) A number is divisible by 125 when the number formed by last three right hand digits is divisible by 125 e.g., 743625, etc.

NOTE : No rule is known till today for divisibility of a number by 7.

RATIONAL NUMBERS BETWEEN ANY TWO RATIONAL NUMBERS :

(i) Average of any two rational number is always a rational number.

(ii) Between any two rational numbers, infinite number of rational numbers can exist.

ILLUSTRATION 1.1

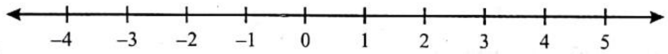
Find two irrational numbers between 0.1 and 0.12.

SOLUTION :

Two rational numbers between 0.1 and 0.12 are 0.102 and 0.103.

REAL NUMBER LINE:

A line is called a Real Number line, if each point on the line exactly corresponds to an unique real number and each real number exactly corresponds to an unique point on the line that is one-to-one correspondence exists between the set of real numbers and the set of points on the line.

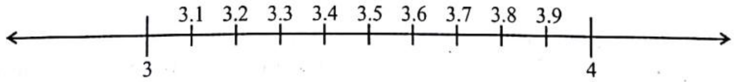


The point corresponds to 0 is called the origin. The arrow on the right end of the line indicates a positive direction. Each point to the right of the origin reperesents a positive real number and each point to the left of origin reperesents a negative real number.

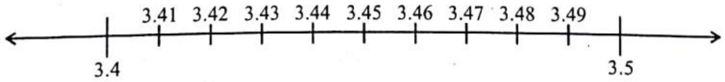
REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE THROUGH SUCCESSIVE MAGNIFICATION:

Let us try to represent 3.47 on the number line.

We know that 3.47 lies between 3 and 4. We divide the portion between 3 and 4 into 10 equal parts as below:



Now, 3.47 lies between 3.4 and 3.5 Again we divide the portion between 3.4 and 3.5 into 10 equal parts



Now, we can easily locate 3.47 on the number line.

In the above method, we have successively magnified different portions to represent 3.47 on the number line.

This method of representation of a real number on the number line is known as method of successive magnification.

ILLUSTRATION 1.2

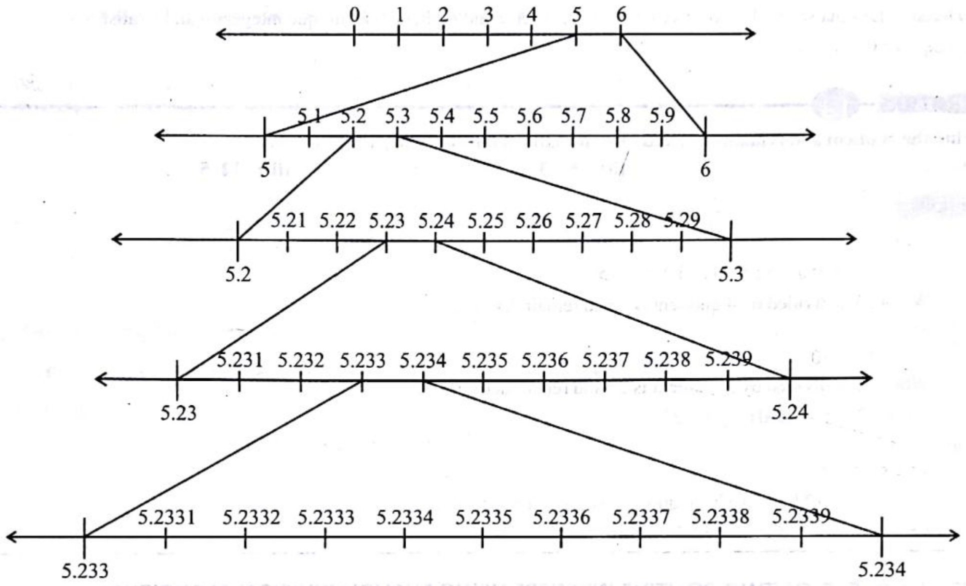
Represent $5.2\bar{3}$ on the number line using successive magnification (upto 4 places of decimal).

SOLUTION :

$5.2\bar{3} = 5.2333$ (upto four decimal places).

$5.2\bar{3}$ lies between 5 and 6

$\therefore$ we divide portion between 5 and 6 into 10 equal parts and go on successively magnifying as follows:



$5.2\bar{3}$ will be located closer to 5.2333. The numbers of times we successively magnify determines the level of accuracy of representetion.

REPRESENTATION OF IRRATIONAL NUMBERS ON THE NUMBER LINE :

By using the Pythagoras theorem we can represent an irrational number on the real number line.

ILLUSTRATION 1.3

Show how $\sqrt{5}$ can be represented on the number line.

SOLUTION :

$\sqrt{5} = \sqrt{(2)^2 + (1)^2}$

Draw number line as shown in Fig. Let the point O represent 0 (zero) and point A represent 2. Draw perpendicular AY at A on the number line and cut-off arc $AB = 1$ unit.

We have, $OA = 2$ units $AB = 1$ unit

Using Pythagoras theorem, we have

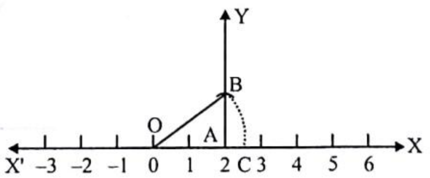
$OB^2 = OA^2 + AB^2 \Rightarrow OB^2 = (2)^2 + 1^2 = 5 \Rightarrow OB = \sqrt{5}$

Taking O as the centre and $OB = \sqrt{5}$

as radius draw an arc cutting real number line at C.

Clearly, $OC = OB = \sqrt{5}$.

Hence, C represents $\sqrt{5}$ on the number line.



NOTE : In the same way other irrational numbers like $\sqrt{2}, \sqrt{3}, \sqrt{6}, \sqrt{7}$ etc. can also be represented on the real number line.

$\sqrt{2} = \sqrt{(1)^2 + (1)^2}$, $\sqrt{3} = \sqrt{(\sqrt{2})^2 + (1)^2}$, $\sqrt{6} = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2}$, $\sqrt{7} = \sqrt{(2)^2 + (\sqrt{3})^2}$

EUCLID's DIVISION LEMMA :

Euclid's Division Lemma states that for given positive integer a and b, there exist unique integers q and r satisfying
$a = bq + r;\ 0 \le r < b$.

ILLUSTRATION 1.4

Find the quotient and remainder q and r for the pairs of positive integers given below :

(i) 23, 4 **(ii) 81, 3** **(iii) 12, 5**

SOLUTION :

(i) 23, 4

$23 = 5 \times 4 + 3;\ q = 5;\ r = 3$ and $0 \le r < 5$

When 23 is divided by 4 quotient is 5 and remainder is 3.

(ii) 81, 3

$81 = 27 \times 3 + 0$

When 81 is divided by 3 quotient is 27 and remainder is 0.

So, $q = 27;\ r = 0$ and $0 \le r < 27$

(iii) 12, 5

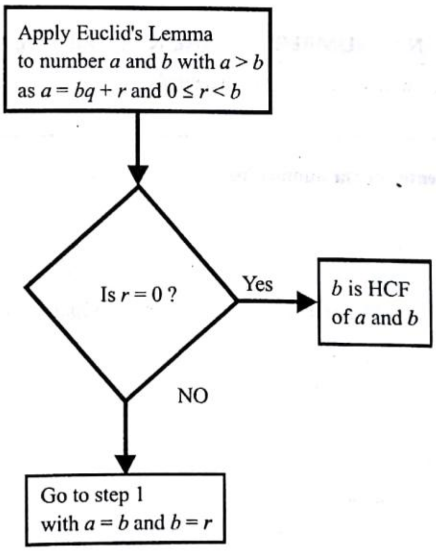
$12 = 5 \times 2 + 2.$

On dividing 12 by 5, we have quotient is 2 and remainder 2.

So, $q = 2;\ r = 2$ and $0 \le r < 5$.

TO FIND THE H. C. F. OF TWO POSITIVE INTEGERS USING EUCLID's DIVISION ALGORITHM :

To obtain the H. C. F. of two positive integers, say a and b, with $a > b$, follow the step below :



Step (i) : Apply Euclid's division lemma, to a and b. So, we find whole numbers, q and r such that $a = bq + r, 0 \le r \le b$

Step (ii) : If $r = 0$, b is the H.C.F. of a and b. If $r \neq 0$, apply the division lemma to b and r.

Step (iii) : Continue the process till the remainder is zero. The divisor at this stage will the required H.C.F.

ILLUSTRATION – 1.5

Let us find the H.C.F. of 60 and 108 using the Euclid's Division Algorithm.

SOLUTION :

Step (i) : Since, $108 > 60$ applying Euclid's Lemma to 60 and 108, we have
$108 = 60 \times 1 + 48$ where $0 < 48 < 60$
Step (ii) : Since, remainder $48 \neq 0$
So, again applying the division lemma to 60 and 48, we have
$60 = 48 \times 1 + 12$ where $0 \leq 12 < 48$.
Step (iii) : Again remainder $12 \neq 0$ so applying division lemma to 48 and 12, we get
$48 = 12 \times 4 + 0$, Here remainder is zero.
Therefore, 12 is the required HCF.

ILLUSTRATION – 1.6

To find the H.C.F. of 1071 and 1029, using Euclid 's division algorithm.

SOLUTION :

Since, $1071 > 1029$, we apply the division lemma to 1071 and 1029, to get
$1071 = 1029 \times 1 + 42$
since, remainder $42 \neq 0$ so again applying division lemma in 1029 and 42, we get
$1029 = 42 \times 24 + 21$ again $21 \neq 0$
Applying Euclid's Lemma again in 42 and 21, we get
$42 = 21 \times 2 + 0$
Since, remainder is zero so H.C.F. is 21.

FUNDAMENTAL THEOREM OF ARITHMETIC :

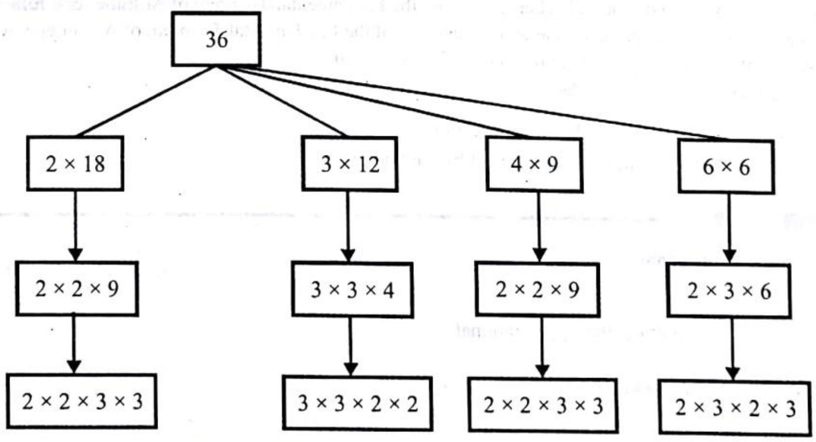
Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

ILLUSTRATION – 1.7

Express following number as the product of prime factors. (i) 36 (ii) 156

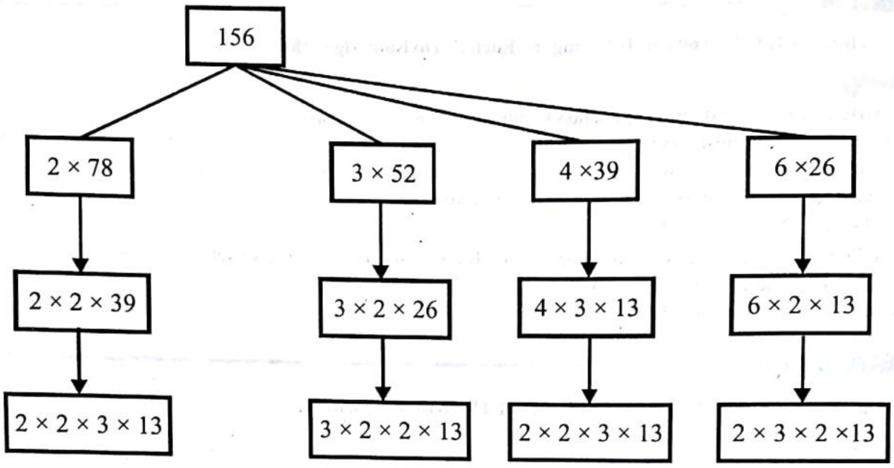
SOLUTION :

(i) 36



In each of the cases product of prime factors of 36 is $2 \times 2 \times 3 \times 3$.

(ii) 156



So, in each of the cases prime factors of 156 is $2 \times 2 \times 3 \times 13$

So, the prime factorisation of a number is unique.

TO FIND THE H. C. F. AND L. C. M. BY PRIME FACTORISATION METHOD :

(i) H. C. F. = Product of each common prime factor(s) with smallest power involved in the numbers.

(ii) L.C. M. = Product of each prime factors with greatest power involved in the numbers.

(iii) For any two positive numbers a and b. H. C. F. $(a, b) \times$ L.C. M. $(a, b) = a \times b$

Note : For any three positive integers p, q and r

H.C. F. $(p, q, r,) \times$ L.C. M. $(p, q, r) \neq p \times q \times r$

Where H. C. F. (a, b) means H. C. F. of a and b and L.C.M. (a, b) means L.C. M. of a and b.

THEOREM : Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Proof : Let the prime factorisation of a be as follow :

$a = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes, not necessarily distinct.

Therefore, $a^2 = (p_1 p_2 \ldots p_n)(p_1 p_2 \ldots p_n) = p_1^2 p_2^2 \ldots p_n^2$

Now, we are given that p divides a^2. Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2. However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are $p_1, p_2, \ldots, p_n$. So p is one of $p_1, p_2, \ldots, p_n$.

Now, since $a = p_1 p_2 \ldots p_n$, p divides a.

We are now ready to give a proof that $\sqrt{3}$ is irrational.

The proof is based on a technique called 'proof by contradiction'.

ILLUSTRATION 1.8

Prove that $\sqrt{2}$ is irrational.

SOLUTION :

Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers r and s $(\neq 0)$ such that $\sqrt{2} = \frac{r}{s}$.

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$, where a and b are co-prime.

So, $b\sqrt{2} = a$

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2. Now, by Theorem 1.3, it follows that 2 divides a.

So, we can write $a = 2c$ for some integer c.

Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2, and so 2 divides b (again using Theorem 1.3 with $p = 2$)

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction, has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

ILLUSTRATION – 1.9

Prove that $\sqrt{3}$ is Irrational.

SOLUTION :

Let us assume, to the contrary, that $\sqrt{3}$ is rational. That is, we can find integers a and b ($\neq 0$) such that $\sqrt{3} = \frac{a}{b}$.

Suppose a and b have a common factor other than 1, then we can divide by the common factor, and assume that a and b are co-prime. So, $b\sqrt{3} = a$

Squaring on both sides, and rearranging, we get $3b^2 = a^2$.

Therefore, a^2 is divisible by 3, and by Theorem it follows that a is also divisible by 3.

So, we can write $a = 3c$ for some integer c

Substituting for a, we get $3b^2 = 9c^2$, that is, $b^2 = 3c^2$.

This means that b^2 is divisible by 3, and so b is also divisible by 3. (using Theorem 1.3 with $p = 3$)

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

So, we conclude that $\sqrt{3}$ is irrational.

ILLUSTRATION – 1.9

Prove that $\sqrt{5}$ is an irrational number.

SOLUTION :

Let us assume on the contrary that $\sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$\sqrt{5} = \frac{a}{b}$

$\Rightarrow$ $5b^2 = a^2$

$\Rightarrow$ $5 \mid a^2$ $[\because 5 \mid 5b^2]$

$\Rightarrow$ $5 \mid a$ [See Theorem 2 on page 1.25 ... (i)

$\Rightarrow$ $a = 5c$ for some positive integer c

$\Rightarrow$ $a^2 = 25c^2$

$\Rightarrow$ $5b^2 = 25c^2$ $[\because a^2 = 5b^2]$

$\Rightarrow$ $b^2 = 5c^2$

$\Rightarrow$ $5 \mid b^2$ $[\because 5 \mid 5c^2]$

$\Rightarrow$ $5 \mid b$ [See Theorem 2 on page 1.25 ... (ii)

From (i) and (ii), we find that a and b have at least 5 as a common factor. This contradicts the fact that a and b are co-prime.

Hence, $\sqrt{5}$ is irrational number.

DECIMAL EXPANSION OF RATIONAL NUMBERS :

(a) If a number in the form $\frac{p}{q}$ is such that p and q are co-prime and prime factorisation of q is of the form $2^n 5^m$, where n and m are non-negative integers then decimal expansion of $\frac{p}{q}$ terminates.

(b) Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form $\frac{p}{q}$ where p and q are co-prime, and the prime factorisation of q is of the form $2^n 5^m$; where n, m are non-negative integers.

(c) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$; where n, m are non-negative integers.
Then, x has a decimal expansion which is non-terminating repeating (recurring).

ILLUSTRATION 1.10

Without performing the long division, state whether the following rational numbers will have a terminating or non-terminating repeating decimal expansion :

(i) $\frac{13}{64}$ (ii) $\frac{7}{80}$ (iii) $\frac{25}{2^3 \times 5^7 \times 7}$ (iv) $\frac{75}{1230}$

SOLUTION :

(i) $\frac{13}{64}$.

Here denominator $q = 64$, Prime factors of $64 = 2^6$, which is of the form $2^n 5^m$, with $n = 6$ and $m = 0$
Therefore, decimal expansion will terminate.

(ii) $\frac{7}{80}$.

Here denominator $= 80$, Prime factors of $80 = 2^4 \times 5^1$, which is given of the form $2^n 5^m$, with $n = 4$ and $m = 1$.
Therefore, decimal expansion will terminate.

(iii) $\frac{25}{2^3 \times 5^7 \times 7} = \frac{5^2}{2^3 \times 5^7 \times 7} = \frac{1}{2^3 \times 5^5 \times 7}$

Denominator of the above rational number is not of the form $2^n 5^m$, hence the number is repeating decimal.

(iv) $\frac{75}{1230} = \frac{3 \times 5 \times 5}{2 \times 5^4} = \frac{3}{2 \times 5^2}$

Since, the prime factorisation of denominator is of form $2^n 5^m$, with $n = 1$, $m = 2$.
So, the decimal expansion will terminate.

ILLUSTRATION 1.11

Without performing the long division, find the decimal expansion of the

(i) $\frac{3}{8}$ (ii) $\frac{13}{125}$ (iii) $\frac{7}{80}$ (iv) $\frac{14588}{625}$

SOLUTION :

(i) $\frac{3}{8} = \frac{3}{2^3} = \frac{3 \times 5^3}{2^3 \times 5^3} = \frac{375}{10^3} = 0.375$

(ii) $\frac{13}{125} = \frac{13}{5^3} = \frac{13 \times 2^3}{2^3 \times 5^3} = \frac{104}{10^3} = 0.104$

(iii) $\frac{7}{80} = \frac{7}{2^4 \times 5} = \frac{7 \times 5^3}{2^4 \times 5^4} = \frac{875}{10^4} = 0.0875$

(iv) $\frac{14588}{625} = \frac{2^2 \times 7 \times 521}{5^4} = \frac{2^6 \times 7 \times 521}{2^4 \times 5^4} = \frac{233408}{10^4} = 23.3408$

ILLUSTRATION-1.12

Convert 6.235 in the form of $\frac{p}{q}$, where p and q are co-prime. Also show that q is in the form $2^n \times 5^m$; where n, m are non-negative integers.

SOLUTION:

$6.235 = \frac{6235}{1000} = \frac{1247}{200}$, which is in the form of $\frac{p}{q}$.

Also $q = 200 = 2^3 \times 5^2$, which is in the form $2^n \times 5^m$.

NOTE: We can also convert a rational number which is non-terminating reccuring (i.e., repeating in the form of $\frac{p}{q}$, where p and q are co-prime (but $q \neq 0$).

ILLUSTRATION-1.13

Convert $23.\overline{426}$ in to $\frac{p}{q}$ form, where p and q are coprime (but $q \neq 0$).

SOLUTION:

Let $x = 23.\overline{426}$

$\Rightarrow \quad x = 23.426\,426$ ...... (i)

Multiply equation (i) by 1000, so that in the right hand side of the equation (i), the point (.) comes just after the first repeating part (426).

$\therefore \quad 1000x = 23426.426\,426$ ....(ii)

Subtracting equation (i) from (ii), we get

$999x = 23403$

$\therefore \quad x = \frac{23403}{999} = \frac{7801}{333}$

$\therefore \quad 23.\overline{426} = \frac{7801}{333}$, which is in the required form of $\frac{p}{q}$.

ILLUSTRATION-1.14

Convert $23.4\overline{26}$ in to $\frac{p}{q}$ from, where p and q are co-prime (but $q \neq 0$).

SOLUTION:

Let $\quad x = 23.4\overline{26}$

$\Rightarrow \quad x = 23.42626$ .... (i)

Multiply equation (i) by 10, so that in the right hand side of the equation (i), the point (.) comes just before the first repeating part (26).

$\therefore \quad 10x = 234.26\,26$ ....(ii)

Multiply equation (ii) by 100, so that in the right hand side of the equation (ii), the point (.) comes just after the first repeating part (26).

$\therefore \quad 1000x = 23426.2626$ ....(iii)

Subtracting equation (ii) from (iii), we get

$990x = 23192$

$\therefore \quad x = \frac{23192}{990} = \frac{3866}{165}$

$\therefore \quad 23.4\overline{26} = \frac{3866}{165}$, which is in the required form of $\frac{p}{q}$.

HCF AND L.C.M. OF POLYNOMIALS :

To find H.C.F of two or more given polynomials follow the following steps :

Step (i) : Express each polynomial as a product of powers irreducible factors (simple factors). Numerical factors, if any, are expressed as product of powers of primes.

Step (ii) : If there is no common factor, then the H.C.F is 1. If there are common simple factors find the smallest exponents of these simple factors in the factorised form of the polynomials.

Step (iii) : Raise the common simple factors to the smallest exponents found in step 2 and multiply to get the H.C.F.

ILLUSTRATION–1.15

Find the H.C.F of the polynomials.
$150\,(6x^2+x-1)(x-3)^3$ and $84\,(x-3)^2\,(8x^2+14x+5)$

SOLUTION :

Let $f(x)=150\,(6x^2+x-1)(x-3)^3$
and $g(x)=84\,(x-3)^2\,(8x^2+14x+5)$
Now, $f(x)=150\,(6x^2+x-1)(x-3)^3=2\times3\times5^2\,(2x-1)(3x-1)(x-3)^3$
$g(x)=84\,(x-3)^2\,(8x^2+14x+5)=2^2\times3\times7\,(x-3)^2\,(2x+1)(4x+5)$
Hence, required H.C.F $=2^1.\,3^1\,(2x+1)^1.\,(x-3)^2=6\,(2x+1)(x-3)^2$

Common simple factor	Least exponent
2	1
3	1
$2x+1$	1
$x-3$	2

L.C.M. OF POLYNOMIALS :

To find L.C.M. of polynomial follow the following steps :

Step (i) : Express each polynomial as a product of powers of irreducible factors. Express numerical factors, if any, as product of powers of primes.

Step (ii) : Consider all the irreducible factors occurring in the given polynomials each one once only. Find the greatest exponent of each of these simple factors in the factorised form of the given polynomials.

Step (iii) : Raise each irreducible factor to the greatest exponent found in step 2, and multiply to get the LCM.

ILLUSTRATION–1.16

Find the L.C.M of the polynomials
$90\,(x^2-5x+6)(2x+1)^2$ and $140\,(x-3)^3\,(2x^2+15x+7)$

SOLUTION :

Let $f(x)=90\,(x^2-5x+6)(2x+1)^2$
and $g(x)=140\,(x-3)^3\,(2x^2+15x+7)$
Then $f(x)=2\times3^2\times5\,(x-2)(x-3)(2x+1)^2$
and $g(x)=2^2\times5\times7\,(x-3)^3\,(2x+1)\,(x+7)$

Irreducible factor	Greatest exponent
2	2
3	2
5	1
7	1
$x-2$	1
$x-3$	3
$2x+1$	2
$x+7$	1

L.C.M $=2^2.\,3^2.\,5^1.\,7^1.\,(x-2)^1.\,(x-3)^3.\,(2x+1)^2.\,(x+7)^1$
L.C.M $=1260\,(x-2)(x-3)^3\,(2x+1)^2\,(x+7)$

MISCELLANEOUS SOLVED EXAMPLES

1. Find 4 rational numbers between 1/5 and 1/6.

Sol. One rational number between 1/5 and 1/6 is $\frac{1}{2}\left(\frac{1}{5}+\frac{1}{6}\right)=\frac{11}{60}$

$\therefore \frac{1}{6}<\frac{11}{60}<\frac{1}{5}$.

Now, a rational number between 1/6 and 11/60 is $\frac{1}{2}\left(\frac{1}{6}+\frac{11}{60}\right)=\frac{1}{2}\left(\frac{10+11}{60}\right)=\frac{21}{120}$

$\frac{1}{6}<\frac{21}{120}<\frac{11}{60}<\frac{1}{5}$

Now, a rational number between $\frac{11}{60}$ and $\frac{1}{5}$ is $\frac{1}{2}\left(\frac{11}{60}+\frac{1}{5}\right)=\frac{1}{2}\times\frac{11+12}{60}=\frac{23}{120}$

$\therefore \frac{1}{6}<\frac{21}{120}<\frac{11}{60}<\frac{23}{120}<\frac{1}{5}$

Again, one rational numbers between $\frac{23}{120}$ and $\frac{1}{5}$ is $\frac{1}{2}\left(\frac{23}{120}+\frac{1}{5}\right)=\frac{1}{2}\times\frac{23+24}{120}=\frac{47}{240}$

$\therefore \frac{1}{6}<\frac{21}{120}<\frac{11}{60}<\frac{23}{120}<\frac{47}{240}<\frac{1}{5}$

Hence, four rational numbers between $\frac{1}{6}$ and $\frac{1}{5}$ are $\frac{21}{120},\frac{11}{60},\frac{23}{120},\frac{47}{240}$.

2. Find the L.C.M. of the polynomials $12(x^4-x^3)$ and $8(x^4-3x^3+2x^2)$ is

Sol. Let the given polynomials be denoted as $p(x)$ and $q(x)$ respectively

We have: $p(x)=2^2\times 3\times x^3\times(x-1)$

$q(x)=2^3\times x^3\times(x-1)\times(x-2)$

Irreducible factors are $2, 3, x, x-1$ and $x-2$.

The respective highest exponents are 3, 1, 3, 1 and 1.

$\therefore$ L.C.M. $=2^3\times 3\times x^3\times(x-1)\times(x-2)=24x^3(x-1)(x-2)$.

3. If $(x+m)$ is the H.C.F of the functions x^2+ax+b and x^2+cx+d, then show that their L.C.M. is $x^3+(a+c-m)x^2+(ac-m^2)x+m(a-m)(c-m)$.

Sol. Let $x^2+ax+b=(x+m)(x+p)$

$x^2+cx+d=(x+m)(x+q)$ where $(x+p$ and $(x+q)$ are prime to each other.

$x^2+ax+b=(x+m)(x+p)$

$=x^2+x(m+p)+mp$ ....(i)

$x^2+cx+d=(x+m)(x+q)=x^2+x(m+q)+mp$ ....(ii)

Equating coefficients of x and constant terms in the two equations

$\therefore\quad a=m+p;\ mp=b$ ....(i)

$c=m+q;\ mq=d$ ....(ii)

From (i) $p=a-m$

From (ii) $q=c-m$

The L.C.M. of expression i and ii is $(x+m)(x+p)(x+q)$

$$= (x+m)(x+a-m)(x+c-m)$$
$$= x^3 + x^2[m+a-m+c-m] + x[m(a-m)+m(c-m)+(a-m)(c-m)] + m(a-m)(c-m)$$
$$= x^3 + x^2(a+c-m)) + x(am-m^2+cm-m^2+ac-cm-am+m^2) + m(a-m)(c-m)$$
$$= x^3 + (a+c-m)x^2 + (ac-m^2)x + m(a-m)(c-m).$$

4. If a number 774958A96B is to be divisible by 8 and 9, then find the values of A and B.

Sol. According to the question, the number is divisible by 8 and 9. For the number to be divisible by 8, its last three digits have to be divisible by 8.

This 96$\underline{0}$ and 96$\underline{8}$ can be the possibilities. For the number to be divisible by 9, the sum of the digits of the number should be divisible by 9.

Hence, it can be possible if $B=8$ and $A=9$ and if $B=0$ and $A=8$.

Hence, (8, 0) is the possible values of A and B.

5. If n is an integer, how many values of n will give an integral value of $(16n^2+7n+6)/n$?

Sol. $\dfrac{16n^2+7n+6}{n}$; (n is an integer)

$$= \underbrace{16n+7}_{\text{Integer}} + \frac{6}{n}$$

Hence, to become the entire expression an integer $\left(\dfrac{6}{n}\right)$ should be an integer and $\left(\dfrac{6}{n}\right)$ can be an integer

for $n=1, n=2, n=3$ and $n=6$

Hence, n will have only four values.

6. Three wheels can complete respectively 60, 36, 24 revolutions per minute. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?

Sol. 1st wheel makes 1 revolutions per sec

2nd wheel makes $\dfrac{6}{10}$ revolutions per sec

3rd wheel makes $\dfrac{4}{10}$ revolutions per sec

In other words 1st, 2nd and 3rd wheel take 1, $\dfrac{5}{3}$ and $\dfrac{5}{2}$ seconds respectively to complete one revolution.

L.C.M of 1, $\dfrac{5}{3}$ and $\dfrac{5}{2} = \dfrac{\text{L.C.M. of } 1,5,5}{\text{H.C.F. of } 1,3,2} = 5$

Hence, after every 5 seconds the red spots on all the three wheels touch the ground.

7. Let D be a recurring decimal of the form $D = 0.a_1\,a_2\,a_1a_2a_1a_2$............where a_1 and a_2 lie between 0 and 9. Further at most one of them is zero. Which of the following numbers necessarily produces an integer when multiplied by D?

(a) 18 **(b)** 198 **(c)** 100 **(d)** 288

Sol. (b) $D = 0.\overline{a_1a_2}$

Multiplied by 100 on both side

$100D = a_1a_2.\overline{a_1a_2}$

$100D = a_1a_2.D$

$\therefore \quad 99D = a_1a_2 \Rightarrow D = \dfrac{a_1a_2}{99}$

Required number should be the multiple of 99. So we can get an integer when multiplied by D.

Hence, 198 is the required number.

8. If p be a number between 0 and 1, which one of the following will be true ?

(a) $p > \sqrt{p}$ (b) $\frac{1}{p^2} > \sqrt{p}$ (c) $p < \frac{1}{p}$ (d) $p^3 > p^2$

Sol. (b,c) Here, $0 < p < 1$, so let $p = \frac{1}{2}$

Clearly, $p < \frac{1}{p}$ $\left(\because \frac{1}{2} < \frac{1}{1/2} \text{ or } \frac{1}{2} < 2\right)$

Also, $\frac{1}{p^2} > \sqrt{p}$

$\because \frac{1}{\left(\frac{1}{2}\right)^2} > \sqrt{\frac{1}{2}}$ or $4 > 0.707$

9. If p is a prime number greater than 3, then find the number by which $(p^2 - 1)$ is always divisible :

Sol. Let $p = 5, 7, 11, 13,$ ....

For $p = 5$, $(p^2 - 1) = 24$

For $p = 7$, $(p^2 - 1) = 48$

For $p = 11$, $(p^2 - 1) = 120$

For $p = 13$, $(p^2 - 1) = 168$

....

....

....

Clearly, all the above numbers are divisible by 24.

10. What is the value of M and N if M39048458N is divisible by 8 and 11, where M and N are single digit integers?

Sol. A number is divisible by 8 if the number formed by the last three digits is divisible by 8.
i.e., $58N$ is divisible by 8.
Clearly, $N = 4$
Again, a number is divisible by 11 if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or is divisible by 11.

i.e., $(M + 9 + 4 + 4 + 8) - (3 + 0 + 8 + 5 + N) = M + 25 - (16 + N)$

$= M - N + 9$ must be zero or it must be divisible by 11

i.e. $M - N = 2 \Rightarrow M = 2 + 4 = 6$

Hence, $M = 6$, $N = 4$

11. Show that one and only one out of $n, n+2$ or, $n+4$ is divisible by 3, where n is any positive integer.

Sol. We know that any positive integer is of the form $3q$ or, $3q + 1$ or, $3q + 2$ for some non-negative integer q.
So, we have following cases :
Case I : When $n = 3q$
In this case, we have
$n = 3q$, which is divisible by 3
Now, $n = 3q \Rightarrow n + 2 = 3q + 2 \Rightarrow n + 2$ leaves remainder 2 when divided by 3
$\Rightarrow n + 2$ is not divisible by 3.

Again, $n = 3q \Rightarrow n + 4 = 3q + 4 = 3(q + 1) + 1 \Rightarrow n + 4$ leaves remainder 1, when divided by 3

$\Rightarrow n + 4$ is not divisible by 3.

Thus, n is divisible by 3 but $n + 2$ and $n + 4$ are not divisible by 3.

Case II : When $n = 3q + 1$

In this case, we have $n = 3q + 1$

$\Rightarrow n$ leaves remainder 1 when divided by $3 \Rightarrow n$ is not divisible by 3.

Now, $n = 3q + 1$

$\Rightarrow n + 2 = (3q + 1) + 2 = 3(q + 1) \Rightarrow n + 2$ is divisible by 3.

Again, $n = 3q + 1$

$\Rightarrow n + 4 = 3q + 1 + 4 = 3q + 5 = 3(q + 1) + 2$

$\Rightarrow n + 4$ leaves remainder 2 when divided by $3 \Rightarrow n + 4$ is not divisible by 3.

Thus, $n + 2$ is divisible by 3 but n and $n + 4$ are not divisible by 3.

Case III : When $n = 3q + 2$

In this case, we have $n = 3q + 2$

$\Rightarrow n$ leaves remainder 2 when divided by $3 \Rightarrow n$ is not divisible by 3.

Now, $n = 3q + 2$

$\Rightarrow n + 2 = 3q + 2 + 2 = 3(q + 1) + 1$

$\Rightarrow n + 2$ leaves remainder 1 when divided by $3 \Rightarrow n + 2$ is not divisible by 3.

Again, $n = 3q + 2$

$n + 4 = 3q + 2 + 4 = 3(q + 2) \Rightarrow n + 4$ is divisible by 3.

Thus, $n + 4$ is divisible by 3 but n and $n + 2$ are not divisible by 3.

12. When 2^{256} is divided by 17 then find the remainder.

Sol. When 2^{256} is divided by 17 then,

$$\Rightarrow \frac{2^{256}}{2^4 + 1} \Rightarrow \frac{(2^4)^{64}}{(2^4 + 1)}$$

By remainder theorem when $f(x)$ is divided by $x + a$ the remainder $= f(-a)$

Here $f(x) = (2^4)^{64}$ and $x = 2^4$ and $a = 1$

$\therefore$ Remainder $= f(-1) = (-1)^{64} = 1$

13. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Sol. Clearly, the required number is the H.C.F of the numbers

$398 - 7 = 391$; $436 - 11 = 425$, and $542 - 15 = 527$.

First we find the H.C.F of 391 and 425 by Euclid's algorithm as given below :

$425 = 391 \times 1 + 34$

$391 = 34 \times 11 + 17$

$34 = 17 \times 2 + 0$

Clearly, H.C.F of 391 and 425 is 17.

Let us now the H.C.F of 17 and the third number 527 by Euclid's algorithm:

$527 = 17 \times 31 + 0$

The H.C.F of 17 and 527 is 17. Hence, H.C.F of 391, 4250 and 527 is 17.

Hence, the required number is 17.

14. Two bills of ₹ 6075 and ₹ 8505 respectively are to be paid separately by cheques of same amount. Find the largest possible amount of each cheque.

Sol. Largest possible amount of cheque will be the HCF (6075, 8505).

Applying Euclid's division lemma to 8505 and 6075, we have,

$8505 = 6075 \times 1 + 2430$

Since, remainder $2430 \neq 0$ again applying division lemma to 6075 and 2430

$6075 = 2430 \times 2 + 1215$

Again remainder $1215 \neq 0$
So, again applying the division lemma to 2430 and 1215
2430 = 1215 ñ 2 + 0
Here the remainder is zero
So, H.C.F = 1215
Therefore, the largest possible amount of each cheque will be 1215.

15. Prove that if x and y are odd positive integers, then x^2+y^2 is even but not divisible by 4.

Sol. We know that any odd positive integer is of the form $2q+1$ for some integer q.
So, let $x = 2m+1$ for some integers m and n.
\ $x^2+y^2=(2m+1)^2+(2n+1)^2$
$\Rightarrow$ $x^2+y^2=4(m^2+n^2)+4(m+n)+2$
$\Rightarrow$ $x^2+y^2=4q+2$, where $q=(m^2+n^2)+(m+n)$
$\Rightarrow$ x^2+y^2 is even and leaves remainder 2 when divided by 4
$\Rightarrow$ x^2+y^2 is even but not divisible by 4

16. Write the decimal expansion using prime factorisation :

(i) $\frac{35}{16}$ **(ii)** $\frac{17}{8}$ **(iii)** $\frac{327}{500}$

Sol. (i) $\frac{35}{16}=\frac{35\times5^4}{2\times5^4}=\frac{35\times625}{(10)^4}=\frac{21875}{10000}=2.1875$

(ii) $\frac{17}{8}=\frac{17\times5^3}{2^3\times5^3}=\frac{17\times125}{(10)^3}=\frac{2125}{1000}=2.125$

(iii) $\frac{327}{500}=\frac{327}{5\times5\times5\times2\times2}=\frac{327}{5^3\times2^2}=\frac{327\times2}{5^3\times2^3}=\frac{654}{(10)^3}=0.654$

17. If d is the H.C.F of 56 and 72, find x, y satisfying $d = 56x + 72y$. Also, show that x and y are not unique.

Sol. Applying Euclid's division lemma to 56 and 72, we get

72 = 56 ñ 1 + 16 .....(i) $\left[\because 56\overline{)72}(1 \quad \frac{56}{16}\right]$

Since the remainder $16 \neq 0$. So, we consider the divisor 56 and the remainder 16 and apply division lemma to get

56 = 16 ñ 3 + 8 .....(ii) $\left[\because 16\overline{)56}(3 \quad \frac{48}{8}\right]$

We consider the divisor 16 and the remainder 8 and apply division algorithm to get

16 = 8 ñ 2 + 0 ....(iii) $\left[\because 8\overline{)16}(2 \quad \frac{16}{0}\right]$

We observe that the remainder at this stage is zero. Therefore, last divisor 8 (or the remainder at the earlier stage) is the H.C.F of 56 and 72.
From (ii), we get
8 = 56 $ 16 ñ 3
$\Rightarrow$ 8 = 56 $ (72 $ 56 ñ 1) ñ 3 [$\therefore$ 16 = 72 $ 56 ñ 1 (from (i))]
$\Rightarrow$ 8 = 56 $ 3 ñ 72 + 56 ñ 3
$\Rightarrow$ 8 = 56 ñ 4 + ($3) ñ 72
$\therefore$ x = 4 and y = $3.

Now, $8 = 56 \times 4 + (-3) \times 72$

$8 = 56 \times 4 + (-3) \times 72 - 56 \times 72 + 56 \times 72$

$\Rightarrow \quad 8 = 56 \times 4 - 56 \times 72 + (-3) \times 72 + 56 \times 72$

$\Rightarrow \quad 8 = 56 \times (4 - 72) \{(-3) + 56\} \times 72$

$\Rightarrow \quad 8 = 56 \times (-68) + (53) \times 72$

$\therefore \quad x = -68$ and $y = 53$.

Hence, x and y are not unique.

18. Show that any positive odd integer is of the form $8q+1, 8q+3, 8q+5, 8q+7$, where q is some integer.

Sol. Let a and $b = 8$ be two positive integers where a is odd.

Applying division lemma $a = 8q + \text{r}$ where $0 \le r < 8$

So, r can take any of the values 0, 1, 2, 3, 4, 5, 6, 7

Therefore, $a = 8q, 8q+1, 8q+2, 8q+3, 8q+4, 8q+5, 8q+6, 8q+7, 8q+8$

Since, a is odd.

Therefore, a cannot take values $8q, 8q+2, 8q+4, 8q+8$ since they can expressed as multiples of 2.

So, a will take values $8q+1, 8q+3, 8q+5, 8q+7$.

Also, $8q+5 = 8q+8-3 = 8(q+1)-3 = 8q'-3$

where $q' = q + 8q + 7 = 8q + 8 - 3 = 8q' - 1$

So, every positive odd integer is of the form $8q \pm 1, 8q \pm 3$.

19. If $a^2 - b^2$ is a prime number, show that $a^2 - b^2 = a + b$, where a, b are natural number.

Sol. $a^2 - b^2 = (a-b)(a+b)$ .....(1)

$\because \quad a^2 - b^2$ is a prime number

$\because \quad$ one of the two factors = 1

$\because \quad a - b = 1$ $[\because a - b < a + b]$

$\because \quad$ The only divisor of a prime number are 1 and itself.

(1) become $a^2 - b^2 = 1(a+b)$

or $\quad a^2 - b^2 = a + b$

e.g., $3^2 - 2^2 = 5$ (which is prime)

$\Rightarrow \quad 3^2 - 2^2 = 3 + 2, 3, 2 \in \text{N}$

20. For any positive real number x, prove that there exists an irrational number y such that $0 < y < x$.

Sol. If x is irrational, then $y = \frac{x}{2}$ is also an irrational number such that $0 < y < x$.

If x is rational, then $\frac{x}{\sqrt{2}}$ is an irrational number such that $\frac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

$\because \quad y = \frac{x}{2}$ is an irrational number such that $0 < y < x$.

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. $\sqrt{5}$ is a/ an .... number
2. $\frac{1}{\sqrt{2}}$ is a/ an .... number
3. $3+2\sqrt{5}$ a/ an .... number
4. $7\sqrt{5}$ is a/ an .... number
5. $6+\sqrt{2}$ is a/ an .... number
6. An .... is a series of well defined steps which gives a procedure for solving a type of problem.
7. A .... is a proven statement used for proving another statement.
8. L.C.M of 96 and 404 is ....
9. H.C.F of 96, 72 and 120 is ....
10. 156 as a product of its prime factors ....
11. L.C.M of 26 and 91 is ....
12. H.C.F of 26 and 91 is ....
13. $\frac{35}{50}$ is a .... decimal expansion.

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r$, $0 \le r < b$.
2. Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
3. $\sqrt{2}, \sqrt{3}$ are irrationals.
4. If let $x = p/q$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
5. If $x = p/q$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is terminating.
6. Any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.
7. cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.
8. The quotient of two integers is always a rational number
9. 1/0 is not rational.
10. After rationalising the denominator of $\frac{5}{3\sqrt{2}-2\sqrt{3}}$, we get denominator as 7.
11. The number of irrational numbers between 15 and 18 is infinite.
12. An irrational number between $7.40\overline{312}$ and $7.4\overline{04}$ is $7.403\overline{45}$.
13. π is an irrational and $\frac{22}{7}$ is a rational.
14. Every fraction is a rational number.

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Match the following columns :

Column I	Column II
(A) Irrational number is always	(p) rational number
(B) Rational number is always	(q) irrational number
(C) $\sqrt[3]{6}$ is not a	(r) Non-terminating non-repeating
(D) $2+\sqrt{2}$ is an	(s) terminating decimal

2. Match the column

Column I	Column II
(A) H.C.F of the smallest composite number and the smallest prime number	(p) 6
(B) H.C.F of 32 and 54 is	(q) 5
(C) H.C.F of 336 and 54	(r) 2
(D) H.C.F of 475 and 495	(s) 2

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Let x be a real variable, and let $3 < x < 4$. Name five values that x might have.
2. What is a real number ?
3. What are the two main categories of real numbers?
4. What is a real variable?
5. Which numbers have rational square roots?
6. A rational number can always be written in what form?
7. Which of the following numbers are rational ?

   $1, \ -6, \ 3\frac{1}{2}, \ -\frac{2}{3}, \ 0, \ 5.8,$
   3.14159265358979323846264 33
8. What are the rational numbers?
9. Prove that $\sqrt{3}+\sqrt{2}$ is irrational.
10. Given that H.C.F of $(306, 657) = 9$, find the L.C.M. of $(306, 657)$.
11. Find the H.C.F of 12576 and 4052 by using the fundamental theorem of Arithmetic.
12. Find the largest which divides 245 and 1029 leaving remainder 5 in each case.
13. The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.
14. A certain type of wooden board is sold only in lengths of multiples of 25 cm from 2 to 10 metres. A carpenter needs a large quantity of this type of boards in 1.65 meter length. For the minimum waste, find the lengths to be purchased.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the H.C.F of 96 and 404 by the prime factorisation method. Hence, find their L.C.M.
2. If the sum of two numbers is 1215 and their HCF is 81, find the number of such pairs.
3. Find the H.C.F of 300, 540, 890 by applying Euclid's alogrithm.
4. Find the L.C.M and H.C.F of 336 and 54 by the prime factorization method.
5. Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer.
6. A, B and C starts cycling around a circular path in the same direction at same time. Circumference of the path is 1980 m. If the speed of A is 330 m/min, speed of B is 198 m/min and C is 220 m/min and they start from the same point, then after what time interval they will be together at the starting point?
7. Prove that $3+2\sqrt{5}$ is irrational.
8. Show that $5\sqrt{3}$ is an irrational number.
9. If n is any odd number greater than 1, then find $n(n^2-1)$.
10. $(BE)^2 = MPB$, where B, E, M and P are distinct integers, then find M.
11. A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.
12. A is the set of positive integers such that when divided by 2, 3, 4, 5 and 6 leaves the remainders 1, 2, 3, 4 and 5 respectively. How many integer(s) between 0 and 100 belongs to set A?
13. P is the product of all the prime numbers between 1 to 100. Then find the number of zeroes at the end of P.
14. x_n is either -1 or 1 and $n \geq 4$; If

    $$x_1x_2x_3x_4 + x_2x_3x_4x_5 + x_3x_4x_5x_6 + \ldots.. x_nx_1x_2x_3 = 0$$

    then find the value of n.
15. There are two integers 34041 and 32506, when divided by a three-digit integer n, leave the same remainder. What is the value of n?
16. At a book store, "MODERN BOOK STORE" is flashed using neon lights. The words are individually flashed at intervals of $2\frac{1}{2}, 4\frac{1}{4}, 5\frac{1}{8}$ seconds respectively, and each word is put off after a second. Find the least time after which the full name of the bookstore can be read again?
17. What is the remainder when 4^{96} is divided by 6?
18. Four bells begin to toll together and toll respectively at intervals of 6, 5, 7, 10 and 12 seconds. How many times they will toll together in one hour excluding the one at the start ?
19. H.C.F of 3240, 3600 and a third number is 36 and their L.C.M is $2^4 \times 3^5 \times 5^2 \times 7^2$. Find the third number.
20. The numbers 1 to 29 are written side by side as follows 1234567891011........................... 28 29.

    If the number is divided by 9, then what is the remainder ?

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Use Euclid's algorithm to find the H.C.F of 4052 and 12576.
2. Show that $5-\sqrt{3}$ is irrational.
3. A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of barfis that can be placed in each stack for this purpose?
4. Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.
5. Show that $3\sqrt{2}$ is irrational.
6. Show that n^2-1 is divisible by 8, if n is an odd positive integer.
7. If the H.C.F of 210 and 55 is expressible in the form $210 \times 5 + 55y$, find y.
8. Find the H.C.F of 81 and 237 and express it as a linear combination of 81 and 237.
9. Prove that $\sqrt{2}+\sqrt{5}$ is irrational.
10. Find the unit's digit in the product $7^{35} \times 3^{71} \times 11^{55}$.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The nearest integer to 58701 which is divisible by 567 is
   (a) 58968 (b) 58434
   (c) 58401 (d) None
2. The least perfect square number which is divisible by 8, 15, 20, 22 is
   (a) 435600 (b) 43560
   (c) 39600 (d) None
3. When 2^{256} is divided by 17 the remainder would be
   (a) 1 (b) 16
   (c) 14 (d) None of these
4. The sum of three non-zero prime numbers is 100. One of them exceeds the other by 36. Then the largest number is
   (a) 73 (b) 91
   (c) 67 (d) 57
5. If N is the sum of first 13,986 prime numbers, then N is always divisible by
   (a) 6 (b) 4
   (c) 8 (d) None of these
6. H.C.F. of (x^3-3x+2) and (x^2-4x+3) is
   (a) $(x-1)$ (b) $(x-2)^2$
   (c) $(x-1)(x+2)$ (d) $(x-1)(x-3)$
7. If two numbers when divided by a certain divisor give remainder 35 and 30 respectively and when their sum is divided by the same divisor, the remainder is 20, then the divisor is
   (a) 40 (b) 45
   (c) 50 (d) 55
8. In order that the number $1y3y6$ be divisible by 11, the digit y should be
   (a) 1 (b) 2
   (c) 5 (d) 6
9. The rational number of the form $\frac{q}{q}$, $q \neq 0$, p and q are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434....) is
   (a) $\frac{134}{999}$ (b) $\frac{134}{990}$
   (c) $\frac{133}{999}$ (d) $\frac{133}{990}$
10. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is
    (a) 240 (b) 1600
    (c) 2400 (d) 3600
11. Find the least number which when divided by 12, leaves a remainder of 7, when divided by 15, leaves a remainder of 10 and when divided by 16, leaves a remainder of 11
    (a) 115 (b) 235
    (c) 247 (d) 475
12. If n is an even natural number, then the largest natural number by which $n(n+1)(n+2)$ is divisible is
    (a) 6 (b) 8
    (c) 12 (d) 24
13. Find the least number which when divided by 15, leaves a remainder of 5, when divided by 25, leaves a remainder of 15 and when divided by 35 leaves a remainder of 25
    (a) 515 (b) 525
    (c) 1040 (d) 1050
14. If $(-1)^n+(-1)^{4n}=0$, then n is
    (a) any positive integer
    (b) any negative integer
    (c) any odd natural number
    (d) any even natural number

15. The number $3^{13} - 3^{10}$ is divisible by
(a) 2 and 3 (b) 3 and 10
(c) 2, 3 and 10 (d) 2, 3 and 13

16. A number lies between 300 and 400. If the number is added to the number formed by reversing the digits, the sum is 888 and if the unit's digit and the ten's digit change places, the new number exceeds the original number by 9. Then the number is
(a) 339 (b) 341
(c) 378 (d) 345

17. Number that has to be added to 345670 in order to make it divisible by 6 is
(a) 2 (b) 4
(c) 5 (d) 6

18. Which of the following will have a terminating decimal expansion
(a) $\frac{77}{210}$ (b) $\frac{23}{30}$
(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

19. What is the number x
I. The L.C.M of x and 18 is 36.
II. The H.C.F of x and 18 is 2.
(a) 1 (b) 2
(c) 3 (d) 4

20. The greatest number of five digits exactly divisible 279 is
(a) 99603 (b) 99837
(c) 99882 (d) None

21. The greatest number which can divide 1854, 1866 and 2066 leaving the same remainder 2 in each case is
(a) 4 (b) 6
(c) 12 (d) None

MTOC More than One Correct :

DIRECTIONS : *This section contains 8 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Product of two co-prime numbers is 117.
Their LCM should be
(a) 1 (b) 117
(c) equal to their HCF (d) Lies between 115 to 120

2. Which of the following is always false?
(a) The sum of two distinct irrational numbers is rational
(b) the rationalising factor of a number is unique
(c) Every irrational number is a surd
(d) Any surd of the form $\sqrt[n]{a} + \sqrt[n]{b}$ can be rationalised by a surd of the form $\sqrt[n]{a} - \sqrt[n]{b}$, where $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are surds.

3. The possible numbers that can be formed by using unit digit and ten's digit in $(274 \times 243 \times 131)$ lies between
(a) 23 and 45 (b) 23 and 44
(c) 26 and 46 (d) 25 and 50

4. Which of the following is/are correct?
(a) $\frac{7^3}{5^4}$ is a non terminating repeating decimal.
(b) If $a = 2 + \sqrt{3}$ and $b = \sqrt{2} - \sqrt{3}$, then $a + b$ is irrational
(c) If 19 divides a^3, then 19 divides a, where a is a positive integer
(d) Product of L.C.M and H.C.F of 25 and 625 is 15625.

5. The product of unit digit in $(7^{95} - 3^{58})$ and $(7^{95} + 3^{58})$ is
(a) cube of 2 (b) lies between 6 and 10
(c) 6 (d) lies between 3 and 6

6. Which of the following is/are correct?
(a) Every integer is a rational number.
(b) The sum of a rational number and an irrational number is an irrational number
(c) Every real number is rational
(d) Every point on a number line is associated with a real number

7. What should be the maximum value of Q in the following equation?
$4P8 + 8Q3 + 7R8 = 2079$
(a) lies between $0 \le Q \le 9$
(b) More than or equal to 7
(c) Less than 6
(d) $0 \le Q \le 11$

8. Which of the following pairs of fraction adds up to a number less than 5?
(a) $\frac{5}{3}, \frac{3}{4}$ (b) $\frac{7}{3}, \frac{11}{5}$
(c) $\frac{11}{4}, \frac{8}{3}$ (d) $\frac{13}{5}, \frac{11}{6}$

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE - I

If p is prime, then $\sqrt{p}$ is irrational and if a, b are two odd prime numbers, then $a^2 - b^2$ is composite. Read the above PASSAGE and mark the correct answer to the following questions.

1. $\sqrt{7}$ is
(a) a rational number (b) an irrational number
(c) not a real number (d) terminating decimal

2. $119^2 - 111^2$ is
(a) prime number
(b) composite
(c) an odd prime number
(d) an odd composite number

PASSAGE - II

LCM of several fractions = $\dfrac{\text{LCM of their numerators}}{\text{HCF of their denominators}}$

HCF of several fractionl = $\dfrac{\text{HCF of their numerators}}{\text{LCM of their denominotors}}$

1. The L.C.M. of the fractions $\frac{5}{16}, \frac{15}{24}$ and $\frac{25}{8}$ is

(a) $\frac{5}{48}$ (b) $\frac{5}{8}$ (c) $\frac{75}{48}$ (d) $\frac{75}{8}$

2. The H.C.F. of $\frac{2}{5}, \frac{6}{25}$, and $\frac{8}{35}$ is

(a) $\frac{2}{5}$ (b) $\frac{24}{5}$ (c) $\frac{2}{175}$ (d) $\frac{24}{175}$

3. The H.C.F. of the fractions $\frac{8}{21}, \frac{12}{35}$, and $\frac{32}{7}$ is

(a) $\frac{4}{105}$ (b) $\frac{192}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{109}$

A&R Assertion & Reason

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect.**

(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** $\frac{13}{3125}$ is a terminating decimal fraction.
   Reason : If $q = 2^n.5^m$ where n, m are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.
2. **Assertion :** Denominator of 34.12345 is of the form $2^m \times 5^n$, where m, n are non-negative integers.
   Reason : 34.12345 is a terminating decimal fraction.
3. **Assertion :** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M = 162.
   Reason : If a, b are two positive integers, then H.C.F × L.C.M. = $a \times b$.
4. **Assertion :** 2 is a rational number.
   Reason : The square roots of all positive integers are irrationals.
5. **Assertion :** If L.C.M. $\{p, q\} = 30$ and H.C.M $\{p, q\} = 5$, then $p.q = 150$.
   Reason : L.C.M. of $a, b \times$ H.C.F of $a, b = a.b$.
6. **Assertion :** $n^2 - n$ is divisible by 2 for every positive integer.
   Reason : $\sqrt{2}$ is not a rational number.
7. **Assertion :** $n^2 + n$ is divisible by 2 for every positive integer n.
   Reason : If x and y are odd positive integers, from $x^2 + y^2$ is divisible by 4.

MMQ Muliple Matching Questions

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column-I	Column-II
(A) $\dfrac{551}{2^3 \times 5^6 \times 7^9}$	(p) a prime number
(B) Product of $(\sqrt{5} - \sqrt{3})$ and $(\sqrt{5} + \sqrt{3})$ is	(q) is an irrational number
(C) $\sqrt{5} - 4$	(r) is a terminating decimal representation
(D) $\dfrac{422}{2^3 \times 5^4}$	(s) a rational number
	(t) is a non-terminating but repeating decimal representation
	(u) is non-terminating and non recurring decimal representation

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Show that there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.
2. If d is the HCF of 56 and 72, find x, y satisfying $d = 56x + 72y$. Also, show that x and y are not unique.
3. Show that one and only one out of $n, n+2$ or, $n+4$ is divisible by 3, where n is any positive integer.
4. If n is a positive integer, let $S(n)$ denote the sum of the positive divisors of n, including n, $G(n)$ is the greatest divisor of n. If $H(n)=\frac{G(n)}{S(n)}$ then which of the following is the largest $H(2010)$ or $H(2011)$?
5. If P is a Prime number, show that $\sqrt{P}$ is not a rational number.
6. Show that any positive odd integer is of the form $6q+1$ or $6q+5$, where q is some integer.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1.	irrational	2.	irrational	3.	rational
4.	irrational	5.	irrational	6.	algorithm
7.	lemma	8.	9696	9.	6
10.	$2^2 \times 3 \times 13$	11.	182	12.	13
13.	terminating				

TRUE / FALSE :

1.	True	2.	True	3.	True	4.	True
5.	False	6.	True	7.	True	8.	False
9.	True	10.	False	11.	True	12.	False
13.	True	14.	True				

MATCH THE FOLLOWING :

1. (A) $\to$ (r) [$\because$ $12 = 3 \times 4$ $\because$ it is a composite number
   (B) $\to$ (s) [$\because$ g.c.d. between 2 and 7 = 1]
   (C) $\to$ (p) [$\because$ 2 is a prime number]
   (D) $\to$ (q) [$\because$ $\sqrt{2}$ is not a rational number]
2. (A) $\to$ (r, s); (B) $\to$ (r, s); (C) $\to$ (p); (D) $\to$ (q)

VERY SHORT ANSWER QUESTIONS :

1. $3.1, 3.2, \sqrt{10}, \sqrt{11}, \pi$
2. Any number that you would expect to find on the number line. It is a number required to label any point on the number line. It is a number whose absolute value names the distance of any point from 0
3. Rational and irrational
4. A variable whose values are real numbers.
5. Only the square roots of the perfect square numbers.
6. As a fraction $\frac{a}{b}$, where a and b are integers ($b \neq 0$).
7. All of them. All decimals are rational. That long one is an approximation to π.
8. They are the numbers of arithmetic: The whole numbers, fractions, mixed numbers, and decimals; together with their negative images.
9. Let $\sqrt{3}+\sqrt{2} = r$ be a rational number.
   $\Rightarrow 3+2+2\sqrt{6}=r^2 \Rightarrow 2\sqrt{6}=r^2-5$
   $\Rightarrow \sqrt{6}=\frac{r^2-5}{2}$
   As R.H.S is rational, $\sqrt{6}$ should be rational which is incorrect.
10. 22338
11. H.C.F $= 2^2 = 4$, L.C.M $= (12576, 4052) = 1,27,39,488$
12. 240, 1024 13. 75 cm.
14. 1.65 m = 165 cm
    Required length = L.C.M of 25 and 165 = 825 cm = 8.25 m

SHORT ANSWER QUESTIONS :

1. The prime factorisation of 96 and 404 gives:
   $96 = 2^5 \times 3, 404 = 2^2 \times 101$
   Therefore, the H.C.F of these two integers is $2^2 = 4$.
   Also, L.C.M (96, 404)
   $$=\frac{96\times 404}{\text{H.C.F}(96,404)}=\frac{96\times 404}{4}=9696$$
2. 1134, 81, 1053, 162, 891, 324, 648, 567
3. H.C.F of (300, 540, 890) = 0
4. We have, $336 = 2^4 \times 3^1 \times 7^1$
   $54 = 2^1 \times 3^3$
   $\therefore$ H.C.F of $(336, 54) = 2^1 \times 3^1 = 6$
   L.C.M. of $(336, 54) = 2^4 \times 3^3 \times 7 = 3024$

5. Let a be any positive integer and $b = 2$. Then, by Euclid's algorithm, $a = 2q + r$, for some integer $q \geq 0$, and $r = 0$ or $r = 1$, because $0 \leq r < 2$. So, $a = 2q$ or $2q + 1$.

If a is of the form $2q$, then a is an even integer. Also, a positive integer can be either even or odd. Therefore, any positive odd integer is of the form $2q + 1$.

6. They will meet after 90 min.

7. Let us assume on the contrary that $3+2\sqrt{5}$ is rational. Then there exist co-prime positive integers a and b such that

$$3+2\sqrt{5}=\frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{5}=\frac{a}{b}-3$$

$$\Rightarrow \quad \sqrt{5}=\frac{a-3b}{2b}$$

$\Rightarrow \quad \sqrt{5}$ is rational

$$\left[\because a,b \text{ are integers } \therefore \frac{a-3b}{2b} \text{ is a rational}\right]$$

This contradicts the fact that $\sqrt{5}$ is irrational. So, our supposition is incorrect.

Hence, $3+2\sqrt{5}$ is an irrational number.

8. If possible, let $5\sqrt{3}$ be a rational number.

So, $5\sqrt{3} = p/q$ where p and q are co-prime integers and $q \neq 0$

So, $\sqrt{3}=\frac{p}{5q}$

So, R.H.S is a rational number and hence $\sqrt{3}$ is also rational which is a contradiction.

So our supposition is wrong. Hence, 5 is an irrational number

9. n is an odd no. > 1

$\therefore$ The minimum possible value of $n = 3$

$n(n^2-1) = 3 \times 8 = 24$

Hence, $n(n^2-1)$ is divisible by 24 always

10. $M = 3$

11. Dividend = Divisor × Quotient + Remainder

Hence, remainder = 5 when same no. is divided by 29.

12. Only one integer between 0 and 100 belongs to A.

13. There are only 2 prime numbers 5 and 2 between 1 and 100 which when multiplied will give zero in the end.

Thus, there will be only one zero at the end of the product of given number.

14. Every term in the question is either 1 or –1. In order to have zero the number of terms must be even. Note that there are n number of terms. (since the first term in each product varies from x_1 to x_n).

So n has to be even.

15. Let the common remainder be x. Then numbers $(34041 - x)$ and $(32506 - x)$ would be completely divisible by n.

Hence the difference of the numbers $(34041 - x)$ and $(32506 - x)$ will also be divisible by n

or $(34041 - x - 32506 + x) = 1535$ will also be divisible by n.

Now, using options we find that 1535 is divisible by 307.

16. Full name of the bookstore can be read again by taking L.C.M of the times $\frac{5}{2},\frac{17}{4},\frac{41}{8}$

$$=\frac{\text{L.C.M of }(5, 17, 41)}{\text{H.C.F of }(2, 4, 8)}=\frac{3485}{2}=1742.5 \text{ seconds}$$

17. From this we know that remainder for any power of 4 will be 4 only.

18. L.C.M of 6, 5, 7, 10 and 12 = 420 seconds

$$=\frac{420}{60}=7 \text{ minutes}.$$

Therefore, in one hour (60 minutes), then will fall together 8 times $\left(\frac{60}{7}\right)$ excluding the one at the start.

19. H.C.F $= 2^2 \times 3^2$

L.C.M $= 2^4 \times 3^5 \times 5^2 \times 7^2$

Ist number $= 2^3 \times 3^4 \times 5$

2nd number $= 2^4 \times 3^2 \times 5^2$

observing the above situation, we conclude that the third number must be

$$x = 2^2 \times 3^2 \times 3^3 \times 7^2 = 2^2 \times 3^5 \times 7^2$$

20. Hence, the given number remains 3 as remainder when divided by 9.

LONG ANSWER QUESTIONS :

1. **Step 1:** Since, 12576 > 4052, we apply the division lemma to 12576 and 4052, to get $12576 = 4052 \times 3 + 420$

Step 2: Since, the remainder 420 ? 0, we apply the division lemma to 4052 and 420, to get $4052 = 420 \times 9 + 272$

Step 3: We consider the new divisor 420 and the new remainder 272, and apply the division lemma to get

$420 = 272 \times 1 + 148$

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

$272 = 148 \times 1 + 124$

We consider the new divisor 148 and the new remainder 124, and apply the division lemma to get

$148 = 124 \times 1 + 24$

We consider the new divisor 124 and the new remainder 24, and apply the division lemma to get

$124 = 24 \times 5 + 4$

We consider the new divisor 24 and the new remainder 4, and apply the division lemma to get

$24 = 4 \times 6 + 0$

The remainder has now become zero, so our procedure stops. Since, the divisor at this stage is 4, the H.C.F of 12576 and 4052 is 4.

Notice that 4 = H.C.F (24, 4) = H.C.F (124, 24)
= H.C.F (148, 124) = H.C.F (272, 148) = H.C.F (420, 272)
= H.C.F (4052, 420) = H.C.F (12576, 4052).

2. Let us assume, to the contrary, that $5-\sqrt{3}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that

$5-\sqrt{3}=\frac{a}{b}$

Therefore, $5-\frac{a}{b}=\sqrt{3}$

Rearranging this equation, we get $\sqrt{3}=5-\frac{a}{b}=\frac{5b-a}{b}$

Since a and b are integers, we get $5-\frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5-\sqrt{3}$ is rational.

So, we conclude that $5-\sqrt{3}$ is irrational.

3. Now, let us use Euclid's algorithm to find their H.C.F.

We have :

$420 = 130 \times 3 + 30$
$130 = 30 \times 4 + 10$
$30 = 10 \times 3 + 0$

So, the H.C.F of 420 and 130 is 10.

Therefore, the sweetseller can make stacks of 10 for both kinds of barfi.

4. There is no natural number n for which 4^n ends with the digit zero.

5. Let us assume, to the contrary, that $3\sqrt{2}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that

$3\sqrt{2}=\frac{a}{b}$

Rearranging, we get $\sqrt{2}=\frac{a}{3b}$

Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $3\sqrt{2}$ is irrational.

6. We know that any odd positive integer is of the form $4q+1$ or, $4q+3$ for some integer q.

So, we have the following cases :

Case I : When $n = 4q+1$

In this case, we have

$n^2-1=(4q+1)^2-1=16q^2+8q+1-1$
$=16q^2+8q=8q(2q+1)$

$\Rightarrow$ n^2-1 is divisible by 8 [$\because$ $8q(2q+1)$ is divisible by 8]

Case II : When $n = 4q+3$

In the case, we have

$n^2-1=(4q+3)^2-1=16q^2+24q+9-1$
$=16q^2+24q+8$

$\Rightarrow$ $n^2-1=8(2q^2+3q+1)=8(2q+1)(q+1)$

$\Rightarrow$ n^2-1 is divisible by 8

[$\because$ $8(2q+1)(q+1)$ is divisible by 8]

Hence, n^2-1 is divisible by 8.

7. Let us first find the H.C.F of 210 and 55.

Applying Euclid's division lemma on 210 and 55, we get

$210 = 55 \times 3 + 45$ ..... (i)

Since, the remainder $45 \neq 0$. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

$55 = 45 \times 1 + 10$ ..... (ii)

We consider the divisor 45 and the remainder 10 and apply division lemma to get

$45 = 4 \times 10 + 5$ ..... (iii)

We consider the divisor 10 and the remainder 5 and apply division lemma to get

$10 = 5 \times 2 + 0$ ..... (iv)

We observe that the remainder at this stage is zero. So, the last divisor i.e., 5 is the H.C.F of 210 and 55.

$\therefore$ $5 = 210 \times 5 + 55y$

$\Rightarrow$ $y=\frac{-1045}{55}=-19$

8. Given integers are 81 and 237 such that $81 < 237$.

Applying division lemma to 81 and 237, we get

$237 = 81 \times 2 + 75$ ..... (i)

Since, the remainder $75 \neq 0$. So, consider the divisor 81 and the remainder 75 and apply division lemma to get

$81 = 75 \times 1 + 6$ ..... (ii)

We consider the new divisor 75 and the new remainder 6 and apply division lemma to get

$75 = 6 \times 12 + 3$ ..... (iii)

We consider the new divisor 6 and the new remainder 3 and apply division lemma to get

$6 = 3 \times 2 + 0$ ..... (iv)

The remainder at this stage is zero. So, the divisor at this stage or the remainder at the earlier stage i.e., 3 is the H.C.F of 81 and 237.

To represent the H.C.F as a linear combination of the given two numbers, we start from the last but one step and successively eliminate the previous remainders as follows:

From (iii), we have

$3 = 75 - 6 \times 12$

$\Rightarrow$ $3 = 75 - (81 - 75 \times 1) \times 12$

[Substituting $6 = 81 - 75 \times 1$ obtained from (ii)]

$\Rightarrow\ 3 = 75 - 12 \times 81 + 12 \times 75$

$\Rightarrow\ 3 = 13 \times 75 - 12 \times 81$

[Substituting $75 = 237 - 81 \times 2$ obtained from (i)]

$\Rightarrow\ 3 = 13 \times (237 - 81 \times 2) - 12 \times 81$

$\Rightarrow\ 3 = 13 \times 237 - 26 \times 81 - 12 \times 81$

$\Rightarrow\ 3 = 13 \times 237 - 38 \times 81$

$\Rightarrow\ 3 = 237x + 81y$, where $x = 13$ and $y = -38$.

9. Let us assume on the contrary that $\sqrt{2} + \sqrt{5}$ is rational number. Then, there exist co-prime positive integers a and b such that

$$\sqrt{2} + \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow\ \frac{a}{b} - \sqrt{2} = \sqrt{5}$$

$$\Rightarrow\ \left(\frac{a}{b} - \sqrt{2}\right)^2 = (\sqrt{5})^2 \quad \text{[Squaring both sides]}$$

$$\Rightarrow\ \frac{a^2 - 3b^2}{2ab} = \sqrt{2}$$

$\Rightarrow\ \sqrt{2}$ is a rational number

$$\left[\because a, b \text{ are integers} \therefore \frac{a^2 - 3b^2}{2ab} \text{ is rational}\right]$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

Hence, $\sqrt{2} + \sqrt{5}$ is irrational.

10. Units digit in $(7^4) = 1$. Therefore, units digit in $(7^4)^8$ i.e., 7^{32} will be 1. Hence, units digit in

$$(7)^{35} = 1 \times 7 \times 7 \times 7 = 3$$

Again, units digit in $(3)^4 = 1$

Therefore, units digit in the expansion of

$$(3^4)^{17} = (3)^{68} = 1$$

$\Rightarrow$ Units digit in the expansion of

$$(3^{71}) = 1 \times 3 \times 3 \times 3 = 7$$

and units digit in the expanison of $(11^{35}) = 1$

Hence, units digit in the expansion of

$$7^{35} \times 3^{71} \times 11^{55} = 3 \times 7 \times 1 = 1$$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (a) 2. (a)

3. (a) When 2^{256} is divided by 17 then,

$$\Rightarrow \frac{2^{256}}{2^4 + 1} \Rightarrow \frac{(2^2)^{64}}{(2^4 + 1)}$$

By remainder theorem when $f(x)$ is divided by $x + a$ the remainder $= f(-a)$

Here $f(a) = (2^2)^{64}$ and $x = 2^4$ and $a = 1$

$\therefore$ Remainder $= f(-1) = (-1)^{64} = 1$

4. (c) Largest number is 67.

5. (d) N will be an odd number because N is sum of one even number (2) and 13985 odd numbers. Hence, N will not be divisible by even number.

6. (a)

7. (b) Division $= r_1 + r_2 - r_3 = 35 + 30 - 20 = 45$

8. (c) Use test of 11 after putting $y = 5$

9. (d) $0.\overline{134} = \frac{134 - 1}{990} = \frac{133}{990}$

10. (d) The L.C.M of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choice (1) and (3) since they are not perfect number.

11. (b)

12. (d) Out of n and $n + 2$, one is divisible by 2 and the other by 4, hence $n(n + 2)$ is divisible by 8. Also $n, n + 1, n + 2$ are three conseceutive numbers, hence one of them is divisible by 3. Hence $n(n + 1)(n + 2)$ must be divisible by 24. This will be true for any even number n.

13. (a) The number is short by 10 for complete division by 15, 25 or 35.

14. (c) $(-1)^{4n} = [(-1)^2]^{2n} = 1$

For $(-1)^n + (-1)^{4n} = 0$, $(-1)^n = -1$

15. (d) $3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26) = 2 \times 13 \times 3^{10}$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13

16. (d) Sum is 888 $\Rightarrow$ unit's digit should add up to 8. This is possible only for D option as "3" + "5" = "8".

17. (a) On dividing the given number 345670 by 6, we get 4 as the remainder.

So, 2 must be added to the given number.

18. (d)

19. (c) L.C.M × H.C.F = First number × second number

Hence, required number $= \frac{36 \times 2}{18} = 4$.

20. (c) 21. (a)

MORE THAN ONE CORRECT :

1. (b,d) Product of two co-prime numbers is equal to their LCM.
   So, LCM = 117
2. (a,b,c,d)
3. (a, b) In the product of $(274 \times 243 \times 131)$
   Unit digit will be 2
   Ten's digit will be 4
   So the numbers are 24 or 42.
4. (b,c,d)
5. (a,b) Unit digit in (7^{95})
   $=$ Unit digit in $[(7^4)^{23} \times 7^3]$
   $=$ Unit digit in 7^3 (as unit digit in $7^4 = 1$)
   $=$ Unit digit in 343
   Unit digit in 3^{58} $=$ Unit digit in $(3^4)^4 \times 3^2$
   [as unit digit $3^4 = 1$]
   $=$ Unit digit is 9
   So unit digit in $(7^{95} - 3^{58})$
   $=$ Unit digit in $(343 - 9)$
   $=$ Unit digit in $334 = 4$
   Unit digit in $(7^{95} + 3^{58}) =$ Unit digit in $(343 + 9)$
   $=$ Unit digit in $352 = 2$
   So the product is $4 \times 2 = 8$
6. (a, b, c)
7. (a, b, d) Given equation is
   $4P8 + 8Q3 + 7R8 = 2079$
   Sum of unit digits in $(4P8 + 8Q3 + 7R8)$ is 19, so 1 will carry to ten's.

$$\begin{array}{r} 1\,1 \\ 4\;P\;9 \\ 8\;Q\;3 \\ 7\;R\;8 \\ \hline 2\,0\,7\,9 \end{array}$$

   $\therefore 1 + P + Q + R = 17$
   Also $0 + 9 + 7 = 16$
   Hence maximum value of $Q = 9$
8. (a,b,d) $\frac{5}{3} + \frac{3}{4} = \frac{29}{12} < 5$

   $\frac{7}{3} + \frac{11}{5} = \frac{68}{15} < 5$

   $\frac{11}{14} + \frac{8}{3} = \frac{33+32}{12} = \frac{65}{12} > 5$

   $\frac{13}{5} + \frac{11}{6} = \frac{133}{30} < 5$.

PASSAGE BASED QUESTIONS :

Passage-1

1. (b) 2. (b)

Passage-2

1. (d) L.C.M. of $\frac{5}{16}, \frac{15}{24}$ and $\frac{25}{8} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$
   L.C.M. of 5, 15 and 25 is 75.
   H.C.F. of 16, 24 and 8 is 8.
   The HCF of the given fractions $= \frac{75}{8}$
2. (c) H.C.F. of the fractions $= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$
   H.C.F. of 2, 6 and 8 is 2.
   L.C.M. of 5, 25 and 35 is 175.
   Thus, the H.C.F. of the given fractions $= \frac{2}{175}$
3. (a) H.C.F. of given fraction is
   $= \frac{\text{H.C.F. of 8, 12, 32}}{\text{L.C.M. of 21, 35, 7}} = \frac{4}{105}$

ASSERTION & REASON :

1. (a) Reason is correct.
   Since the factors of the denominator 3125 is of the form $2^0 \times 5^5$.
   $\therefore$ $\frac{13}{3125}$ is a terminating decimal $\therefore$ (a) is true.
   Since assertion follows from reason $\therefore$ (a) holds.
2. (a) Reason is clearly true
   Again $34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$
   Its denominator is of the form $2^m \times 5^n$
   $\begin{bmatrix} m = 5,\ n = 4 \text{ are} \\ \text{non-negative integers} \end{bmatrix}$
   $\therefore$ assertion is true. Since reason gives assertion
   $\therefore$ (a) holds.
3. (d) Here reason is true [standard result]
   Assertion is false. $\because \frac{3072}{16} = 192 \neq 162$ $\therefore$ (d) holds
4. (c) Here reason is not true. $\because \sqrt{4} = \pm 2$, which is not an irrational number.
   $\therefore$ reason holds. Clearly, assertion is false $\therefore$ (c) holds.
5. (a) 6. (b) 7. (a)

MULTIPLE MATCHING QUESTIONS :

1. (A) $\rightarrow$ (t, s); (B) $\rightarrow$ (p, s); (C) $\rightarrow$ (q, u); (D) $\rightarrow$ (r, s)

HOTS SUBJECTIVE QUESTIONS :

1. We assume that there is a positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational, and equal to $\frac{a}{b}$ where a and b are positive integers.

So, $\sqrt{n-1}+\sqrt{n+1}=\frac{a}{b}$ ...(1)

$\Rightarrow \frac{b}{a}=\frac{1}{\sqrt{n-1}+\sqrt{n+1}}$,

Rationalising RHS by multiplying N_r and D_r by $\left(\sqrt{n-1}-\sqrt{n+1}\right)$, we get

$\frac{b}{a}=\frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$

or, $\frac{b}{a}=\frac{\sqrt{n+1}-\sqrt{n-1}}{2} \Rightarrow \sqrt{n+1}-\sqrt{n-1}=\frac{2b}{a}$ ...(2)

Adding equation (1) and (2), we get

$\Rightarrow \sqrt{n+1}=\frac{a^2+2b^2}{2ab}$ ...(3)

Subtracting (2) from (1), we get,

$\Rightarrow \sqrt{n-1}=\frac{a^2-2b^2}{2ab}$ ...(4)

From (3) and (4), we have $\sqrt{n+1}$ and $\sqrt{n-1}$ are both rational as a and b are both positive integer.

This is possible only when both $n+1$ and $n-1$ are perfect squares of some positive integer n.

$(n-1)$ and $(n+1)$ differ by 2. But two perfect squares never differ by 2, hence $(n-1)$ and $(n+1)$ both cannot be perfect square. So, there is no positive number n for which $\sqrt{n-1}+\sqrt{n+1}$ is a rational number.

2. Applying *Euclid's division lemma* to 56 and 72, we get

$72=56\times1+16$ ... (i)

Since the remainder $16\neq0$, So, we consider the divisor 56 and the remainder 16 and apply division lemma to get

$56=16\times3+8$ ... (ii)

We consider the divisor 16 and the remainder 8 and apply division lemma to get

$16=8\times2+0$ ... (iii)

We observe that the remainder at this stage is zero. Therefore, last divisor, i.e., 8 is the HCF of 56 and 72.

From (ii), we get $8=56-16\times3$

$\Rightarrow 8=56-(72-56\times1)\times3 \Rightarrow 8=56-3\times72+56\times3$

$\Rightarrow 8=56\times4+(-3)\times72$

On comparing with the given equation, $d=56x+72y$, we get $x=4$ and $y=-3$.

Now, $8=56\times4+(-3)\times72$

$\Rightarrow 8=56\times4+(-3)\times72-56\times72+56\times72$

$\Rightarrow 8=56\times4-56\times72+(-3)\times72+56\times72$

$\Rightarrow 8=56\times(-68)+72\times53$

Clearly, $x=-68$ and $y=53$.

Hence, x and y are not unique.

3. We know that any positive integer is of the form $3q$ or, $3q+1$ or, $3q+2$ for some non-negative integer q.

So, we have following cases :

Case I : When $n=3q$

In this case, we have

$n=3q$, which is divisible by 3

Now, $n=3q \Rightarrow n+2=3q+2 \Rightarrow n+2$ leaves remainder 2 when divided by 3

$\Rightarrow n+2$ is not divisible by 3.

Again, $n=3q \Rightarrow n+4=3q+4=3(q+1)+1 \Rightarrow n+4$ leaves remainder 1, when divided by 3

$\Rightarrow n+4$ is not divisible by 3.

Thus, n is divisible by 3 but $n+2$ and $n+4$ are not divisible by 3.

Case II : When $n=3q+1$

In this case, we have $n=3q+1$

$\Rightarrow n$ leaves remainder 1 when divided by 3 $\Rightarrow n$ is not divisible by 3.

Now, $n=3q+1$

$\Rightarrow n+2=(3q+1)+2=3(q+1) \Rightarrow n+2$ is divisible by 3.

Again, $n=3q+1$

$\Rightarrow n+4=3q+1+4=3q+5=3(q+1)+2$

$\Rightarrow n+4$ leaves remainder 2 when divided by 3

$\Rightarrow n+4$ is not divisible by 3.

Thus, $n+2$ is divisible by 3 but n and $n+4$ are not divisible by 3.

Case III : When $n=3q+2$

In this case, we have $n=3q+2$

$\Rightarrow n$ leaves remainder 2 when divided by 3

$\Rightarrow n$ is not divisible by 3.

Now, $n=3q+2$

$\Rightarrow n+2=3q+2+2=3(q+1)+1$

$\Rightarrow n+2$ leaves remainder 1 when divided by 3

$\Rightarrow n+2$ is not divisible by 3.

Again, $n=3q+2$

$n+4=3q+2+4=3(q+2) \Rightarrow n+4$ is divisible by 3.

Thus, $n+4$ is divisible by 3 but n and $n+2$ are not divisible by 3.

4. Let the positive divisors of n be $1, a, b, c,$ .... and n itself.

Let $S(n)$ = sum of positive divisors of n, including n.

$=1+a+b+c+\text{.........}+n.$

and $G(n)$ = greatest divisor of $n=n$ (itself)

Given : $H(n)=\frac{G(n)}{S(n)}=\frac{n}{1+a+b+c+\text{......}+n}$

Now, For $H(n)$ to be largest, $1+a+b+c$ .... $+n$ should be minimum. In this series 1 and n can never be removed so if we ignore $a, b, c,$ ..... and so $a+b+c+$.... Hence, we get the minimum value of $S(n)$ as $1+n$.

Hence, $1+(n)$ is maximum when n is a prime number because it has only two positive divisors 1 and n.

Now we have given two values 2010 and 2011 of n, of which only 2011 is prime. Hence $H(2011)$ is the largest.

5. Let $\sqrt{P}$ be a rational number

Then $\sqrt{P} = \frac{a}{b}$ ...(1)

$a, b \in I,\ b \neq 0$ and $(a, b) = 1$

Squaring,

$\therefore\ P = \frac{a^2}{b^2}$

$\Rightarrow\ a^2 = Pb^2$ ...(2)

Now, P is a factor of R.H.S of (2)

$\therefore$ P is a factor of L.H.S of (2)

$\therefore\ \frac{P}{a^2} \Rightarrow \frac{P}{a}$

let $a = Pk$, where $k \in I$

$P^2 K^2 = a^2$

$P^2 K^2 = Pb^2 \Rightarrow b^2 = PK^2$ ...(3)

Now P is a factor of R.H.S of (3)

$\Rightarrow$ P is a factor is L.H.S of (3)

$\therefore\ P/b^2 \Rightarrow P/b.$

$\therefore$ P is a factor of boths a & b which is contradiction that $(a, b) = 1$.

$\therefore$ our assumption is wrong.

Hence $\sqrt{P}$ is not a rational number.

$\sqrt{P}$ is a irrational number.

6. Let a be any positive integer and $b = 6$

Then by Euclid's algorithem, $a = bq + r,\ \ 0 \leq r < b$

We have $a = 6q + r$ ...(1) $\quad 0 \leq r < 6$

$\therefore$ From (1), for $r = 0,\ a = 6q$

for $r = 1,\ a = 6q + 1$

for $r = 2,\ a = 6q + 2$

for $r = 3,\ a = 6q + 3$

for $r = 4,\ a = 6q + 4$

for $r = 5,\ a = 6q + 5$

Since $6q$ is divisible by 2,

$\therefore$ $6q$ is even.

$6q + 1$ is not divisible by 2

$6q + 2$ is divisible by 2

$\therefore$ $6q + 2$ is even.

$6q + 3$ is not divisible by 2.

$6q + 4$ is divisible by 2.

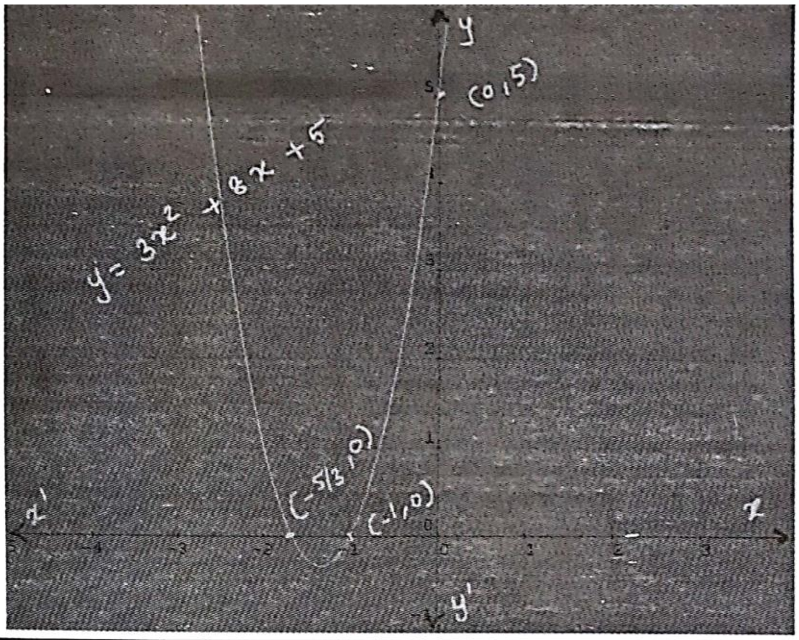
$\therefore$ $6q + 4$ is even.

$6q + 5$ is not divisible by 2

So, we see that $6q, 6q + 2, 6q + 4$ are even. Since the number which are not divisible by 2 are odd integer.

$\therefore$ $6q + 1,\ 6q + 3,\ 6q + 5$ are odd integer. Hence any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

chapter 2



POLYNOMIALS

Introduction

In earlier classes, we have studied little about polynomials. Let us first review some basic concepts and then we will learn about geometrical meaning of the zeroes of the polynomials and relation between zeroes and coefficient of polynomials.

An algebraic expression $f(x)$ of the form $f(x) = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a^nx^n$; where $a_0, a_1, a_2, \dots\dots\dots, a_n$ are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if $a_n \neq 0$. Here, a_0, a_1x, a_2x^2, ... and a_nx^n are called the terms of the polynomial and $a_0, a_1, a_2 \dots\dots\dots a_n$ are called various coefficients of the polynomial $f(x)$. A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

STANDARD FORM OF A POLYNOMIAL :

Hence, $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ or $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$ are standard forms of the same polynomial in variable x.

A symbol which takes various numerical values is known as a variable.

Terms having same variable with same indices are called like terms, otherwise they are called unlike terms.

Here, $a_0, a_1, a_2, ..., a_n$ are real numbers. But if $a_0, a_1, a_2, ..., a_n$ are all integers then the above polynomial is said to be a polynomial over integers.

If $a_n \neq 0$, then degree of the above polynomial is n.

If a polynomial involves two or more variables, then the sum of the powers of all the variables in each term is taken up and the Highest sum so obtained is the degree of the polynomial.

Examples :

(i) $6x^7 - 5x^4 + 2x + 3$ is a polynomial of degree 7.

(ii) $2 + 5x^{3/2} + 7x^2$ is an expression but not a polynomial , since it contains a term in which power of x is 3/2, which is not a non-negative integer.

(iii) $3xy^2 - 4\ x\sqrt{y}\ + 5y^3$ is an expression but not a polynomial, as it contains two variables and a term in which the sum of the powers of the variables is $\frac{3}{2}$, which is not a non-negative integer.

(iv) In $-9y^2$, the -9 is the numerical coefficient of y; y is the variable and 2 is the index of y.

(v) $5a^2, -7a^2$ and $\sqrt{2}\ a^2$ are like terms.

(vi) $6x^2, -8y^2$ and $-4ab$ are unlike terms.

DIFFERENT TYPES OF POLYNOMIALS :

(A) There are some important types of polynomials based on degrees. These are listed below :

(i) **Linear polynomials :** A polynomial of degree one is called a linear polynomial. The general formula of linear polynomial is $ax + b$, where a and b are any real constant and $a \neq 0$.
Example : $3 + 5x$ is a linear polynomial.

(ii) **Quadratic polynomials :** A polynomial of degree two is called a quadratic polynomial. The general form of quadratic polynomial is $ax^2 + bx + c$, where $a \neq 0$.
Example : $2y^2 + 3y - 1$ is a quadratic polynomial.

(iii) **Cubic polynomials :** A polynomial of degree three is called a cubic polynomial. The general form of a cubic polynomial is $ax^3 + bx^2 + cx + d$, where $a \neq 0$.
Example : $6x^3 - 5x^2 + 2x + 1$ is a cubic polynomial.

(iv) **Biquadratic polynomials :** A polynomial of degree four is called a biquadratic polynomial. The general form of a biquadratic polynomial is $ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$.
A polynomial of degree five or more than five does not have any particular name. Such a polynomial is usually called a polynomial of degree five or six or ...., etc.

(v) **Zero degree polynomial :**
Any non-zero number is regarded as a polynomial of degree zero or zero degree polynomial.
For example, $f(x) = a$, where $a \neq 0$ is a zero degree polynomial, since we can write $f(x) = a$ as $f(x) = ax^0$.

(vi) **Zero polynomial :**
A polynomial whose coefficient are all zeros is called a zero polynomial i.e., $f(x) = 0$ but we cannot talk about the degree of a zero polynomial.

(B) Names of the polynomials are also based on number of terms. These are as follows :

(i) **Monomial :** A polynomial is said to be a monomial if it has only one term.
Example : $3x^2,\ 5x^3,\ 10x$ are monomials.

(ii) **Binomial :** A polynomial is said to be a binomial if it contains two terms.
Example : $(2x^2 + 5), (3x^3 - 7), (6x^2 + 8x)$ are binomials.

(iii) **Trinomials :** A polynomial is said to be a trinomial if it contains three terms.

Example : $3x^3 - 8x + \frac{5}{2},\ \sqrt{7}x^{10} + 8x^4 - 3x^2,\ 5 - 7x + 8x^9$ are trinomials.

No specific name is given to those polynomials which have more than three terms.

ILLUSTRATION 2.1

Which of the following functions are polynomials ?

(a) $5x^2 - 3x + 9$ **(ii)** $\frac{4}{3}x^7 - 5x^4 + 3x^2 - 1$

SOLUTION :

Both are polynomials.

ILLUSTRATION 2.2

Write down the power of the following polynomials.

(a) $3x + 5$ **(b)** $3t^2 - 5t + 9t^4$. **(c)** $2 - y^2 - y^3 + 2y^8$

SOLUTION :

(a) The highest power term is 3x and its exponent is 1.

$\therefore$ degree of polynomial $3x + 5 = 1$

(b) The highest power term is $9t^4$ and its exponent is 4. So, degree of the given polynomial is 4.

(c) The highest power of the variable is 8. So, the degree of the polynomial is 8.

ILLUSTRATION 2.3

Write the following polynomial in standard form :

$$x^6 - 3x^4 + \sqrt{2}x + \frac{5}{2}x^2 + 7x^5 + 4$$

SOLUTION :

The given polynomial in standard form :

$$x^6 + 7x^5 - 3x^4 + \frac{5}{2}x^2 + \sqrt{2}x + 4$$

VALUE OF A POLYNOMIAL :

The value of a polynomial $f(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

Consider the polynomial : $p(x) = 5x^3 - 2x^2 + 3x - 2$

If we replace x by 1 everywhere in $p(x)$, we get

$$p(1) = 5 \times (1)^3 - 2 \times (1)^2 + 3 \times (1) - 2$$
$$= 5 - 2 + 3 - 2 = 4$$

So, we say that the value of $p(x)$ at $x = 1$ is 4.

ZERO(ES)/ ROOT(S) OF POLYNOMIALS :

$x = r$ is a root or zero of a polynomial, $P(x)$, if $P(r) = 0$.

In other words, $x = r$ is a root or zero of a polynomial if it is a solution to the equation $P(x) = 0$

The process of finding the zeros of $P(x)$ is nothing more than solving the equation $P(x) = 0$

Let's first find the zeroes for $P(x) = x^2 + 2x - 15$. To do this we simply solve the following equation.

$$x^2 + 2x - 15 = 0 \Rightarrow (x + 5)(x - 3) = 0 \Rightarrow x = -5, x = 3$$

So, this second degree polynomial has two zeroes or roots –5 and 3.

ILLUSTRATION 2.4

If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

SOLUTION :

$f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0 \Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0 \Rightarrow 128 - 176 + 12k - 180 = 0$$

$\Rightarrow 12k + 128 - 356 = 0 \Rightarrow 12k = 228 \Rightarrow k = 19$

ILLUSTRATION 2.5

If $x = 2$ and $x = 0$ are roots of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$. Find the values of a and b.

SOLUTION :

Since, 2 is a root, therefore

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$.

$\Rightarrow 16 - 20 + 2a + b = 0 \Rightarrow \quad 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow b = 0 \quad \Rightarrow \quad 2a = 4 \quad \Rightarrow a = 2, b = 0$

GEOMETRICAL MEANING OF THE ZERO(ES) OF A POLYNOMIAL :

Zero(es) of a polynomial is/are the x-coordinate of the point(s) where the graph of $Y = f(x)$ intersects the X-axis.

GRAPHS OF POLYNOMIALS :

In geometrical or in graphical language the graph of a polynomial $f(x)$ is a smooth free hand curve passing through points (x_1, y_1), (x_2, y_2), (x_3, y_3), ... etc., where y_1, y_2, y_3, ... are the values of the polynomial $f(x)$ at x_1, x_2, x_3, ... respectively. In order to draw the graph of a polynomial $f(x)$, we may follow the following alogrithm.

Algorithm to draw the graph of the polynomials :

Step 1 : Find the values y_1, y_2, ...y_n, ... of polynomial $f(x)$ at x_1, x_2, ...x_n, ... and prepare a table that gives values of y or $f(x)$ for various values of x.

x	x_1	x_2	.....	x_n	x_{n+1}	.....
$y = f(x)$	$y_1 = f(x_1)$	$y_2 = f(x_2)$	.....	$y_n = f(x_n)$	$y_{n+1} = f(x_{n+1})$	.....

Step 2 : Plot these points (x_1, y_1), (x_2, y_2), (x_3, y_3), ...,(x_n, y_n),......... on rectangular coordinate system. In plotting these points you may use different scales on the X and Y-axis.

Step 3 : Draw a free hand smooth curve passing through the points plotted in step II to get the graph of the polyncmial $f(x)$.

GRAPH ON A LINEAR POLYNOMIAL :

Consider a linear polynomial $f(x) = ax + b$, $a \neq 0$. In class IX we have studied that the graph of $y = ax + b$ is a straight line. That is why $f(x) = ax + b$ is called a linear polynomial. Since two points determine a straight line, so only two points need to plotted to draw the line $y = ax + b$. The line represented by $y = ax + b$ crosses the X-axis at exactly one point, namely $\left(-\frac{b}{a}, 0\right)$.

ILLUSTRATION 2.6

Draw the graph of the polynomial $f(x) = 2x - 5$. Also find its zero.

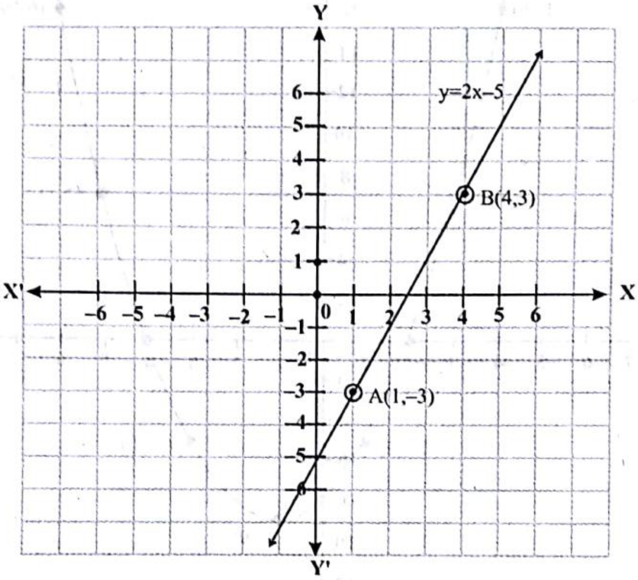
SOLUTION :

Let $y = 2x - 5$

The following table list the values of y corresponding to different values of x.

x	1	4
y	−3	3

The points A (1, –3) and B (4, 3) are plotted on the graph paper on a suitable scale. A line is drawn passing through these points to obtain the graphs of the given polynomial.



The line intersects the X-axis at $x = \frac{5}{2}$. Hence, zero or root of the given polynomial is $\frac{5}{2}$.

GRAPH OF A QUADRATIC POLYNOMIAL :

Let a, b, c be real numbers and $a \neq 0$. Then the $f(x) = ax^2 + bx + c$ is known as quadratic polynomial in x. Graph of a quadratic polynomial is always a parabola.

ILLUSTRATION 2.7

Draw the graph of the polynomial $f(x) = x^2 - 2x - 8$. Also find its zeroes.

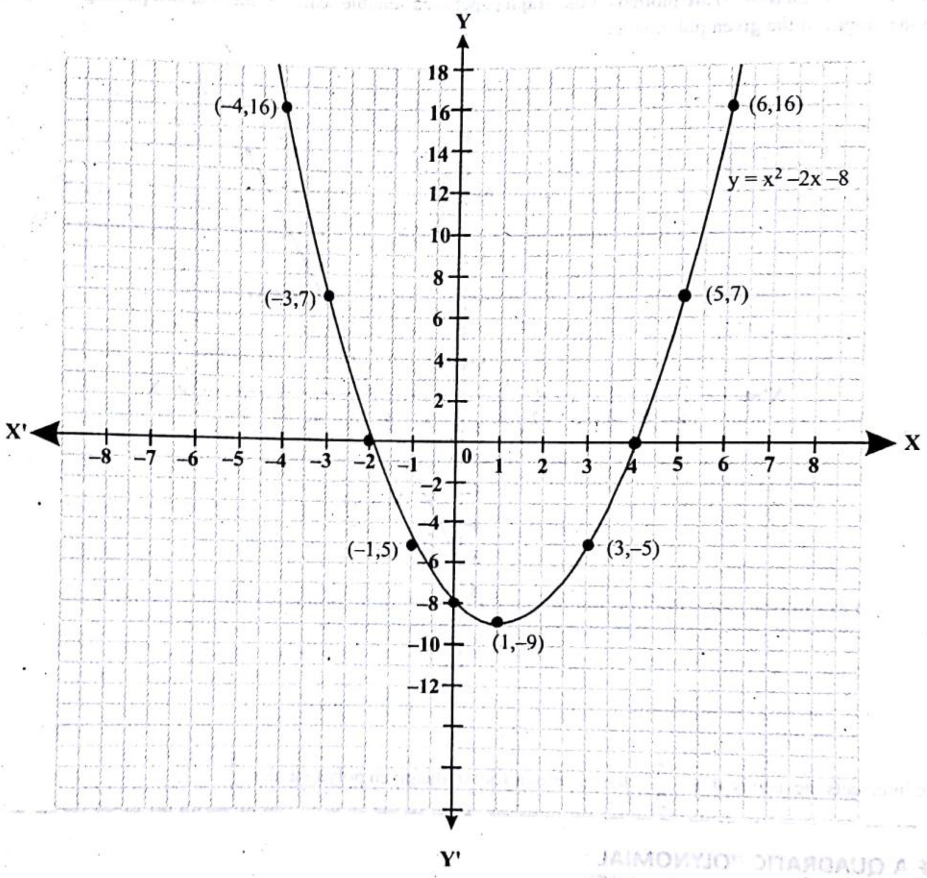
SOLUTION:

Let $y = x^2 - 2x - 8$

The following table gives the values of y or $f(x)$ for various values of x.

x	–4	–3	–2	–1	0	1	2	3	4	5	6
$y = x^2 - 2x - 8$	16	7	0	–5	–8	–9	–8	–5	0	7	16

Let us now plot the points (– 4, 16), (–3, 7), (–2, 0), (–1, –5), (0, –8), (1, –9), (2, –8), (3, –5), (4, 0), (5, 7) and (6, 16) on a graph paper and draw a smooth free hand curve passing through these points. The curve thus obtained represents the graph of the polynomial $f(x) = x^2 - 2x - 8$. Which is a parabola.



The parabola intersects the X-axis at $x = -2$ and 4. Hence, zeroes or roots of the polynomial are -2 and 4.

ILLUSTRATION 2.8

Draw the graphs of the quadratic polynomial $f(x) = 3 - 2x - x^2$. Also find its zeroes.

SOLUTION :

Let $y = f(x)$ or $y = 3 - 2x - x^2$

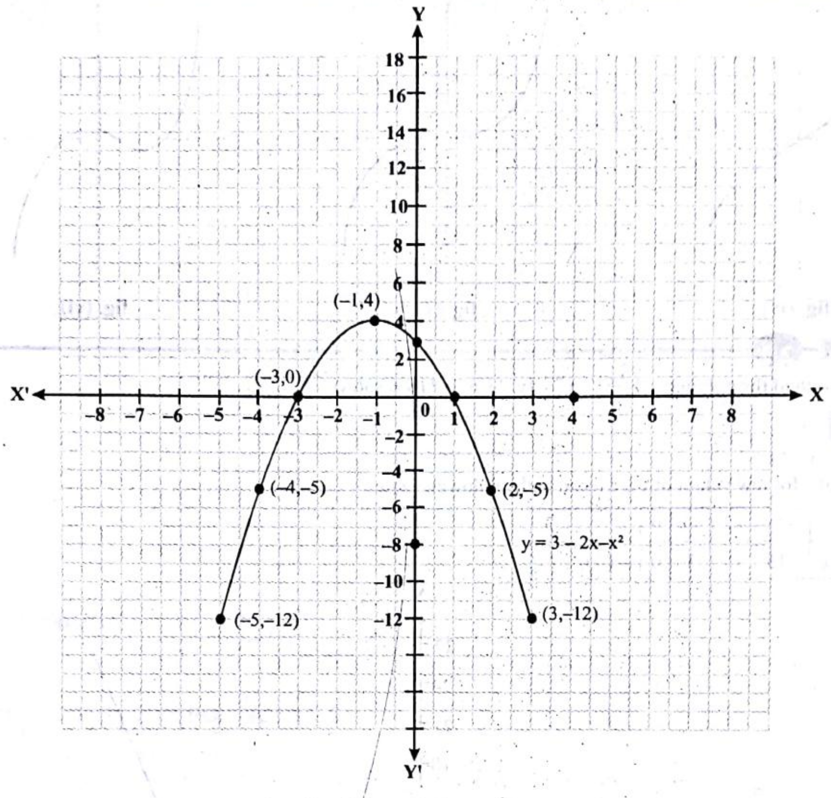
Let us list a few values of $y = 3 - 2x - x^2$ corresponding to a few values of x as follows :

x	-5	-4	-3	-2	-1	0	1	2	3
$y = 3 - 2x - x^2$	-12	-5	0	3	4	3	0	-5	-12

Thus, the following points lie on the graph of polynomial $y = 3 - 2x - x^2$:

$(-5, -12), (-4, -5), (-3, 0), (-2, 3), (-1, 4), (0, 3), (1, 0), (2, -5)$, and $(3, -12)$

Let us plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graph of $y = 3 - 2x - x^2$. The curve thus obtained is a parabola.



The parabola intersects X-axis at $x = -3$ and 1. Therefore, zeroes or roots of the polynomial are −3 and 1.

GRAPHS OF A CUBIC POLYNOMIAL :

A cubic polynomial is a function of the form $y = ax^3 + bx^2 + cx + d$

where $a \neq 0$, and a, b, c and d are real constants.

If $a > 0$, then graph of a cubic function looks similar to one of the graphs in Figures (i), (ii) and (iii).

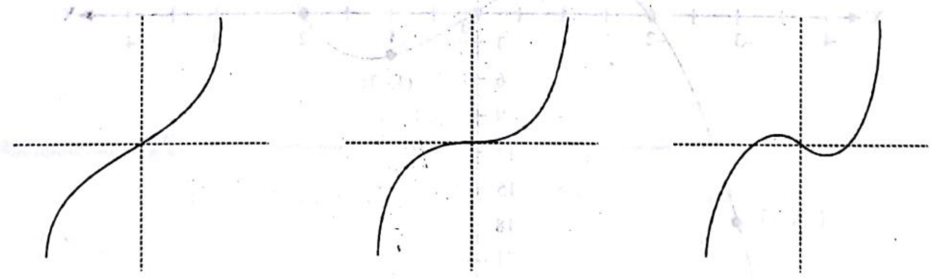


fig. (i) fig. (ii) fig. (iii)

If $a < 0$, then graph of the cubic function looks similar to one of the graphs in Figures (iv), (v) and (vi).

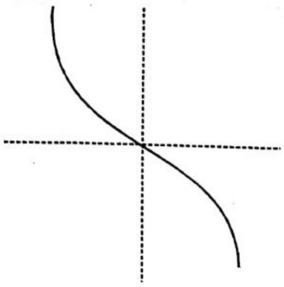


fig. (iv)

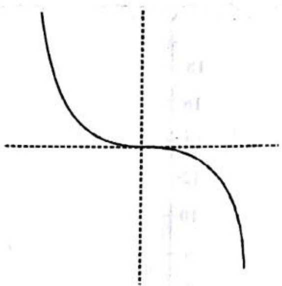


fig. (v)

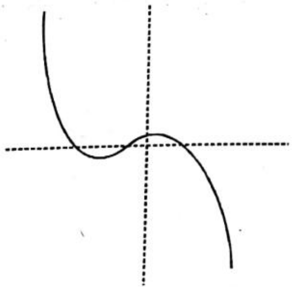


fig. (vi)

ILLUSTRATION 2.9

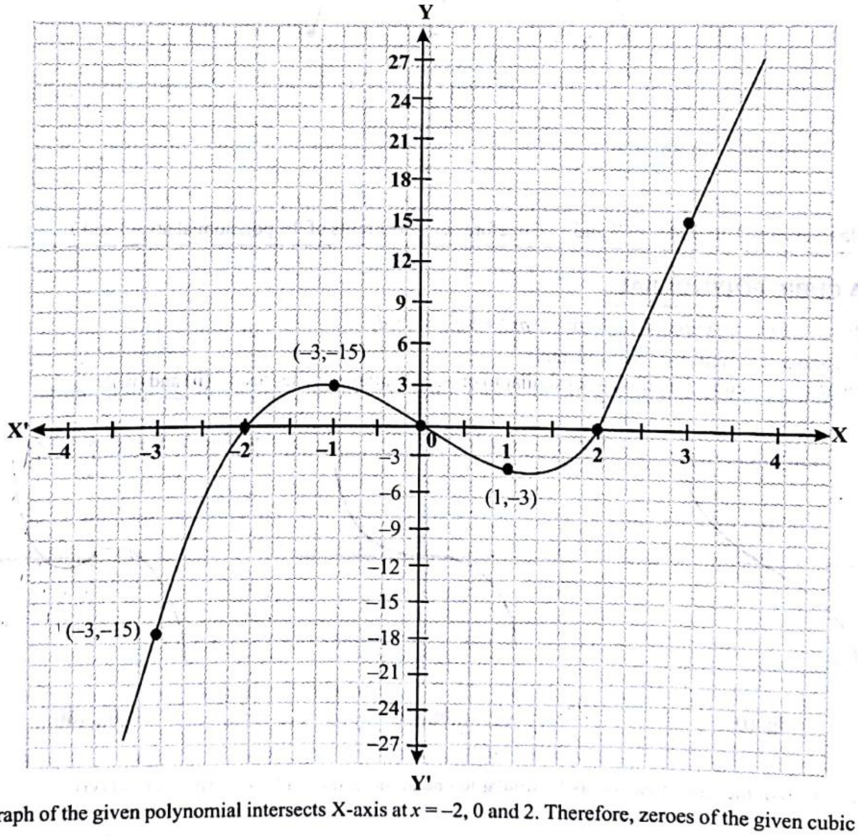
Draw the graph of the polynomial $f(x) = x^3 - 4x$. Also find its zero(es).

SOLUTION:

Let $y = f(x) = x^3 - 4x$.

The values of y for few values of x are listed in the following table :

x	−3	−2	−1	0	1	2	3
$y = x^3 - 4x$	−15	0	3	0	−3	0	15



Since the graph of the given polynomial intersects X-axis at $x = -2$, 0 and 2. Therefore, zeroes of the given cubic polynomial are −2, 0 and 2.

RELATIONSHIP BETWEEN ZERO(ES) AND COEFFICIENT OF A POLYNOMIAL :

(a) Zero of a linear polynomial $ax + b$, is $x = -\frac{b}{a}$

(b) If quadratic polynomial $ax^2 + bx + c = k(x - \alpha)(x - \beta)$, where k is any real constant; then α and β are zeroes of quadratic polynomial $ax^2 + bx + c$
where $a, b, c \in R$ and $a \neq 0$.

$$\alpha + \beta = -\frac{b}{a},$$

i.e., sum of zeroes $= -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$ And $\alpha \cdot \beta = \dfrac{c}{a}$

i.e., Product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

(c) If cubic polynomial $ax^3 + bx^2 + cx + d = k(x - \alpha)(x - \beta)(x - \gamma)$ where k is any real constant, then α, β and γ are zeroes of cubic polynomial
$ax^3 + bx^2 + cx + d$ where $a, b, c, d \in R$ and $a \neq 0$

$$\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \text{and} \quad \alpha\beta\gamma = -\frac{d}{a}$$

COMPLEX CONJUGATE THEOREM :

If $P(x)$ is a polynomial function with real coefficients, and $a + b$ is a solution of the equation $P(x) = 0$, then $a - b$ is also a solution.

BASIC OPERATIONS WITH POLYNOMIALS :

Sum and Difference of two Polynomials :

The sum and difference of two polynomials can be found by grouping like power terms, taking the variable with same index common if any and then taking algebraic sum of the coefficients of like terms.

ILLUSTRATION – 2.10

Add : $(4a^3 - 5a^2 + 6a - 3), (2 + 8a^2 - 3a^3), (9a - 3a^2 + 2a^3 + a^4), (1 - 2a - 3a^3)$

SOLUTION :

$(4a^3 - 5a^2 + 6a - 3) + (2 + 8a^2 - 3a^3) + (9a - 3a^2 + 2a^3 + a^4)$
$= a^4 + (4a^3 - 3a^3 + 2a^3 - 3a^3) + (-5a^2 + 8a^2 - 3a^2) + (6a + 9a - 2a) + (-3 + 2 + 1)$
$= a^4 + (4 - 3 + 2 - 3)a^3 + (-5 + 8 - 3)a^2 + (6 + 9 - 2)a$
$= a^4 + 13a$
$\therefore$ Required sum $= a^4 + 13a$

ILLUSTRATION – 2.11

If $p(y) = y^6 - 3y^4 + 2y^2 + 6$ and $q(y) = y^5 - y^3 + 2y^2 + y - 6$, find $p(y) + q(y)$ and $p(y) - q(y)$.

SOLUTION :

$p(y) + q(y) = (y^6 - 3y^4 + 2y^2 + 6) + (y^5 - y^3 + 2y^2 + y - 6)$
$= y^6 + y^5 - 3y^4 - y^3 + (2 + 2)y^2 + y + (6 - 6)$
$= y^6 + y^5 - 3y^4 - y^3 + 4y^2 + y$

$p(y) - q(y) = (y^6 - 3y^4 + 2y^2 + 6) - (y^5 - y^3 + 2y^2 + y - 6)$
$= y^6 - y^5 - 3y^4 + y^3 + (2y^2 - 2y^2) - y + (6 + 6)$
$= y^6 - y^5 - 3y^4 + y^3 - y + 12$

Multiplication of Monomials :

Product of monomials = (Product of their numerical coefficients) × (Product of their variable parts)

Multiplication of Two Polynomials :

Multiply each term of the multiplicand by each term of the multiplier and take the algebraic sum of these products.

ILLUSTRATION–2.12

Find the product of $(x+3)$ and (x^2+4x+5).

SOLUTION :

$$\begin{aligned}(x+3)(x^2+4x+5) &= x(x^2+4x+5)+3(x^2+4x+5).\\ &= x^3+4x^2+5x+3x^2+12x+15\\ &= x^3+(4+3)x^2+(5+12)x+15\\ &= x^3+7x^2+17x+15\end{aligned}$$

ILLUSTRATION–2.13

Multiply : $p(t)=t^4-6t^3+5t-8$ and $q(t)=t^3+2t^2+7$.
Also, find the degree of $p(t).q(t)$.

SOLUTION :

$$\begin{aligned}p(x).q(x) &= (t^4-6t^3+5t-8).(t^3+2t^2+7)\\ &= t^4(t^3+2t^2+7)-6t^3(t^3+2t^2+7)+5t(t^3+2t^2+7)-8(t^3+2t^2+7)\\ &= t^7+2t^6+7t^4-6t^6-12t^5-42t^3+5t^4+10t^3+35t-8t^3-16t^2-56\\ &= t^7+(2t^6-6t^6-6t^6)-12t^5+(7t^4+5t^4)+(-42t^3+10t^3-8t^3)-16t^2+35t-56\\ &= t^7-4t^6-12t^5+12t^4-40t^3-16t^2+35t-56\end{aligned}$$

The highest power term is t^7, and its exponent is 7.

$\therefore$ degree of $p(t).q(t)$ is 7.

Division of Polynomials :

On dividing a polynomial $p(x)$ by a polynomial $d(x)$, let the quotient be $q(x)$ and the remainder be $r(x)$, then

$p(x)=d(x).q(x)+r(x)$, where either $r(x)=0$ or deg. $r(x)<$ deg.$d(x)$

Here, Dividend $=p(x)$, Divisor $=d(x)$, Quotient $=q(x)$ and Remainder $=r(x)$.

Division algorithm of a polynomial by a polynomial :

Step 1 : Arrange the terms of the dividend and the divisor in descending order of their degrees.
Step 2 : Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.
Step 3 : Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.
Step 4 : Consider the remainder as new dividend and proceed as before.
Step 5 : Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

ILLUSTRATION–2.14

Divide $x^3+x^2-2x-30$ by $x-3$

SOLUTION :

$$\begin{array}{r|l}
x-3 & x^3+x^2-2x-30 \;(\; x^2+4x+10\\
 & \underline{\overset{-}{}x^3 \overset{+}{-} 3x^2}\\
 & \quad 4x^2-2x-30\\
 & \quad \underline{4x^2 \overset{+}{-} 12x}\\
 & \qquad 10x-30\\
 & \qquad \underline{10x \overset{+}{-} 30}\\
 & \qquad\quad \times
\end{array}$$

$\therefore (x^3+x^2-2x-30) \div (x-3)=x^2+4x+10$

ILLUSTRATION 2.15

Divide : $(15x^2 - 32y^2 + 38xy)$ by $(3x - 2y)$

SOLUTION :

Arranging the terms of the dividend and the divisor in descending order of powers of x and then dividing, we get

$$
\begin{array}{r|l|l}
3x - 2y & 15x^2 + 38xy - 32y^2 & 5x + 16y \\
 & 15x^2 - 10xy & \\
 & - \quad + & \\
\hline
 & 48xy - 32y^2 & \\
 & 48xy - 32y^2 & \\
 & - \quad + & \\
\hline
 & \times & \\
\hline
\end{array}
$$

$\therefore\ (15x^2 - 32y^2 + 38xy) \div (3x - 2y) = 5x + 16y$

ILLUSTRATION 2.16

Find the remainder obtained on dividing $x^3 + 3$ by $x + 1$.

SOLUTION :

$$
\begin{array}{r|l|l}
x + 1 & x^3 + 3 & x^2 - x + 3 \\
 & x^3 + x^2 & \\
 & - \quad - & \\
\hline
 & -x^2 + 3 & \\
 & -x^2 - x & \\
 & + \quad + & \\
\hline
 & x + 3 & \\
 & x + 1 & \\
 & - \quad - & \\
\hline
 & 2 & \\
\end{array}
$$

Hence, remainder = 2

ILLUSTRATION 2.17

Find all the zero(es) of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zero(es) are $\sqrt{2}$ and $-\sqrt{2}$.

SOLUTION :

Two zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Now, $(x - \sqrt{2}), (x + \sqrt{2}) = x^2 - 2$, is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 2$

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & 2x^4 \qquad\quad - 4x^2 \\
 & - \qquad\qquad + \\
\hline
 & -3x^3 + x^2 + 6x - 2 \\
 & -3x^3 \qquad + 6x \\
 & + \qquad\qquad - \\
\hline
 & x^2 \quad - 2 \\
 & x^2 \quad - 2 \\
 & - \quad + \\
\hline
 & 0 \\
\end{array}
$$

(First term of quotient is $\frac{2x^4}{x^2} = 2x^2$)

(Second term of quotient is $\frac{-3x^3}{x^2} = -3x$)

(Third term of quotient is $\frac{x^2}{x^2} = 1$)

So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$.

Now, by splitting $-3x$, we factorise $2x^2 - 3x + 1$ as $(2x - 1)(x - 1)$.

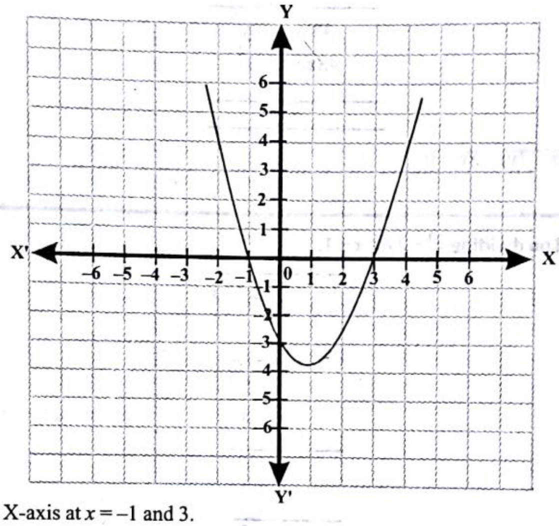
So, the remaining two zero(es) are given by $x = \frac{1}{2}$ and $x = 1$. Therefore, the zero(es) of the given polynomial are $\sqrt{2}, -\sqrt{2}, \frac{1}{2}$ and 1.

MISCELLANEOUS SOLVED EXAMPLES

1. $f(x) = x^2 - 2x - 3$. Find the roots of $f(x)$, and sketch the graph of $y = f(x)$.

Sol. $x^2 - 2x - 3 = (x+1)(x-3)$.

Therefore, the roots are -1 and 3.

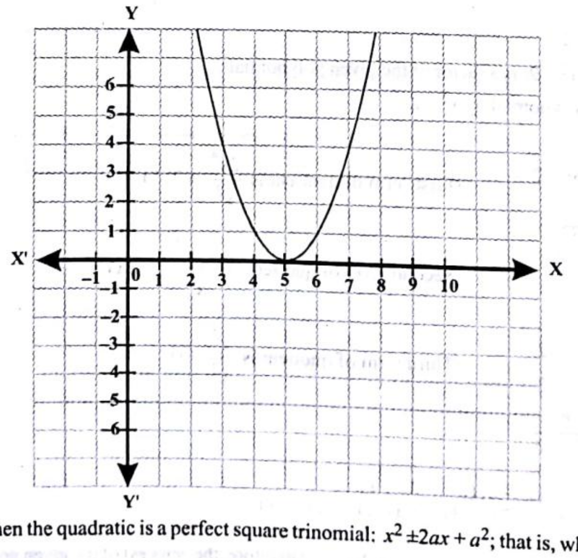


The graph intersects the X-axis at $x = -1$ and 3.

2. $f(x) = x^2 - 10x + 25$. Find the roots of $f(x)$, and sketch the graph of $y = f(x)$.

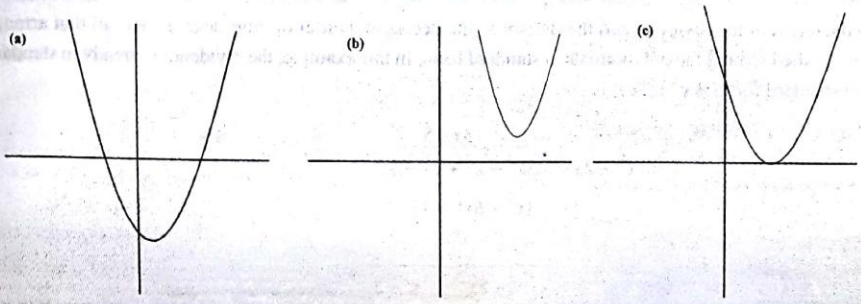
Sol. $x^2 - 10x + 25 = (x-5)(x-5) = (x-5)^2$. The "two" roots are 5, 5.

5 is called a double root. At a double root, the graph does not cross the x-axis. It just touches it.



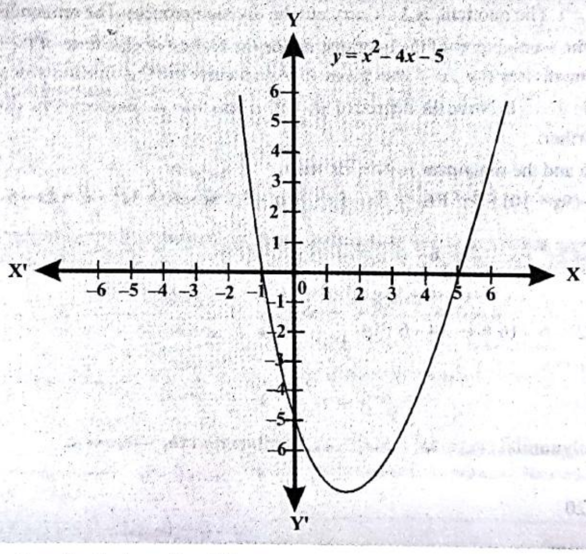
A double root occurs when the quadratic is a perfect square trinomial: $x^2 \pm 2ax + a^2$; that is, when it is the square of a binomial: $(x \pm a)^2$.

3. How many real roots, i.e., roots that are real numbers, has the parabola of each graph ?



Sol. Graph (a) has two real roots. It has two x-intercepts (i.e., the graph intersects the X-axis at two different points)
Graph (b) has no real roots. It has no x-intercepts. Both roots are complex.
Graph (c) has two real roots. But they are a double root.

4. Inspect the following graph to solve this equation : $x^2-4x-5=0$



Sol. The quadratic will have the value 0 at $x=-1$ and 5.

5. Find the zero(es) of the polynomial x^2-3 and verify the relationship between the zero(es) and the coefficients.

Sol. Recall the identity $a^2-b^2=(a-b)(a+b)$. Using it, we can write:

$$x^2-3=\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)$$

So, the value of x^2-3 is zero when $x=\sqrt{3}$ or $x=-\sqrt{3}$

Therefore, the zeroes of x^2-3 are $\sqrt{3}$ and $-\sqrt{3}$

Now, sum of zeroes $=\sqrt{3}-\sqrt{3}=0=\dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\left(\sqrt{3}\right)\left(-\sqrt{3}\right)=-3=\dfrac{-3}{1}=\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

6. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

Sol. We first arrange the terms of the dividend and the divisor in the decreasing order of their degrees. Recall that arranging the terms in this order is called writing the polynomials in standard form. In this example, the dividend is already in standard form, and the divisor, in standard form, is $x^2 + 2x + 1$.

$$\begin{array}{r|l} & \quad 3x - 5 \\ x^2+2x+1 & 3x^3 + x^2 + 2x + 5 \\ & 3x^3 + 6x^2 + 3x \\ & - \quad - \quad - \\ \hline & \quad -5x^2 - x + 5 \\ & \quad -5x^2 - 10x - 5 \\ & \quad + \quad + \quad + \\ \hline & \qquad 9x + 10 \end{array}$$

Step 1 : To obtain the first term of the quotient, divide the highest degree term of the dividend (i.e., $3x^3$) by the highest degree term of the divisor (i.e., x^2). The quotient is $3x$. Carry out the division process. The remainder is $-5x^2 - x + 5$.

Step 2 : Now, to obtain the second term of the quotient, divide the highest degree term of the new dividend (i.e., $-5x^2$) by the highest degree term of the divisor (i.e., x^2). This gives -5. Again carry out the division process.

Step 3 : The remainder is $9x + 10$. Now, the degree of $9x + 10$ is less than the degree of the divisor $x^2 + 2x + 1$. So, we cannot continue the division further.

So, the quotient is $3x - 5$ and the remainder is $9x + 10$. Also,

$(x^2 + 2x + 1) \times (3x - 5) + (9x + 10) = 3x^3 + 6x^2 + 3x - 5x^2 - 10x - 5 + 9x + 10 = 3x^3 + x^2 + 2x + 5$

7. Show that $x = 2$ is a root of $2x^3 + x^2 - 7x - 6$

Sol. $p(x) = 2x^3 + x^2 - 7x - 6$

$p(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 - 6 = 0$

Hence $x = 2$ is a root of $p(x)$.

8. If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$\Rightarrow \quad 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow \quad 128 - 176 + 12k - 180 = 0$

$\Rightarrow \quad 12k + 128 - 356 = 0 \Rightarrow \quad 12k = 228 \Rightarrow k = 19$

9. If $x = 2$ and $x = 0$ are roots of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$. Find the values of a and b.

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 16 - 20 + 2a + b = 0 \Rightarrow \quad 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\Rightarrow b = 0 \Rightarrow \quad 2a = 4 \Rightarrow \quad a = 2, b = 0$

10. Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by $g(x) = 1 - 2x$.

Sol. $1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Remainder $= f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4 = \frac{1}{8} - \frac{3}{2} + 1 - 4 = \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}$

11. The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $x + 2$ if the remainder in each case is the same, find the value of a.

Sol. Let $p(x) = ax^3 + 3x^2 - 13$ and $q(x) = 2x^3 - 5x + a$

When $p(x)$ and $q(x)$ are divided by $x + 2 = 0 \Rightarrow x = -2$

$\therefore p(-2) = q(-2) \Rightarrow a(-2)^3 + 3(-2)^2 - 13 = 2(-2)^3 - 5(-2) + a \Rightarrow -8a + 12 - 13 = -16 + 10 + a$

$\Rightarrow -9a = -5 \Rightarrow a = \frac{5}{9}$

12. Show that $x + 1$ and $2x - 3$ are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that $(x + 1)(2x - 3)$ are factors of $2x^3 - 9x^2 + x + 12$ it is sufficient to show that $p(-1)$ and $p(3/2)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = -2 - 9 - 1 + 12 = -12 + 12 = 0$

and $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12$

$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0$

13. Find α and β if $x + 1$ and $x + 2$ are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$

Sol. When we put $x + 1 = 0$ or $x = -1$ but $x + 2 = 0$ or $x = -2$ in $p(x)$

Then, $p(-1) = 0$ and $p(-2) = 0$

Therefore $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ .... (1)

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ .... (2)

By equalising both of the above question

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow \quad 2\alpha = -2 \Rightarrow \quad \alpha = -1$

put $\alpha = -1$ in eq. (1) $\Rightarrow \beta = -2(-1) - 2 = 2 - 2 = 0$

Hence, $\alpha = -1$, $\beta = 0$

14. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol. Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know if $p(x)$ is divided by $q(x)$ which is quadratic polynomial therefore if $p(x)$ is not exactly divisible by $q(x)$ then the remainder be $r(x)$ and degree of $r(x)$ is less than $q(x)$ or Divisor.

By long division method

Let, we added $ax + b$ (linear polynomial) in $p(x)$, so that $p(x) + ax + b$ is exactly divisible by $3x^2 + 7x - 6$

Hence, $p(x) + ax + b = s(x) = 3x^3 + x^2 - 22x + 9 + ax + b$

$$= 3x^3 + x^2 - x(22 - a) + (9 + b)$$

$$3x^2 + 7x - 6 \overline{)\, 3x^3 + x^2 - x(22 - a) + 9 + b}$$

$$3x^3 + 7x^2 - 6x$$
$$- \quad - \quad +$$

$$-6x^2 + 6x - (22 - a)x + 9 + b$$

or

$$-6x^2 + x(-16 + a) + 9 + b$$
$$-6x^2 - 14x + 12$$
$$+ \quad + \quad -$$

$$x(-2 + a) + (b - 3) = 0$$

Hence, $x(a - 2) + b - 3 = 0.x + 0$

$\Rightarrow$ $a - 2 = 0$ and $b - 3 = 0$

$\Rightarrow$ $a = 2$ or $b = 3$

Hence, if in $p(x)$ we added $ax + b$ or $2x + 3$ then it is exactly divisible by $3x^2 + 7x - 6$.

15. Factorise : $x^2 - 31x + 220$

Sol. $x^2 - 31x + 220 = x^2 - 2.\frac{31}{2}x + \left(\frac{31}{2}\right)^2 - \left(\frac{31}{2}\right)^2 + 220$

$$= \left(x - \frac{31}{2}\right)^2 - \frac{964}{4} + 220 = \left(x - \frac{31}{2}\right)^2 - \frac{81}{4}$$

$$= \left(x - \frac{31}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = \left(x - \frac{31}{2} + \frac{9}{2}\right)\left(x - \frac{31}{2} - \frac{9}{2}\right) = (x - 11)(x - 20)$$

16. Factorise : $-10x^2 + 31x - 24$

Sol. $-10x^2 + 31x - 24$

$$= -[10x^2 - 31x + 24] = -10\left[x^2 - \frac{31}{10}x + \frac{24}{10}\right] = -10\left[x^2 - 2\left(\frac{31}{20}\right)x + \left(\frac{31}{20}\right)^2 - \left(\frac{31}{20}\right)^2 + \frac{24}{10}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{961}{400} + \frac{24}{10}\right] = -10\left[\left(x - \frac{31}{20}\right)^2 - \frac{1}{400}\right]$$

$$= -10\left[\left(x - \frac{31}{20}\right)^2 - \left(\frac{1}{20}\right)^2\right] = -10\left[x - \frac{31}{20} + \frac{1}{20}\right]\left[x - \frac{31}{20} - \frac{1}{20}\right]$$

$$= -10\left(\frac{2x - 3}{2}\right)\left(\frac{5x - 8}{5}\right) = -(2x - 3)(5x - 8) = (3 - 2x)(5x - 8)$$

17. Factorise: $2x^2+12\sqrt{2}x+35$

Sol. Product $ac=70$ and $b=12\sqrt{2}$

$\therefore$ Split $12\sqrt{2}$ as $7\sqrt{2}, 5\sqrt{2}$

$\Rightarrow\ 2x^2+12\sqrt{2}x+35=2x^2+7\sqrt{2}x+5\sqrt{2}x+35$

$=\sqrt{2}x\left[\sqrt{2}x+7\right]+5\left[\sqrt{2}x+7\right]=\left[\sqrt{2}x+5\right]\left[\sqrt{2}x+7\right]$

18. Factorise: $x^2-14x+24$

Sol. Product $ac=24$ and $b=$ ş 14

$\therefore$ Split the middle term as ş 12 and ş 2

$\Rightarrow x^2$ ş $14x+24=x$ ş $12x$ ş $2x+24=x(x$ ş $12)$ ş $2(x$ ş $12)=(x$ ş $12)(x$ ş $2)$

19. Factorise: $x^2-\frac{13}{24}x-\frac{1}{12}$

Sol. $x^2-\frac{13}{24}x-\frac{1}{12}=\frac{1}{24}[24x^2-13x-2]$

Product $ac=$ ş 48 and $b=$ ş 13

$\therefore$ We split the middle term as ş $16x+3x=\frac{1}{24}[24x^2-16x+3x-2]$

$=\frac{1}{24}[8x(3x-2)+1(3x-2)]=\frac{1}{24}(3x-2)(8x+1)$

20. Factorise: $\frac{3}{2}x^2-8x-\frac{35}{2}$

Sol. $\frac{3}{2}x^2-8x-\frac{35}{2}=\frac{1}{2}(3x^2-16x-35)=\frac{1}{2}(3x^2-21x+5x-35)$

$=\frac{1}{2}[3x(x-7)+5(x-7)]=\frac{1}{2}(x-7)(3x+5)$

21. If $f(x)=2x^3-13x^2+17x+12$ then find out the value of $f(-2)$ and $f(3)$

Sol. $f(x)=2x^3$ ş $13x^2+17x+12$

$f($ş$2)=2($ş$2)^3$ ş $13($ş$2)^2+17($ş$2)+12=$ ş16 ş 52 ş $34+12=$ ş 90

$f(3)=2(3)^3$ ş $13(3)^2+17(3)+12=54$ ş $117+51+12=0$

22. Using factor theorem, factorize:
$p(x)=2x^4-7x^3-13x^2+63x-45$

Sol. If we put $x=1$ in $p(x)$

$p(1)=2(1)^4$ ş $7(1)^3$ ş $13(1)^2+63(1)$ ş 45

$=2$ ş 7 ş $13+63$ ş $45=65$ ş $65=0$

$\therefore x=1$ or x ş 1 is a factor of $p(x)$.

Similarly, if we put $x = 3$ in $p(x)$

$p(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

$162 - 189 - 117 + 189 - 45 = 162 - 162 = 0$

Hence, $x = 3$ or $x - 3 = 0$ is the factor of $p(x)$

$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

$\therefore \quad p(x) = 2x^3(x-1) - 5x^2(x-1) - 18x(x-1) + 45(x-1)$

$2x^4 - 2x^3 - 5x^3 + 5x^2 - 18x^2 + 18x + 45x - 45$

$p(x) \quad = (x-1)(2x^3 - 5x^2 - 18x + 45)$

$= (x-1)[(2x^2(x-3) + x(x-3) - 15(x-3)]$

$= (x-1)[(2x^3 - 6x^2 + x^2 - 3x - 15x + 45]$

$= (x-1)(x-3)(2x^2 + x - 15)$

$= (x-1)(x-3)(2x^2 + 6x - 5x - 15)$

$= (x-1)(x-3)[2x(x+3) - 5(x+3)]$

$= (x-1)(x-3)(x+3)(2x-5)$

23. What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$.

Sol. In above problem, we see that if in $p(x) = x^3 - 6x^2 - 15x + 80$ we subtracted $ax + b$ so that is exactly divisible by $x^2 + x - 12$

$s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$

$= x^3 - 6x^2 - (15 + a)x + (80 - b)$

Dividend = Divisor × quotient + remainder

But remainder will be zero.

$\therefore$ Dividend = Divisor × quotient

$s(x) = (x^2 + x - 12) \times$ quotient

$s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$x(x^2 + x - 12) - 7(x^2 + x - 12)$

$= x^3 + x^2 - 7x^2 - 12x - 7x + 84$

$= x^3 - 6x^2 - 19x + 84$

Hence, $x^3 - 6x^2 - 19x + 84 = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$-15 - a = -19 \quad \Rightarrow a = +4$

and $80 - b = 84 \quad \Rightarrow b = -4$

Hence, if in $p(x)$ we subtracted $4x - 4 = (ax + b)$ then it is exactly divisible by $x^2 + x - 12$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Polynomials of degrees 1, 2 and 3 are called ...., ... and .... polynomials respectively.
2. The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points, where the graph of $y = p(x)$ intersects the ....axis.
3. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most .... zeroes.
4. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $\alpha+\beta = \frac{-b}{\ldots\ldots}$ & $\alpha\beta = \frac{c}{\ldots\ldots}$
5. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then $\alpha+\beta+\gamma = \frac{-b}{\ldots\ldots}$
6. Zero of a polynomial is always ....
7. A polynomial of degree n has at the most .... zeros.

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. For polynomials $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\,q(x) + r(x)$, where $r(x) = 0$ or degree $r(x) <$ degree $g(x)$.
2. Sum of zeroes of quadratic polynomial $= -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$
3. $\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$ is a polynomial
4. $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}}$ is a polynomial, $x \neq 0$
5. Product of zeroes of quadratic polynomial $= -\frac{\text{constant term}}{(\text{coefficient of } x^2)}$
6. Every polynomial is a binomial
7. A polynomial cannot have more than one zero
8. The degree of the sum of two polynomials each of degree 5 is always 5.
9. Dividend = Divisor × Quotient + Remainder
10. 3, –1, 1/3 are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$.
11. Zeroes of quadratic polynomial $x^2 + 7x + 10$ are 2 and –5
12. Sum of zeroes of $2x^2 - 8x + 6$ is – 4

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II gives polynomial (quadratic) for zeroes given in column I, match them correctly.

Column I	Column II
(A) 3 and –5	(p) $x^2 - 25$
(B) $5+\sqrt{2}$ and $5-\sqrt{2}$	(q) $x^2 + 2x - 15$
(C) – 9 and 1/9	(r) $x^2 + (80/9)x - 1$
(D) 5 and – 5	(s) $x^2 - 10x + 21$

2. Column II give remainder for division of polynomial given in column I, match them correctly.

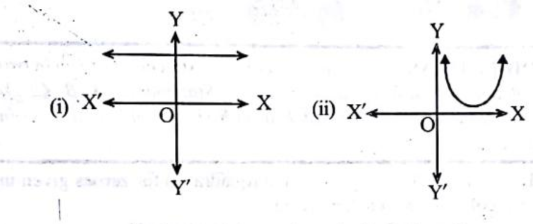
Column I	Column II
(A) $\frac{x^3 - 3x^2 + x + 2}{x^2 - x + 1}$	(p) 8
(B) $\frac{x^3 - 3x^2 + 5x - 3}{x + 2}$	(q) $x - 5$
(C) $\frac{x^4 - 6x^3 + 16x^2 - 25x + 10}{x^2 - 2x + 5}$	(r) –33
(D) $\frac{x^4 - 3x^2 + 4x + 5}{x^2 - x + 1}$	(s) $-2x + 4$

VSAQ Very Short Answer Questions :

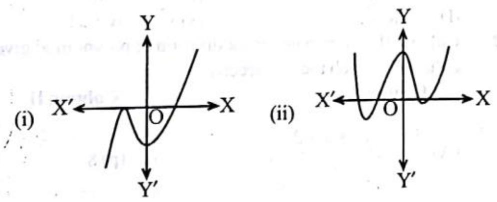
DIRECTIONS : *Give answer in one word or one sentence.*

1. Factorise : $x^4 + x^2y^2 + y^4$
2. Factorise : $a^6 + 4a^3 - 1$
3. Find the quadratic polynomial with the sum and the product of its zeros as $\frac{1}{4}$ and -1 respectively.
4. Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

5. Factorise : $81\,a^2b^2c^2 + 64\,a^6b^2 - 144\,a^4b^2c$
6. Factorise : $4\,(2a + 3b - 4c)^2 - (a - 4b + 5c)^2$
7. Let α, β, γ be the roots of $2x^3 - 3x^2 + 6x + 1$. Find the value of $\Sigma\alpha\beta$.
8. Let α, β, γ be the roots of $2x^3 - 3x^2 + 6x + 1$. Find the value of $\Sigma\alpha^2$.
9. The graph of $y = p(x)$ are given in figure, below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



10. The graph of $y = p(x)$ are given in fig. below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



11. Find out the degrees of following polynomials.
   (i) $p(x) = 7x + 5x^2 - \sqrt{3}$
   (ii) $q(x) = 5x^4 - 32x^2 + 5x - 8$
   (iii) $r(x) = x^3 - x^6 - 5\sqrt{2}$
   (iv) $h(x) = \frac{1}{2} - 3x$
12. Find the zeroes of the polynomial
   $p(x) = x^2 - 10x - 75$
13. Write a quadratic polynomial, the sum and product of whose zeroes are -7 and 10 respectively.
14. Give example of polynomials $p(x), g(x), q(x), r(x)$ which satisfy the division algorithm and degree $q(x) = \deg r(x)$
15. Check whether $(3x - 5)$ is a factor of polynomial.
   $6x^4 - 22x^3 + 41x^2 - 38x + 5$?
16. Show that 2 is not a zero of the polynomial,
   $P(x) = x^2 + 2x + 5$

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.
2. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.
3. Factorise :
$$\left(3a - \frac{1}{b}\right)^2 - 6\left(3a - \frac{1}{b}\right) + 9 + \left(c + \frac{1}{b} - 2a\right)\left(3a - \frac{1}{b} - 3\right)$$
4. Find the polynomial having $5 \pm \sqrt{3}$ as its zeroes.
5. Factorise : $4x^2 + \frac{1}{4x^2} + 2 - 9y^2$
6. Factorise : $a^4 + \frac{1}{a^4} - 3$
7. Factorise : $64\,a^{13}b + 343ab^{13}$
8. Factorise : $x^3 - 6x^2 + 32$
9. Factorise : $a^3 + b^3 + c^3 - 3abc$
10. Factorise : $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$
11. Factorise : $y^4 + y^2 - 2ay + 1 - a^2$
12. Find the condition which must be satisfied by the coefficients of the polynomial $f(x) = x^3 - \ell x^2 + mx - n$. Given that the sum of the two zeroes is zero.
13. Given that the sum of the zeroes of the polynomial $(a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1. Find the product of its zeroes.
14. If $ax^3 + bx + c$ has a factor of the form $x^2 + px + 1$, show that $a^2 - c^2 = ab$.
15. If α, β, γ are the zeroes of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.
16. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?
17. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, find a and b.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Verify that 2, 1, –1 are the zeros of $x^3 - 2x^2 - x + 2$. Also verify the relationship. between the zeros and the coefficients.
2. Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

3. Factorise : $p^3q^2x^4 + 3p^2qx^3 + 3px^2 + \frac{x}{q} - q^2r^3x$

4. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

6. If the two zeroes of the polynomial $f(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, then obtain the other two.

7. Let α, β be the zeroes of the polynomial $ax^2 + bx + c$, then find other one polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

8. Find the zeroes of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$
Also, verify the relationship between the zeros and its coefficients.

9. Find the family of cubic polynomial having zeroes $-3, -1$ and 2.

10. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial h(x), the quotient and the remainder are $(x - 2)$ and $(-2x + 4)$ respectively. Determine h(x).

11. Draw the graphs of the polynomial $f(x) = x^3 - 4x$.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. When $(x^5 + 1)$ is divided by $(x - 2)$, the remainder is –
(a) 5 (b) 17 (c) 31 (d) 33

2. If the expression $(x^2 - x + c)$ when divided by $(x + 1)$ leaves remainder 3, then the value of c is –
(a) 0 (b) 1 (c) 2 (d) 3

3. If $(x + 1)$ and $(x - 2)$ are the factors of the expression $(2x^3 - px^2 + x + q)$, then the values of p and q are given by –
(a) $p = 5, q = 2$ (b) $p = 7, q = 8$
(c) $p = 7, q = 10$ (d) $p = 15, q = 12$

4. The value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously, is –
(a) 2 (b) –2 (c) –1 (d) 1

5. The value of k for which the polynomial $2x^3 - x^2 + 3x - k$ is divisible by $(x - 1)$ is –
(a) 1 (b) 2 (c) 3 (d) 4

6. A positive integer is said to be a prime if it is not divisible by any positive integer other than itself and one. Let p be a prime number strictly greater than 3. Then, when $p^2 + 17$ is divided by 12, the remainder is –
(a) 6 (b) 1 (c) 0 (d) 8

7. The value of the polynomial $x^8 - x^5 + x^2 - x + 1$ is –
(a) positive for all the real numbers
(b) negative for all the real numbers
(c) 0
(d) depends on value of x

8. If $(x - 3)$, $(x - 3)$ are factors of $x^3 - 4x^2 - 3x + 18$; then the other factor is :
(a) $x + 2$ (b) $x + 3$
(c) $x - 2$ (d) $x + 6$

9. If $f\left(\frac{-3}{4}\right) = 0$; then for $f(x)$, which of the following is a factor?
(a) $3x - 4$ (b) $4x + 3$
(c) $-3x + 4$ (d) $4x - 3$

10. The quotient when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ divided by $(x - 3)$ is
(a) $3x^3 + 4x^2 + 22x + 77$ (b) $77x^3 + 22x^2 + 4x + 3$
(c) $3x^2 + 4x^3 + 22x + 77$ (d) None of these

11. If $x = 0.\overline{7}$, then 2x is –
(a) $1.\overline{4}$ (b) $1.\overline{5}$ (c) $1.\overline{54}$ (d) $\overline{1.45}$

12. Lowest value of $x^2 + 4x + 2$ is –
(a) 0 (b) –2 (c) 2 (d) 4

13. If $a^3 - 3a^2b + 3ab^2 - b^3$ is divided by $(a - b)$, then the remainder is
(a) $a^2 - ab + b^2$ (b) $a^2 + ab + b^2$
(c) 1 (d) 0

14. The remainder when $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$ is :
(a) –1 (b) 1 (c) 0 (d) –2

15. If $(x - 1)$, $(x + 1)$ and $(x - 2)$ are factors of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of p is
(a) 1 (b) 2 (c) 3 (d) 4

16. Maximum value of $2 - 4x - x^2$ is –
(a) 2 (b) 4 (c) 6 (d) 8

17. A quadratic polynomial when divided by $x + 2$ leaves a remainder of 1 and when divided by $x - 1$, leaves a remainder of 4. What will be the remainder if it is divided by $(x + 2)(x - 1)$?

(a) 1 (b) 4 (c) $x + 3$ (d) $x - 3$

18. If α, β, γ be the zeroes of the polynomial $ax^2 + bx^2 + cx + d$, then the value of $\alpha\beta + \beta\gamma + \gamma\alpha$ is –

(a) $-b/a$ (b) c/a (c) $-c/a$ (d) d/a

19. α, β, γ are the zeroes of the cubic polynomial $x^3 - 2x^2 + qx - 6$ is 4, then a is equal to –

(a) 3/2 (b) –3/2 (c) 2/3 (d) –2/3

MTOC *More than One Correct :*

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following given options is/are not correct?

(a) $\frac{2}{x} + 3$ is a polynomial

(b) $\sqrt{x} + 5$ is a polynomial

(c) $\frac{2}{3x-4}$ is a polynomial

(d) $\sqrt{5}x^2 + \frac{1}{2}x + \frac{3}{7}$ is a polynomial

2. Which of the following given options is/are not correct?

(a) Degree of a zero polynomial is '0'

(b) Degree of a zero polynomial is not defined

(c) Degree of a constant polynomial is not defined.

(d) A polynomial of degree n must have n zeroes

3. Which of the following given options is/are incorrect?

If $p(x) = q(x)\, g(x) + r(x)$ (By Division Algorithm) where $p(x)$, $g(x)$ are any two polynomials with $g(x) \neq 0$, then

(a) $r(x) = 0$ always

(b) degree of $r(x) <$ degree of $g(x)$ always

(c) either $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

(d) $r(x) = g(x)$

4. Which of the following is a Binomial ?

(a) $x + 2$ (b) $x^2 + 2x + 3$

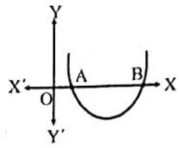
(c) $4x^2$ (d) $x^2 + 8$

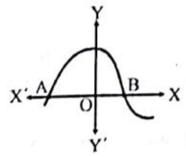
5. Which of the following is not a Polynomial?

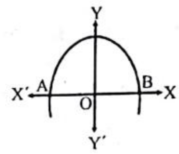
(a) $x^2 + \frac{1}{x}$ (b) $2x^2 - 3\sqrt{x} + 1$

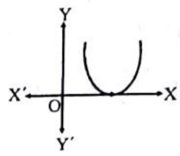
(c) $x^3 - 3x + 1$ (d) $2x^{\frac{3}{2}} - 5x$

6. Which of the following is/are not graph of a quadratic?

(a) 

(b) 

(c) 

(d) 

7. Which of the following is/are incorrect?

(a) $x^2 + 7x + 5$ is a linear polynomial

(b) $4x^2$ is a monomial

(c) $x + 2$ is monomial

(d) $x^2 + 2x + 3$ is a bionomial

PBQ *Passage Based Questions :*

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE I

If α, β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

1. If α, β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(a) $p - q$ (b) $p + q$

(c) $\frac{p}{q}$ (d) $\frac{q}{p}$

2. If α, β are the zeroes of the quadratic polynomial $x^2 + x - 2$, then $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 =$

(a) $\frac{9}{4}$ (b) $\frac{-9}{4}$

(c) $\frac{2}{5}$ (d) $\frac{-2}{5}$

3. If α, β are the zeroes of the quadratic polynomial $f(x) = x^2 - 5x + 4$, then $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta =$

(a) $\frac{27}{4}$ (b) $-\frac{27}{4}$

(c) $\frac{4}{27}$ (d) $-\frac{4}{27}$

PASSAGE II

If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$, then

$$\sum\alpha = -\frac{b}{a},\ \sum\alpha\beta = \frac{c}{a},\ \alpha\beta\gamma = -\frac{d}{a}$$

1. If α, β, γ are the zeroes of $x^3 - 5x^2 - 2x + 24$ and $\alpha\beta = 12$, then $\gamma =$
   (a) 2 (b) −2 (c) 3 (d) −3
2. If $a - b, a, a + b$ are the roots of $x^3 - 3x^2 + x + 1$, then $a + b^2 =$
   (a) 3 (b) 4 (c) 5 (d) 2
3. If two zeroes of the polynomial $x^3 - 5x^2 - 16x + 80$ are equal in magnitude but opposite in sign, then zeroes are
   (a) 4, −4, 5 (b) 3, −3, 5
   (c) 2, −2, 5 (d) 1, −1, 5

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** If α, β, γ are the zeroes of $x^3 - 2x^2 + qx - r$ and $\alpha + \beta = 0$, then $2q = r$
   Reason : If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$, then
   $$\alpha+\beta+\gamma = -\frac{b}{a},\ \alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a},\ \alpha\beta\gamma = -\frac{d}{a}$$
2. **Assertion :** If one zero of polynomial $p(x) = (k^2+4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$
   Reason : If $x - \alpha$ is a factor of $p(x)$, then $p(\alpha) = 0$ i.e. α is a zero of $p(x)$
3. **Assertion :** The polynomial $x^4 + 4x^2 + 5$ has four zeroes.
   Reason : If $p(x)$ is divided by $(x - k)$, then the remainder = $p(k)$
4. **Assertion :** $x^3 + x$ has only one real zero.
   Reason : A polynomial of nth degree must have n real zeroes.
5. **Assertion :** If 2, 3 are the zeroes of a quadratic polynomial, then polynomial is $x^2 - 5x + 6$.
   Reason : If α, β are the zeroes of a monic quadratic polynomial, then polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.
6. **Assertion :** Degree of a zero polynomial is not defined.
   Reason : Degree of a non-zero constant polynomial is '0'
7. **Assertion :** Zeroes of $f(x) = x^2 - 4x - 5$ are 5, − 1
   Reason : The polynomial whose zeroes are $2+\sqrt{3}, 2-\sqrt{3}$ is $x^2 - 4x + 7$
8. **Assertion :** $x^2 + 4x + 5$ has two zeroes
   Reason : A quadratic polynomial can have at the most two zeroes.

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

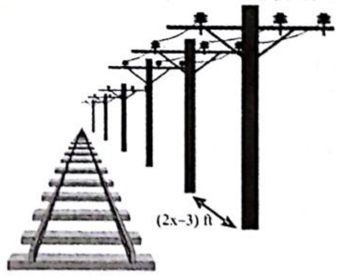
1. **Column II gives zeros of the polynomials given in column-I**

Column-I	Column-II
(A) $4 - x^2$	(p) 7
(B) $x^3 - 2x^2$	(q) −2
(C) $6x^2 - 3 - 7x$	(r) 2
(D) $-x + 7$	(s) 3/2
	(t) 0
	(u) −1/3

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Obtain all zeroes of $x^4 - 3x^3 - 7x^2 + 9x + 12$ if two of its zeroes are $\pm\sqrt{3}$.
2. Find the zeroes of the cubic polynomial $x^3 + 6x^2 + 11x + 6$ and verify the relationship between the zeroes and the coefficient.
3. If α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of
   $$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta.$$
4. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2\alpha+3\beta$ and $3\alpha+2\beta$.
5. Form a cubic polynomial having sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes are 2, −7 and −14 respectively.
6. In the figure, telephone poles were installed at every $(2x - 3)$ m along a stretch of railboard track $(8x^3 - 6x^2 + 5x - 21)$ m long. How many poles were used ?



7. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.
8. Draw the graphs of the quadratic polynomial
   $f(x) = 3 - 2x - x^2$

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. linear, quadratic, cubic
2. x
3. 3
4. a, a
5. a
6. Zero
7. *n*

TRUE / FALSE

1. True
2. True
3. False, because the exponent of the variable is not a whole number.
4. True, because $\frac{6\sqrt{x}+x^{\frac{3}{2}}}{\sqrt{x}} = 6+x$, which is a polynomial.
5. False
6. False, x^3+x+1 is a polynomial but not a binomial.
7. False, a polynomial can have any number of zeroes. It depends upon the degree of the polynomial.
8. False, x^5+1 and $-x^5+2x+3$are two polynomials of degree 5 but the degree of the sum of the two polynomials is 1.
9. True
10. True
11. False
12. False

MATCH THE FOLLOWING :

1. (A) → q; (B) → s; (C) → r; (D) → p
2. (A) → s; (B) → r; (C) → q; (D) → p

VERY SHORT ANSWER QUESTIONS :

1. $x^4+x^2y^2+y^4=(x^2)^2+2.x^2.y^2+(y^2)^2-x^2y^2$
$=(x^2+y^2)^2-(xy)^2=(x^2+y^2+xy)(x^2+y^2-xy)$

2. $a^6+a^3+3a^3-1$
$=(a^2)^3+(a)^3+(-1)^3-3(a^2)(a)(-1)$
$=(a^2+a-1)(a^4+a^2+1+a^2+a-a^3)$
$=(a^2+a-1)(a^4+2a^2-a^3+a+1)$

3. Let the polynomial be ax^2+bx+c and its zeros be α & β.

We have, $\alpha+\beta=\frac{1}{4}=-\frac{b}{a}$, $\alpha\beta=-1=\frac{c}{a}$

$\Rightarrow -\frac{b}{a}=\frac{1}{4}$ and $\frac{c}{a}=-1$

We have, $a+4$, $b=-1$, $c=-4$

The polynomial is $4x^2-x-4$

4. Let the quadratic polynomial be ax^2+bx+c, and its zeroes be α and β.

We have $\alpha+\beta=-3=\frac{-b}{a}$ and $\alpha\beta=2=\frac{c}{a}$

If $a=1$, then $b=3$ and $c=2$.

So, one quadratic polynomial which fits the given conditions is x^2+3x+2.

5. $81\,a^2b^2c^2+64\,a^6b^2-144\,a^4b^2c$
$=[9\,abc]^2-2.[9abc][8a^3b]+[8a^3b]^2$
$=[9abc-8a^3b]=a^2b^2[9c-8a^2]^2$

6. $4(2a+3b-4c)^2-(a-4b+5c)^2$
$=[2(2a+3b-4c)]^2-(a-4b+5c)^2$
$=[4a+6b-8c+a-4b+5c][4a+6b-8c-a+4b-5c]$
$=[5a+2b-3c][3a+10b-13c]$

7. $\Sigma\alpha\beta=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{\text{Coefficient of x}}{\text{Coefficient of }x^3}=\frac{6}{2}=3$

8. $\Sigma\alpha\beta=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{\text{coefficient of x}}{\text{coefficient of }x^3}=\frac{6}{2}=3$

9. (i) No zeroes (ii) No zeroes

10. (i) 2 zeroes (ii) 4 zeroes

11. (i) p(x) is of degree 2 as highest powered term is $5x^2$ .i.e., p(x) is quadratic.
(ii) q(x) is of degree 4.
(iii) r(x) is of degree 6.
(iv) h(x) is of degree 1. i.e. h(x) is linear.

12. We have, $p(x)=x^2-10x-75=x^2-15x+5x-75=x(x-15)+5(x-15)=(x-15)(x+5)$
$\therefore p(x)=(x-15)(x+5)$
So, $p(x)=0$ when $x=15$ or $x=-5$. Therefore required zeroes are 15 and -5.

13. Here, let α, β be zeroes then, $\alpha+\beta=-7$, $\alpha\beta=10$
So, required polynomial p(x) is given by
$p(x)=(x-\alpha)(x-\beta)$
$=x^2-(\alpha+\beta)x+\alpha\beta=x^2-(-7)x+10$
$\therefore p(x)=x^2+7x+10$

14. $2x^3-4x^2+5=(2x^2-1)(x-2)+(x+3)$
Here, $p(x)=2x^3-4x^2+5$
$g(x)=2x^2-1$
$q(x)=x-2$ [degree q(x) = 1]
$r(x)=x+3$ [degree r(x) = 1]

15. **[Hint :** Yes, $(3x-5)$ is a factor of given polynomial as, we get, remainder $r(x)=0$.]

16. $P(x)=x^2+2x+5$

then $P(2)=(2)^2+2(2)+5=4+4+5=13\neq 0$

since, $P(2)\neq 0$, 2 is not a zero of the polynomial $P(x)$.

SHORT ANSWER QUESTIONS :

1. We have, $6x^2-7x-3=6x^2-9x+2x-3$

$=3x(2x-3)+1(2x-3)=(2x-3)(3x+1)$

Value of $6x^2-3-7x$ is zero

when $2x-3=0$ or $3x+1=0$

$\Rightarrow x=\frac{3}{2}$ or $x=-\frac{1}{3}$ $\therefore$ Zeroes are $\frac{3}{2}$ and $-\frac{1}{3}$

Now sum of zeroes

$$=\frac{3}{2}-\frac{1}{3}=\frac{9-2}{6}=\frac{7}{6}=\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of zeroes

$$=\frac{3}{2}\times\frac{-1}{3}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{constant term}}{\text{coefficient of } x^2}$$

2. We have : $x^2+7x+10=(x+2)(x+5)$

So, the value of $x^2+7x+10$ is zero when $x+2=0$ or $x+5=0$, i.e., when $x=-2$ or $x=-5$.

Therefore, the zeroes of $x^2+7x+10$ are -2 and -5. Now, sum of zeroes $=-2+(-5)=-(7)$

$$=\frac{-(7)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

product of zeroes $=-2+(-5)=-(7)$

$$=\frac{10}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

3. $$\left(3a-\frac{1}{b}\right)^2-6\left(3a-\frac{1}{b}\right)+9+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$$

$$=\left(3a-\frac{1}{b}\right)^2-2.3\left(3a-\frac{1}{b}\right)+(3)^2+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$$

$$=\left(3a-\frac{1}{b}-3\right)^2+\left(c+\frac{1}{b}-2a\right)\left(3a-\frac{1}{b}-3\right)$$

$$=\left(3a-\frac{1}{b}-3\right)\left[3a-\frac{1}{b}-3+c+\frac{1}{b}-2a\right]$$

$$=\left(3a-\frac{1}{b}-3\right)[a+c-3]$$

4. Let $\alpha=5+\sqrt{3}$, $\beta=5-\sqrt{3}$

we know, $p(x)$ having zeroes as α, β is given by

$p(x)=(x-\alpha)(x-\beta)$

$\therefore$ $p(x)=[x-(5+\sqrt{3})][x-(5-\sqrt{3})]$

$=x^2-(5+\sqrt{3})x-(5-\sqrt{3})x+(5+\sqrt{3})(5-\sqrt{3})$

$=x^2-(5+\sqrt{3}+5-\sqrt{3})x+(5^2-3)$

$\therefore$ $p(x)=x^2-10x+22$

5. $$4x^2+\frac{1}{4x^2}+2-9y^2=(2x)^2+2.2x\left(\frac{1}{2x}\right)+\left(\frac{1}{2x}\right)^2-(3y)^2$$

$$=\left(2x+\frac{1}{2x}\right)^2-(3y)^2=\left(2x+\frac{1}{2x}+3y\right)\left(2x+\frac{1}{2x}-3y\right)$$

6. $$(a^2)^2+\left(\frac{1}{a^2}\right)^2-2(a^2)\left(\frac{1}{a^2}\right)-1$$

$$=\left(a^2-\frac{1}{a^2}\right)^2-(1)^2=\left(a^2-\frac{1}{a^2}+1\right)\left(a^2-\frac{1}{a^2}-1\right)$$

7. $64a^{13}b+343ab^{13}=ab[64a^{12}+343ab^{12}]$

$=ab[(4a^4)^3+(7b^4)^3]=ab[4a^4+7b^4][(4a^4)^2-(4a^4)(7b^4)+(7b^4)^2]$

$=ab[4a^4+7b^4][16a^8-28a^4b^4+49b^8]$

8. $x^3-6x^2+32=x^3+8+24-6x^3$

$=[(x)^3+(2)^3]+6[4-x^2]$

$=(x+2)[x^2-2x+4]+6[2+x][2-x]$

$=(x+2)[x^2-2x+4+6(2-x)]$

$=(x+2)[x^2-2x+4+12-6x]$

$=(x+2)[x^2-8x+16]$

$=(x+2)(x-4)^2$

9. $a^3+b^3+c^3-3abc$

$=(a^3+b^3)+c^3-3abc=(a+b)^3-3ab(a+b)+c^3-3abc$

$=[(a+b)^3+c^3]-\{3ab(a+b)+3abc\}$

$=(a+b+c)[(a+b)^2+c^2-(a+b)c]-\{3ab(a+b+c)\}$

$=(a+b+c)[a^2+b^2+2ab+c^2-ac-bc-3ab]$

$=(a+b+c)[a^2+b^2+c^2-ab-bc-ca]$

10. Let $a^2-b^2=A$, $B=b^2-c^2$ and $C=c^2-a^2$

$A^3+B^3+C^3=3ABC$ If $A+B+C=0$

$\therefore$ $a^2-b^2+b^2-c^2+c^2-a^2=0$

$\therefore$ $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3=0$

$=3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

11. We arrange the expression in powers of a.

We have the given expression.

$=-a^2-2ay+1+y^2+y^4$

$=-[a^2+2ay-y^2-y^4-1]$

$=-[a^2+2ay+y^2-2y^2-y^4-1]$

$=-[(a+y)^2-(y^2+1)^2]$

$=-[y^2+1+a+y][-y^2-1+a+y]$

$=[y^2+1+a+y][y^2+1-a-y]$

12. Let α, β and γ be the zeroes of the polynomial $f(x)$ such that $\alpha+\beta=0$.

Now, $\alpha+\beta+\gamma=\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

Since, γ is a zero of the polynomial f(x). Therefore, $f(\gamma)=0$

$\Rightarrow \gamma^3-\ell\gamma^2+m\gamma-n=0 \quad [\because \gamma=\ell]$

$\Rightarrow \ell^3-\ell^3+m\ell-n=0 \Rightarrow m\ell=n$

This is the required condition.

13. Since the sum of the zeroes of the quadratic polynomial $(a+1)x^2+(2a+3)x+(3a+4)$ is -1.

$\therefore \dfrac{-(2a+3)}{a+1}=-1$

$\left[\because \text{Sum of zeroes} = \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}\right]$

$\Rightarrow a+1=2a+3 \Rightarrow a=-2$

Now, product of zeroes $=\dfrac{3a+4}{a+1}=\dfrac{3(-2)+4}{-2+1}=2$.

$\left[\because \text{Product of zeroes} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}\right]$

14. By actual division, we get,

$\dfrac{ax^3+bx+c}{x^2+px+1}=ax-ap+\dfrac{R(x)}{x^2+px+1}$

Remainder polynomial, $R(x)=(b-a+ap^2)x+c+ap$

If ax^3+bx+c has a factor of the form x^2+px+1, then R (x) must be identically zero, if $b-a+ap^2=0$ and $c+ap=0$

Eliminating p from these equations, we get

$b-a+a\left(-\dfrac{c}{a}\right)^2=0$ or $a^2-c^2=ab$

15. Polynomial is ax^3+bx^2+cx+d and zeroes are α, β and γ

Let $p=\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma} \Rightarrow p=\dfrac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}$; $\alpha\beta\gamma=-d/a$ and

$\alpha\beta+\beta\gamma+\gamma\alpha=c/a \Rightarrow p=\dfrac{c/a}{-d/a}=-\dfrac{c}{d}$

16. The given polynomial is $f(x)=x^4+2x^3-2x^2+x-1$. To make this polynomial exactly divisible by x^2+2x-3, let us add p then $x^4+2x^3-2x^2+x-1+p$ is exactly divisible by x^2+2x-3.

$$\begin{array}{r|l}
x^2+2x-3 & x^4+2x^3-2x^2+x-1+p \;\big(\, x^2 \\
 & x^4+2x^3-3x^2 \\
 & -\quad -\quad + \\ \hline
 & x^2+x-1+p
\end{array}$$

For remainder to be zero, $x^2+2x-3=x^2+x-1+p$

So, $p=x-2$,

So, $x-2$ to be added to make the polynomial divisible by x^2+2x-3.

17. Zeroes of polynomial $p(x)=x^3-3x^2+x+1$ are $a-b$, a and $a+b$,

So, $a-b+a+a+b=-\dfrac{(-3)}{1} \Rightarrow 3a=3 \Rightarrow a=1$.

$\alpha\beta\gamma=\dfrac{d}{a} \Rightarrow (a-b)a(a+b)=\dfrac{-1}{1}=-1$

$\Rightarrow (1-b)\times 1\times(1+b)=-1$

$1-b^2=-1 \Rightarrow b^2=2 \Rightarrow b=\pm\sqrt{2}$ So, $a=1$, $b=\pm\sqrt{2}$

LONG ANSWER QUESTIONS :

1. Let $p(x)=x^3-2x^2-x+2$

Comparing it with ax^3+bx^2+cx+d,

We get, $a=1, b=-2, c=-1, d=2$

Now, $p(2)=2^3-2(2)^2-2+2=8-8-2+2=0$

$p(1)=1^3-2(1)^2-1+2=1-2-1+2=0$

$p(-1)=(-1)^3-2(-1)^2+1+2=-1-2+1+2=0$

$\Rightarrow$ 2, 1, -1 are the zeros of the polynomial p (n)

Let $\alpha=2, \beta=1, \gamma=-1$

$\alpha+\beta+\gamma=2+1-1=2=\dfrac{(-2)}{1}=-\dfrac{b}{a}$

$\alpha\beta+\beta\gamma+\gamma\alpha=2\times 1+1(-1)+(-1)2$

$=2-1-2=-1=-\dfrac{1}{1}=\dfrac{c}{a}$

$\alpha\beta\gamma=2(1)(-1)=-2=-\dfrac{2}{1}=-\dfrac{d}{a}$

2. Note that the given polynomials are not in standard form. To carry out division, we first write both the dividend and divisor in decreasing orders of their degrees.

So, dividend $=-x^3+3x^2-3x+5$ & divisor $=-x^2+x-1$.

$$\begin{array}{r|l}
 & \quad x-2 \\ \hline
-x^2+x-1 & -x^3+3x^2-3x+5 \\
 & -x^3+x^2-x \\
 & +\quad -\quad + \\ \hline
 & \quad 2x^2-2x+5 \\
 & \quad 2x^2-2x+2 \\
 & \quad -\quad +\quad - \\ \hline
 & \qquad\qquad 3
\end{array}$$

We stop here since degree $(3)=0<2=\text{degree}(-x^2+x-1)$.

So, quotient $=x-2$, remainder $=3$.

Now, Divisor $\times$ Quotient + Remainder

$=(-x^2+x-1)(x-2)+3$

$=-x^3+x^2-x+2x^2-2x+2+3$

$=-x^3+3x^2-3x+5=$ Dividend

In this way, the division algorithm is verified.

3. In above probelm, if we take common then it may become in the form of a^3+b^3

$$\therefore p^3q^2x^4 + 3p^2qx^3 + 3px^2 + \frac{x}{q} - q^2r^3x$$

$$= \frac{x}{q}\,[p^3q^3x^3 + 3p^2q^2x^2 + 3pqx + 1 - q^3r^3]$$

$$= \frac{x}{q}\,[(pqx)^3 + 3(pqx)^2.1 + 3pqx.(1)^2 + (1)^3 - q^3r^3]$$

Let $pqx = A$ and $1 = B$

$$= \frac{x}{q}\,[A^3 + 3A^2B + 3AB^2 + B^3 - q^3r^3]$$

$$= \frac{x}{q}\,[(pqx+1)^3 - (qr)^3] = \frac{x}{q}\,[pqx+1-qr]\,[(pqx+1)^2 + (pqx+1)qr + (qr)^2]$$

$$= \frac{x}{q}\,[pqx+1-qr]\,[p^2q^2x^2 + 1 + 2pqx + pq^2xr + qr + q^2r^2]$$

4. We know that

Dividend = Divisor × Quotient + Remainder

$\Rightarrow$ Dividend – Remainder = Divisor × Quotient

Clearly, RHS of the above result is divisible by the divisor. Therefore, LHS is also divisible by the divisor. Hence, we must subtract remainder from the dividend.

Divide $8x^4 + 14x^3 - 2x^2 + 7x - 8$ by $4x^2 + 3x - 2$, by following process :

$$
\begin{array}{r|l}
4x^2+3x-2 & 8x^4+14x^3-2x^2+7x-8 \quad (2x^2+2x-1 \\
 & 8x^4+6x^3-4x^2 \\
 & -\quad -\quad + \\ \hline
 & 8x^3+2x^2+7x-8 \\
 & 8x^3+6x^2-4x \\
 & -\quad -\quad + \\ \hline
 & -4x^2+11x-8 \\
 & -4x^2-3x+2 \\
 & +\quad +\quad - \\ \hline
 & 14x-10
\end{array}
$$

Clearly, Quotient $= 2x^2 + 2x - 1$ and Remainder $= 14x - 10$

Thus, if we must subtract $14x - 10$ from $8x^4 + 14x^3 - 2x^2 + 7x - 8$.

5. By division algorithm, we have

Dividend = Divisor × Quotient + Remainder

$\Rightarrow$ Dividend – Remainder = Divisor × Quotient

$\Rightarrow$ Dividend – Remainder is always divisible by the divisor.

Hence, $f(x) - (x+a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is exactly divisible by $x^2 - 2x + k$.

Let us now divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$.

$$
\begin{array}{r|l}
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \quad (x^2-4x+(8-k) \\
 & x^4-2x^3+kx^2 \\
 & -\quad +\quad - \\ \hline
 & -4x^3+(16-k)x^2-26x+10-a \\
 & -4x^3+8x^2 \qquad -4kx \\
 & +\quad -\qquad + \\ \hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\qquad +\qquad - \\ \hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

For $f(x) - (x+a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ to be exactly divisible by $x^2 - 2x + k$, we must have

$(-10+2k)x + (10 - a - 8k + k^2) = 0$ for all x.

Equating co-efficients of x and constant terms $-10 + 2k = 0$ and $10 - a - 8k + k^2 = 0$

$\Rightarrow k = 5$ and $10 - a - 40 + 25 = 0 \Rightarrow k = 5$ and $a = -5$.

6. Since, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of (x).

Therefore, $\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are the two factors of f(x).

Product of these two factors must be a factor of f(x).

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) = \frac{1}{3}(3x^2 - 5) \text{ is a factor of } f(x).$$

Also, $3x^2 - 5$ is a factor of f(x).

To find the product of other two factors, we divide f(x) by $3x^2 - 5$ as follows:

$$
\begin{array}{r|l}
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \quad (x^2+2x+1 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\ \hline
 & 6x^3+3x^2-10x-5 \\
 & 6x^3+0x^2-10x \\
 & -\quad -\quad + \\ \hline
 & 3x^2-5 \\
 & 3x^2-5 \\
 & -\quad + \\ \hline
 & 0
\end{array}
$$

Hence, $x^2 + 2x + 1$ is the multiplication of other two factors.

By division algorithm, we have

$$x^2 + 2x + 1 = (x+1)^2 = (x+1)(x+1)$$

Hence, the other two zeroes of f(x) are –1 and –1.

7. Since, α, β are the zeroes of $ax^2 + bx + c$

$\therefore \ \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Sum of the zeroes of required polynomial

$$= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)}{\alpha\beta} = \frac{(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]}{\alpha\beta}$$

$$= \frac{\frac{-b}{a}\left(\frac{b^2}{a^2} - \frac{3c}{a}\right)}{\frac{c}{a}} = -\frac{b}{a^2c}(b^2 - 3ac)$$

And the product of the zeroes of required polynomial

$$= \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = \frac{c}{a}$$

$\therefore$ Consequently required polynomial $= x^2 -$ (Sum of the zeroes)x + Product of the zeroes.

$$= x^2 + \frac{b}{a^2c}(b^2 - 3ac)x + \frac{c}{a}$$

8. We have,

$f(x) = abx^2 + (b^2 - ac)x - bc = abx^2 + b^2x - acx - bc = bx(ax + b) - c(ax + b) = (ax + b)(bx - c)$

Thus, the zeroes of $f(x)$ are $\alpha = -\frac{b}{a}$ and $\beta = \frac{c}{b}$.

Now, $\alpha + \beta = -\frac{b}{a} + \frac{c}{b} = \frac{ac - b^2}{ab}$ and $\alpha\beta = -\frac{b}{a} \times \frac{c}{b} = -\frac{c}{a}$

From the given polynomial f(x), sum of zeroes :

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{b^2 - ac}{ab}\right) = \frac{ac - b^2}{ab}$$

And product of zeroes :

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = -\frac{bc}{ab} = -\frac{c}{a}$$

Hence, the sum and product of the zeroes are same in both cases. Therefore the relationship between the zeros and its coefficients is varified.

9. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$; a, b, c, d, be real constants

Let the zeroes of (1) are α, β, and γ such that $\alpha = -3$, $\beta = -1$, $\gamma = 2$

Now,

$$\alpha + \beta + \gamma = \frac{-b}{a};\ \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a},\ \text{and}\ \alpha\beta\gamma = \frac{-d}{a}.$$

Here, $\alpha + \beta + \gamma = -3 - 1 + 2 = -2 \Rightarrow \frac{-b}{a} = -2 \Rightarrow b = 2a$

Also, $\alpha\beta + \beta\gamma + \gamma\alpha = 3 - 2 - 6 = -5 \Rightarrow \frac{c}{a} = -5 \Rightarrow c = -5a$

And $\alpha\beta\gamma = (-3)(-1)(2) = 6 \Rightarrow \frac{-d}{a} = 6 \Rightarrow \frac{d}{a} = -6$

$\Rightarrow d = -6a$

Substituting the values of b, c, and d in (1), we get the required cubic polynomial as

$$ax^3 + 2ax^2 - 5ax - 6a$$

This is the required family of polynomial satisfying all the given conditions.

10. By division theroem, Dividend = Divisor × Quotient + Remainder

$\therefore\ x^3 - 3x^2 + x + 2 = h(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow h(x) \times (x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4)$

$$\Rightarrow h(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

To find h(x), divide $x^3 - 3x^2 + 3x - 2$ by $x - 2$ as follows

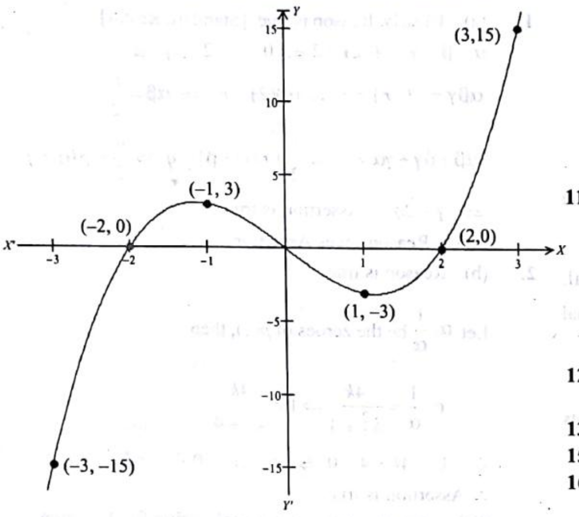
$$\begin{array}{r|l} x-2 & x^3 - 3x^2 + 3x - 2 \ (x^2 - x + 1 \\ & \underline{\begin{matrix} x^3 - 2x^2 \\ - \ \ + \end{matrix}} \\ & -x^2 + 3x \\ & \underline{\begin{matrix} -x^2 + 2x \\ + \ \ - \end{matrix}} \\ & x - 2 \\ & \underline{\begin{matrix} x - 2 \\ - \ \ + \end{matrix}} \\ & 0 \end{array}$$

Hence, $h(x) = x^2 - x + 1$.

11. Let $y = f(x) = x^3 - 4x$.

The values of y for variable value of x are listed in the following table :

x	–4	–3	–2	–1	0	1	2	3	4
$y = x^3 - 4x$	–48	–15	0	3	0	–3	0	15	48



Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. **(d)** $f(x) = x^5 + 1$, divisor $= (x-2)$
Remainder $= f(2) = 2^5 + 1 = 33$

2. **(b)** $f(x) = x^2 - x + c$, divisor $= (x+1)$
Now, $f(-1) = 3$ implies $(-1)^2 - (-1) + c = 3$
$\therefore c = 1$

3. **(c)** $f(x) = 2x^3 - px^2 + x + q$
$f(-1) = -2 - p - 1 + q = 0$
or $p - q = 3$ .... (1)
$f(2) = 16 - 4p + 2 + q = 0$
or $4p - q = +18$ .... (2)
Solving (1) and (2) we have $p = 7, q = 10$

4. **(d)** The expressions $(x-1)(x+1)$ and $(x-1)(x-1)$ which vanish if $x = 1$.

5. **(d)** $f(x) = 2x^3 - x^2 + 3x - k$
Divisor is $(x-1)$
$\therefore f(1) = 0 \Rightarrow 2 - 1 + 3 - k = 0$ or $k = 4$

6. **(a)** The square of any prime greater than 3, when divided by 12 leaves a remainder 1.p^2 when divided by 12 leaves a remainder of 1, and 17 when divided by 12 leaves a remainder of 5. So $p^2 + 17$ when divided by 12 leaves a remainder of 6.

7. **(a)** Let $f(x) = x^8 - x^5 + x^2 - x + 1$. For $x = 1$ or 0
$f(x) = 1 > 0$. For $x < 0$, each term of $f(x)$ is +ve and so first $f(x) > 0$.
Hence $f(x)$ is +ve for all real x.

8. **(a)** **9.** **(b)**

10. **(a)** $f(x) = 3x^4 - 5x^3 + 10x^2 + 11x - 61$. It is divided by $(x-3)$

3	3	–5	10	11	–61
		9	12	66	231
	3	4	22	77	170

11. **(b)** $10x = 7.\overline{7}$ or $x = 0.\overline{7}$
Subtracting, $9x = 7 \quad \therefore x = \frac{7}{9}$

$2x = \frac{14}{9} = 1.555........1.\overline{5}$

12. **(b)** $x^2 + 4x + 2 = (x^2 + 4x + 2) - 2 = (x+2)^2 - 2$
Lowest value $= -2$ when $x + 2 = 0$

13. **(d)** **14.** **(c)**

15. **(d)**

16. **(c)** $2 - 4x - x^2$
$= 6 - 4 - 4x - x^2$
$= 6 - (4 + 4x + x^2) = 6 - (x+2)^2$
Maximum value $= 6$ when $x + 2 = 0$

17. **(c)** **18.** **(b)**

19. **(b)**

MORE THAN ONE CORRECT :

1. **(a, b, c)**
In (a) power of x is -1 i.e. negative $\therefore$ (a) is not true.
In (b) power of $x = \frac{1}{2}$, not an integer. $\therefore$ (b) is not true
In (c) Here also power of x is not an integer $\therefore$ (c) is not true
(d) holds [$\because$ all the powers of x are non-negative integers.]

2. **(a, c, d)**
(a) is not true [By def.]
(b) holds
[$\because$ degree of a zero polynomial is not defined]
(c) is not true
[$\because$ degree of a constant polynomial is '0']
(d) is not true
[$\because$ a polynomial of degree n has at most n zeroes].

3. **(a, b, d)**
(a) If $p(x)$ is not divisible by $g(x)$, then $r(x) \neq 0 \therefore$ (a) is not true
(b) if $p(x)$ is divisible by $g(x)$, then
$r(x) = 0$ for all x i.e., $r(x)$ is a zero polynomial whose degree is not defined.
$\therefore$ (b) is not true
(c) is clearly true [$\because$ division algorithm rule]
(d) Since degree of $r(x) <$ degree of $g(x)$
or $r(x) = 0$, but $g(x) \neq 0$.
$\therefore r(x) = g(x)$ is not true.

4. **(a, d)**
   (a) is a Binomial ∴ (a) is not true [∵ it has two terms]
   (b) is a Trinomial [∵ it has three terms]
   (c) is a Monomial [∵ it has only one term]
   (d) is a Binomial [∵ it has two terms]

5. **(a, b, d)**
   (a) $x^2 \times \frac{1}{x} = x^2 + x^{-1}$ is not a polynomial since the exponent of variable in 2nd term is negative
   (b) $2x^2 - 3\sqrt{x} + 1 = 2x^2 - 3x^{\frac{1}{2}} + 1$ is not a polynomial, since the exponent of variable in 2nd terms is a rational number.
   (c) $x^3 - 3x + 1$ is a polynomial.
   (d) $2x^{\frac{3}{2}} - 5x$ is also not a polynomial, since the exponents of variable in 1st term is a rational number
   (a), (b) and (d)

6. **(a, b, c)**
   (a) Since, the graph meets the x-axis in two distinct points A, B. ∴ it is graph of a quadratic.
   (b) Since, the graph meets the x-axis in two distinct points A, B. ∴ it is graph of a quadratic.
   (c) Since, the graph meets the x-axis at the points A, B (distinct points) ∴ it is graph of a quadratic
   (d) Since, the the graph meets the x-axis at a single point ∴ it is not graph of quadratic.

7. (a, c, d)

PASSAGE BASED QUESTIONS :

Passage-I

1. **(c)** $\alpha+\beta = p, \alpha\beta = q$

$$\therefore \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q}$$

2. **(a)** $\alpha+\beta = -1, \alpha\beta = -2$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2 = \left(\frac{\beta-\alpha}{\alpha\beta}\right)^2 = \frac{(\beta+\alpha)^2 - 4\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(1)^2 - 4(-2)^2}{4} = \frac{9}{4}$$

3. **(b)** $\alpha+\beta = 5, \alpha\beta = 4$

$$\therefore \frac{1}{\alpha}+\frac{1}{\beta} - 2\alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 8 = -\frac{27}{4}$$

Passage-II

1. **(b)** 2. **(a)** 3. **(a)**

ASSERTION & REASON :

1. (a) Clearly, Reason is true. [Standard Result]
$\alpha+\beta+\gamma = -(-2) = 2 \Rightarrow 0 + \gamma = 2 \therefore \gamma = 2.$
$\alpha\beta\gamma = -(-r) = r \therefore \alpha\beta(2) = r \Rightarrow \alpha\beta = \frac{r}{2}$
$\alpha\beta+\beta\gamma+\gamma\alpha = q \Rightarrow \frac{r}{2} + r(\alpha+\beta) = q \Rightarrow \frac{g}{2} + \gamma(0) = q$
$\Rightarrow \gamma = 2q$ ∴ Assertion is true
Since, Reason gives Assertion

2. (b) Reason is true.
Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then
$$\alpha.\frac{1}{\alpha} = \frac{4k}{k^2+4} \Rightarrow 1 = \frac{4k}{k^2+4}$$
$\therefore k^2 - 4k + 4 = 0 \Rightarrow (k-2)^2 = 0 \therefore k = 2$
∴ Assertion is true
Since, Reason is not correct explanation for Assertion.

3. (d) Reason is true by Remainder Theorem.
Again, $x^4 + 4x^2 + 5 = (x^2+2)^2 + 1 > 0$ for all x.
∴ given polynomial has no zero ∴ Assertion is not true

4. (c) Reason is false [∵ a polynomial of nth degree has at most x zeroes.]
Again, $x^3 + x = x(x^2+1)$ which has only one real zero $(x = 0)$
$[\because x^2 + 1 \neq 0$ for all $x \in R]$
∴ Assertion is true

5. (a)
6. (b)
7. (c)
8. (d)

MULTIPLE MATCHING QUESTIONS :

1. (A) → r, q ; (B) → r, t ; (C) → s, u ; (D) → p
(A) $4 - x^2 = 0$
$x = \pm 2$
(B) $x^3 - 2x^2 = 0$
$x^2(x-2) = 0$
$x = 0$ or $x = 2$
(C) $6x^2 - 7x - 3 = 0$
$6x^2 - 9x + 2x - 3 = 0$
$3x(2x-3) + 1(2x-3) = 0$
$(3x+1)(2x-3) = 0$
$x = 3/2$ or $x = -1/3$
(D) $x = 7$

HOTS SUBJECTIVE QUESTIONS :

1. $\pm\sqrt{3}$ are zeroes $\Rightarrow (x+\sqrt{3})(x-\sqrt{3}) = x^2 - 3$ is a divisor of $x^4 - 3x^3 - 7x^2 + 9x + 12$.

$$
\begin{array}{r|l|l}
x^2-3 & x^4-3x^3-7x^2+9x+12 & x^2-3x-4 \\
 & \underline{\underset{-}{x^4} \quad \underset{+}{-3x^2}} & \\
 & -3x^3-4x^2+9x+12 & \\
 & \underline{\underset{+}{-3x^2} \quad \underset{-}{+9x}} & \\
 & -4x^2 \quad +12 & \\
 & \underline{\underset{+}{-4x^2} \quad \underset{-}{+12}} & \\
 & 0 &
\end{array}
$$

$q(x) = x^2-3x-4 = x^2-4x+x-4$

$= x(x-4)+1(x-4)$

$q(x) = (x+1)(x-4)$

zeroes of $q(x)$ are $-1, 4$

$\therefore$ Remaining required zeroes are $-1, 4$.]

2. We have $p(x) = x^3+6x^2+11x+6$

By trial putting $x=-1$, we find that

$p(-1) = (-1)^3+6(-1)^2+11(-1)+6 = -1+6-11+6 = 0$

$\therefore$ By factor theorem $x+1$ is a facrtor of $p(x)$.

Now, $x^3+6x^2+11x+6$

$= x^3+x^2+5x^2+5x+6x+6$

$= x^2(x+1)+5x(x+1)+6(x+1) = (x+1)(x^2+5x+6)$

We could have also got this by dividing $p(x)$ by $(x+1)$

$x^2+5x+6 = x^2+2x+3x+6$ [by splitting middle term]

$= (x+2)(x+3)$

$\therefore x^3+6x^2+11x+6 = (x+1)(x+2)(x+3)$

The value of $p(x) = x^3+6x^2+11x+6$ is zero when $x+1 = 0$ or $x+2=0$ or $x+3=0$

i.e., when $x=-1$, or $x=-2$ or $x=-3$

So, the zeroes of $x^3+6x^2+11x+6$ are $-1, -2$, and -3.

Now, the sum of the zeroes of $x^3+6x^2+11x+6$ is

$$= \frac{-6}{1} = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

Product of the zeros $= (-1)(-2)(-3) = -6 =$

$$\frac{-6}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^3}.$$

Sum of the product of the zeroes, taken two at a time

$(-1)(-2)+(-2)(-3)+(-3)(-1) = 2+6+3 = 11$

$$= \frac{\text{Cofficient of } x}{\text{Cofficient of } x^3}$$

In general, it can be proved that if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d, then,

$$\alpha+\beta+\gamma = \alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} \text{ and } \alpha\beta\gamma = \frac{-d}{a}$$

3. Let α and β are zeroes of quadratic polynomial $p(s) = 3s^2-6s+4$

So, $\alpha+\beta = -\left(\frac{-6}{3}\right) = 2$ and $\alpha\beta = 4/3$

$$\text{Let } x = \frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$$

$$= \frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$$

We know, $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 2^2-2\times\frac{4}{3} = 4-\frac{8}{3} = \frac{4}{3}$

$\alpha^2+\beta^2 = \frac{4}{3}$, $\alpha+\beta = 2$ and $\alpha\beta = 4/3$. So, $x = \frac{4/3}{4/3}+$

$2\left(\frac{2}{4/3}\right)+3\times\frac{4}{3} = 1+4\times\frac{3}{4}+4 = 8$

4. Since, α and β are zeroes of polynomial $f(x) = 2x^2-5x+7$.

So, $\alpha+\beta = -\left(-\frac{5}{2}\right) = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Let S and P denote respectively the sum and product of zeroes of the required polynomial. Then, polynomial is $p(x) = k(x^2-Sx+P)$

$S = (2\alpha+3\beta)+(3\alpha+2\beta) = 5(\alpha+\beta) = 5\times\frac{5}{2} = \frac{25}{2}$ and, P

$= (2\alpha+3\beta)(3\alpha+2\beta)$

$\Rightarrow P = 6(\alpha^2+\beta^2)+13\alpha\beta = 6\alpha^2+6\beta^2+12\alpha\beta+\alpha\beta$

$= 6(\alpha+\beta)^2+\alpha\beta$

$$\Rightarrow P = 6\times\left(\frac{5}{2}\right)^2+\frac{7}{2} = \frac{75}{2}+\frac{7}{2} = 41$$

Hence, the required polynomial is given by $p(x) = k(x^2-Sx+P)$

or, $p(x) = k\left(x^2-\frac{25}{2}x+41\right)$, where k is any non-zero real number.

5. Let the cubic polynomial be ax^3+bx^2+cx+d .... (1)

Let α, β, γ be the zeroes of the required cubic polynomial, then

$$\alpha+\beta+\gamma = \frac{-b}{a};\ \alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a};\ \text{and}\ \alpha\beta\gamma = \frac{-d}{a}.$$

Now, according to the question, $\alpha+\beta+\gamma = 2$

$$\Rightarrow \frac{-b}{a} = 2 \Rightarrow \frac{b}{a} = -2$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = -7 \Rightarrow \frac{c}{a} = -7$$

$$\text{And } \alpha\beta\gamma = -14 \Rightarrow \frac{-d}{a} = -14;\ \frac{d}{a} = 14$$

If $a = 1$, then $b = -2, c = -7, d = 14$

Substituting the values of a, b, c and d in (1), we get the required cubic polynomial
$x^3 - 2x^2 - 7x + 14$

This polynomial satisfies all the given conditions

6. Number of poles $= \dfrac{8x^3 - 6x^2 + 5x - 21}{2x - 3} + 1$ ...(1)

[Since, one pole will be situated at the initial point]

$$
\begin{array}{r|l}
2x-3 & 8x^3 - 6x^2 + 5x - 21 \;(4x^2 + 3x + 7 \\
 & \underline{\overset{-}{8x^3} \overset{+}{-12x^2}} \\
 & 6x^2 + 5x - 21 \\
 & \underline{\overset{-}{6x^2} \overset{+}{-9x}} \\
 & 14x - 21 \\
 & \underline{\overset{-}{14x} \overset{+}{-21}} \\
 & 0
\end{array}
$$

Now, from equation (1), we get, number of poles $= 4x^2 + 3x + 7 + 1 = 4x^2 + 3x + 8$.

7. If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then the remainder should be zero.

On dividing the polynomial by $x^2 + 1$, we get,

$$
\begin{array}{r|l}
x^2+1 & x^4 + x^3 + 8x^2 + ax + b \;(x^2 + x + 7 \\
 & \underline{\overset{-}{x^4} + \overset{-}{x^2}} \\
 & x^3 + 7x^2 + ax + b \\
 & \underline{\overset{-}{x^3} + \overset{-}{x}} \\
 & 7x^2 + x(a-1) + b \\
 & \underline{\overset{-}{7x^2} + \overset{-}{7}} \\
 & x(a-1) + b - 7
\end{array}
$$

$\therefore$ Quotient $= x^2 + x + 7$ and Remainder $= x(a-1) + (b-7)$

Now, since, $x^2 + 1$ is factor of this polynomial,

$\therefore$ Remainder $= 0$

$\Rightarrow x(a-1) + (b-7) = 0 \Rightarrow x(a-1) + (b-7) = 0x + 0$

Equating coefficient of x and constant term, we get,

$a - 1 = 0$ and $b - 7 = 0 \Rightarrow a = 1$ and $b = 7$

8. Let $y = f(x)$ or, $y = 3 - 2x - x^2$

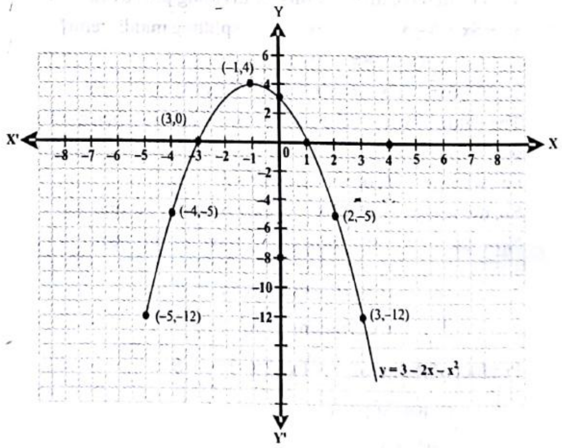
Let us list a few values of $y = 3 - 2x - x^2$ corresponding to a few values of x as follows :

x	–5	–4	–2	–1	0	1	2	3	4
$y = 3 - 2x - x^2$	–12	–5	3	4	3	0	–5	–12	–21

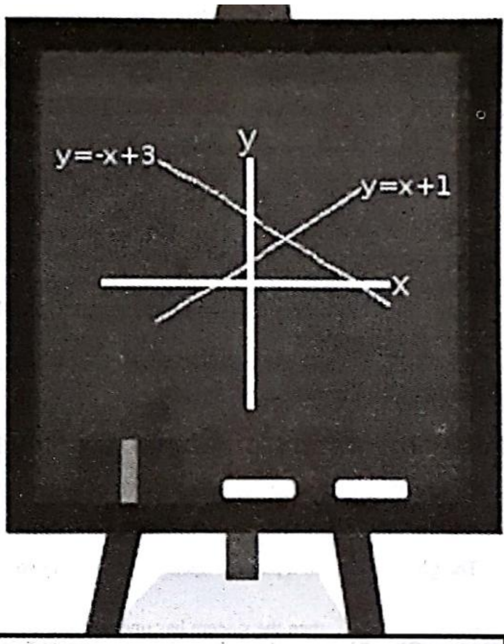
Thus, the following points lie on the graph of polynomial $y = 3 - 2x - x^2$:

(–5, –12), (–4, –5), (–3, 0), (–2, 3), (–1, 4), (0, 3), (1, 0), (2, –5), (3, – 12) and (4, –12)

Let us plot these points on a graph paper and draw a smooth free hand curve passing through these points to obtain the graph of $y = 3 - 2x - x^2$. The curve thus obtained represents a parabola, as shown in figure.



chapter 3



PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Introduction

An equation of the form $Ax + By + C = 0$ is called a linear equation in two variables x and y.

Where A is called coefficient of x, B is called coefficient of y and C is the constant term (i.e. free from x and y)

$A, B, C \in R$, [$\in$ means belongs to R, means set of Real numbers]

But A and B cannot be simultaneously zero

It is called a linear equation because the two unknowns (x and y) occurs only with power one and the product of two unknown quantities does not occur. A linear equation in two variables always represent a straight line.

Since it involves two variables therefore a single equation will have infinite set of solution. But a system of pair of linear equations in two variables have either no solution, unique solution or infinite many set of solutions.

Standard from of a pair of linear equations in two variables :

$$a_1x + b_1y + c_1 = 0 \quad \text{.............. (i)}$$

$$a_2x + b_2y + c_2 = 0 \quad \text{.............. (ii)}$$

Here a_1, b_1, c_1, a_2, b_2 and c_2 all are real constants.

SOLUTION OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES :

To find two numbers such that their sum is 35 and their difference is 5. We could indicate the problem algebraically by letting x represent one number and y the other. Thus, the problem may be indicated by the two equations :

$$x+y=33$$
$$x-y=5$$

Our problem is to find one pair of values that will satisfy both equations. Such a pair of values is said to a graphically solution of both equations at the same time, or simultaneously. The two equations for which we seek a common solution are called simultaneous equations. The two equations, taken together, comprise a system of equations. Each of these equations represents a straight line on a graph.

Graphically solution of a system of a pair of linear equations in two variables is the co-ordinates of the point where the two lines intersect.

CONSISTENT, DEPENDENT AND INCONSISTENT SYSTEM OF EQUATIONS :

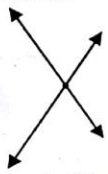
System of a pair of linear equations in two variables :

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 + c_2 = 0$$

The given system of a pair of linear equations in two variables has either one solution, infinite solutions or no solution.

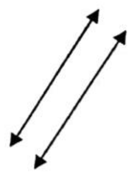
(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the system has one (or unique) solution, and the system is called consistent.

In this case, a pair of straight lines represented by the system intersect each other at only one point.



(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the given system has infinite solution and the system is called dependent. In this case, a pair of lines represented by the system coincides with each other. So the intersect each other at infinite number of points.

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the given system has no solution and hence the system is called inconsistent. In this case, a pair of lines represented by the system are parallel to each other. So they do not intersect each other at any point.



SOLVING SYSTEMS OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES :

We can solve a pair of linear equations in two variables by either by graphically or by algebrically.

GRAPHICAL METHOD TO SOLVE A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES :

Graph both equations using same pair of horizontal and vertical lines called X and Y-axis respectively. Co-ordinates of the point(s) of intersection of the two lines is/are the solution.

ILLUSTRATION 3.1

Check whether the pair of equations

$x + 3y = 6$ .... (1)

and $2x - 3y = 12$ .... (2)

is consistent. If so, solve them graphically.

SOLUTION :

$$\frac{a_1}{a_2}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{3}{-3}=-1 \qquad \therefore\ \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Hence the given system of a pair of equations in two variable is consistent.

Let us draw the graphs of the equations (1) and (2). For this, we find two solutions of each of the equations, which are given below in the table.

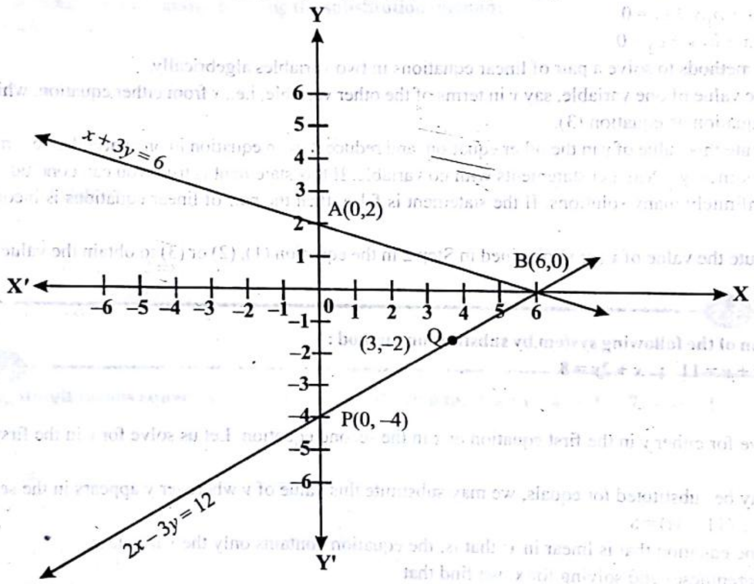
$x+3y=6$

x	0	6
y	2	0

$2x-3y=12$

x	0	3
y	–4	–2

Plot the points $A(0, 2)$, $B(6, 0)$, $P(0, -4)$ and $Q(3, -2)$ on graph paper, and join the points to form the lines AB and PQ.



Point B (6, 0) common to both the lines AB and PQ. So, the solution of the pair of linear equations is $x = 6$ and $y = 0$.

ILLUSTRATION 3.2

In each of the following, find whether the system is consistent, inconsistent or dependent :

(i) $5x+2y=16$ (ii) $5x+2y=16$ (iii) $5x+2y=16$

$7x-4y=2$ $3x+\frac{6}{5}y=2$ $\frac{15}{2}x+3y=24$

SOLUTION :

(i) $\frac{a_1}{a_2}=\frac{5}{7},\ \frac{b_1}{b_2}=\frac{2}{-4}=-\frac{1}{2}$

$\because\ \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Hence the given system is consistent.

(ii) $\frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{10}{6} = \frac{5}{3}, \frac{c_1}{c_2} = \frac{16}{2} = \frac{8}{1}$ $\quad \because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the given system is inconsistent.

(iii) $\frac{a_1}{a_2} = \frac{10}{15} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{16}{24} = \frac{2}{3}$ $\quad \because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore the given system is dependent.

ALGEBRAIC METHODS TO SOLVE A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES :

(A) FIRST METHOD—SUBSTITUTION METHOD:

Algorithm to find the solution :

A system of a pair of linear equations in two variables:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

There are three methods to solve a pair of linear equations in two variables algebrically.

Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient. Consider this equation as equation (3).

Step 2 : Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved. Sometimes, you can get statements with no variable. If this statement is true, you can conclude that the pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent and hence no solution.

Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation (1), (2) or (3) to obtain the value of the other variable.

ILLUSTRATION 3.3

Find the solution of the following system by substitution method :

$$\mathbf{4x + y = 11 \quad ; \quad x + 2y = 8}$$

SOLUTION :

It is easy to solve for either y in the first equation or x in the second equation. Let us solve for y in the first equation. The result is $y = 11 - 4x$

Since equals may be substituted for equals, we may substitute this value of y wherever y appears in the second equation. Thus,

$$x + 2(11 - 4x) = 8$$

We now have one equation that is linear in x; that is, the equation contains only the variable x.

Removing the parentheses and solving for x, we find that

$$x + 22 - 8x = 8 \quad \text{or} \quad -7x = 8 - 22$$
$$-7x = -14 \quad \text{or} \quad x = 2$$

To get the corresponding value of y, we substitute $x = 2$ in $y = 11 - 4x$. The result is

$$y = 11 - 4(2) = 11 - 8 = 3$$

Thus, the solution for the two original equations are $x = 2$ and $y = 9$.

ILLUSTRATION 3.4

Solve the following pair of equations by substitution method:

$$\mathbf{7x - 15y = 2} \quad \text{.............} (1)$$
$$\mathbf{x + 2y = 3} \quad \text{.............} (2)$$

SOLUTION :

Step 1 : We pick either of the equations and write one variable in terms of the other.

Let us consider the Equation (2) :

$x + 2y = 3$ and write it as $x = 3 - 2y$ .... (3)

Step 2 : Substitute the value of x in Equation (1), we get

$$7(3 - 2y) - 15y = 2$$

i.e., $21-14y-15y=2$ i.e., $-29y=-19$

Therefore, $y=\frac{19}{29}$

Step 3 : Substituting this value of y in Equation (3), we get

$$x=3-2\left(\frac{19}{29}\right)=\frac{49}{29}$$

Therefore, the solution : $x=\frac{49}{29},\ y=\frac{19}{29}$

Verification : Substituting $x=\frac{49}{29}$ and $y=\frac{19}{29}$, you can verify that both the Equations (1) and (2) are satisfied.

ILLUSTRATION 3.5

Solve the following simultaneous equations by using the substitution method:

$x+y=15$; $y=x+3$

SOLUTION :

Label the equations as follows:

$x+y=15$ .... (1)

$y=x+3$ .... (2)

substituting $y=x+3$ in (1) gives

$x+x+3=15$

$2x+3=15$

$2x=15-3$ or $2x=12 \Rightarrow x=6$

When $x=6, y=6+3=9$ [From (2)]

So, the solution : $x=6, y=9$

ILLUSTRATION 3.6

Solve the following simultaneous equat ns by using the substitution method: $x+4y=14$; $7x-3y=5$.

SOLUTION :

$x+4y=14$ .... (1)

$7x-3y=5$ .... (2)

From eq. (1), $x=14-4y$ .... (3)

Substitute the value of x in equation (2)

$7(14-4y)-3y=5$

$\Rightarrow 98-28y-3y=5 \Rightarrow 98-31y=5$

$\Rightarrow 93-31y \Rightarrow y=\frac{93}{31} \Rightarrow y=3$

Now substitute value of y in eq. (3)

Solution : $x=2, y=3$

(B) SECOND METHOD—ELIMINATION MEHOD:

Algorithm to find the solution:

Step 1 : If coefficients of one variable (either x or y) in both the equations are not equal, then first multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2 : Add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3.

If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.

If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is the given system is inconsistent.

Step 3 : Solve the equation in one variable (x or y) so obtained to get its value.

Step 4 : Substitute this value of variable (x or y) in either of the original equations to get the value of the other variable.

ILLUSTRATION 3.7

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

SOLUTION :

Let the income of the two person by ₹ $9x$ and ₹ $7x$ and their expenditures be ₹ $4y$ and ₹ $3y$ respectively. Then the equations formed for the given situation are given by :

$9x-4y=2000$ .... (1)

and $7x-3y=2000$ .... (2)

Step 1 : Multiply Equation (1) by 3 and Equation (2) by 4 to make the coefficients of y equal. Then we get the equations:

$27x-12y=6000$ .... (3)

$28x-12y=8000$ .... (4)

Step 2 : Subtract Equation (3) from Equation (4) to eliminate y, because the coefficients of y are the same. So, we get

$(28x-27x)-(12y-12y)=8000-6000$

i.e., $x=2000$

Step 3 : Substituting this value of x in (1), we get

$9(2000)-4y=2000$

i.e., $y=4000$

So, the solution of the equations: $x=2000, y=4000$. Therefore, the monthly incomes of the two persons are ₹ 18,000 and ₹ 14,000, respectively.

Verification : $18000:14000=9:7$.

Also, the ratio of their expenditures $=18000-2000:14000-2000=16000:12000=4:3$

(C) THIRD METHOD-CROSS-MULTIPLICATION METHOD:

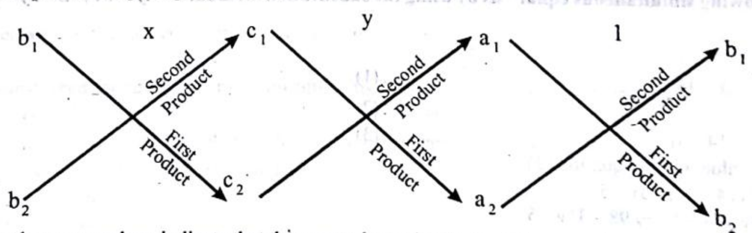
Algorithm to find the solution:

Step 1 : Write the given equations in the form :

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Step 2 : Taking the help of the diagram given below :



The arrows between the two numbers indicate that they are to be multiplied and the second product is to be subtracted from the first product.

Write equation as

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

Step 3 : Find x and y, provided $a_1b_2-a_2b_1 \neq 0$

ILLUSTRATION 3.8

Solve the following equations by cross-multiplication method:

$3x+2y+25=0,\ x+y+15=0$

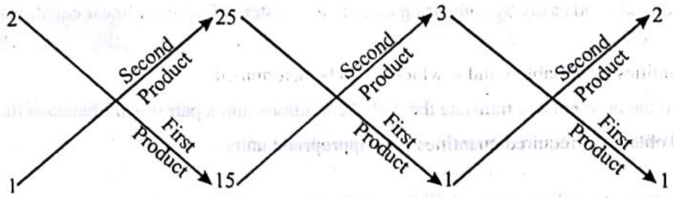
SOLUTION :

$3x+2y+25=0$ .... (1)

$x+y+15=0$ .... (2)

Here, $a_1=3; b_1=2, c_1=25$

$a_2 = 1, b_2 = 1, c_2 = 15$



$\therefore$

$$\frac{x}{2\times15-25\times1}=\frac{y}{25\times1-15\times3}=\frac{1}{3\times1-2\times1}$$

$$\frac{x}{30-25}=\frac{y}{25-45}=\frac{1}{3-2}$$

$$\frac{x}{5}=\frac{y}{-20}=\frac{1}{1} \qquad \text{.........(1)}$$

$$\frac{x}{5}=1,\ \frac{y}{-20}=1 \Rightarrow x=5, y=-20$$

SOLUTION OF A SYSTEM OF A PAIR OF EQUATIONS REDUCIBLE TO THE SYSTEM OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES :

By using the suitable substitution or simplification first we convert the given system into the system of a pair of linear equations in two variables. Then after using any algebraic or graphical method we solve the system.

ILLUSTRATION 3.9

Solve the equations : $\frac{2x+1}{3}+\frac{3y+2}{5}=2$ **and** $\frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1$

SOLUTION :

Given equations are : $\frac{2x+1}{3}+\frac{3y+2}{5}=2$ ...(1)

and $\frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1$ ...(2)

Let $\frac{2x+1}{3}=u$ and $\frac{3y+2}{5}=v$

Then the equations become

$u+v=2$ ... (3)

$2u-3v=-1$ ... (4)

Multiplying (3) by 3,

$3u+3v=6$ ... (5)

Adding (4) and (5), $5u=5 \Rightarrow u=1$

Substituting this value of u in (3), $1+v=2 \Rightarrow v=2-1=1$

Then $\frac{2x+1}{3}=u=1$ and $\frac{3y+2}{5}=v=1$

$\Rightarrow$ $2x+1=3$ and $3y+2=5$

$\Rightarrow$ $2x=3-1=2$ and $3y=5-2=3$

$\Rightarrow$ $x=1$ and $y=1$

SOLVING THE WORD PROBLEMS :

Many problems can be solved quickly and easily by converting them in to a system of a pair of linear equations in two variables as follows :

(i) Represent the unknown quantities by variable x and y, which are to be determined.

(ii) Find the conditions given in the problem and translate the verbal conditions into a pair of simultaneous linear equations.

(iii) Solve these equations and obtain the required quantities with appropriate units.

ILLUSTRATION 3.10

Find the two numbers such that half the first equals one third of the second and twice their sum exceeds three times the second by 4.

SOLUTION :

If we let x and y be the first and second numbers respectively, we can write two equations almost directly from the statement of the problem as

$\frac{x}{2} = \frac{y}{3}$ and $2(x+y) = 3y+4$

Solving for x in the first equation and substituting this value in the second, we have

$$x = \frac{2y}{3} \Rightarrow 2\left(\frac{2y}{3} + y\right) = 3y+4$$

$$\Rightarrow \frac{4y}{3} + 2y = 3y+4 \Rightarrow 4y + 6y = 9y + 12 \Rightarrow y = 12 \text{ (Second number)}$$

$$\frac{x}{2} = \frac{12}{3} \Rightarrow x = 8 \text{ (first number)}$$

MISCELLANEOUS SOLVED EXAMPLES

1. If $2x+3y=19$ and $5x+4y=37$, then find the values of x and y.

Sol. In this method, the two equations are reduced to a single variable equation by eliminating one of the variables.

Step 1: Here, let us eliminate the y term, and in order to eliminate the y term, we have to multiply the first equation with the coefficient of y in the second equation and the second equation with the coefficient of y in the first equation so that the coefficients of y term in both the equations become equal.

$(2x+3y=19)4 \Rightarrow 8x+12y=76$ ...(1)

$(5x+4y=37)3 \Rightarrow 15x+12y=111$ ...(2)

Step 2: Subtract equation (3) from (4).

$(15x+12y)-(8x+12y)=111-76$

$\Rightarrow 7x=35 \Rightarrow x=5$

Step 3: Substitute the value of x in equatin (1) or (2) to find the value of y. Substituting the value of x in the first equation, we get,

$2(5)+3y=19$

$\Rightarrow 3y=19-10 \Rightarrow 3y=9 \Rightarrow y=3$

$\therefore$ The solution of the given pair of equation is $x=5; y=3$.

2. Solve for x and y: $\frac{3}{x}+\frac{4}{y}=1;\ \frac{4}{x}+\frac{2}{y}=\frac{11}{12}$

Sol. $\frac{3}{x}+\frac{4}{y}=1$ ...(1)

$\frac{4}{x}+\frac{2}{y}=\frac{11}{12}$ ...(2)

Multiplying (2) by 2 $\Rightarrow \frac{8}{x}+\frac{4}{y}=\frac{22}{12}$ ...(3)

Subtracting (1) and (3) $\Rightarrow \frac{5}{x}=\frac{10}{12}$

$\therefore x=\frac{5\times 12}{10}=6$

Substituting $x=6$ in (1)

$\Rightarrow \frac{3}{6}+\frac{4}{y}=1$

$\Rightarrow \frac{4}{y}=1-\frac{1}{2}=\frac{1}{2}$

$\therefore y=8$ Hence, $x=6$ and $y=8$

3. Solve the following system of equations for x and y:

$$\frac{b^2x}{a}-\frac{a^2y}{b}=ab(a+b) \text{ and } b^2x-a^2y=2a^2b^2$$

Sol. The given system of equation is

$\frac{b^2x}{a}-\frac{a^2y}{b}=ab(a+b)$ ...(1)

and $b^2x-a^2y=2a^2b^2$ ...(2)

Dividing (2) by a, we get

$$\frac{b^2x}{a} - ay = 2ab^2 \quad ...(3)$$

Subtracting (3) from (1), we get $\left(\frac{b^2x}{a} - \frac{a^2y}{b}\right) - \left(\frac{b^2x}{a} - ay\right) = ab(a+b) - 2ab^2$

$\Rightarrow \quad ay - \frac{a^2y}{b} = a^2b + ab^2 - 2ab^2 \Rightarrow \frac{aby - a^2y}{b} = a^2b - ab^2 \Rightarrow y.a\left(\frac{b-a}{b}\right) = ab(a-b)$

$\Rightarrow \quad y = ab\,(a-b) \times \frac{b}{a\,(b-a)} = -b^2$

Substituting this value of y in (2), we get

$\frac{b^2x}{a} - a(-b^2) = 2ab^2 \Rightarrow b^2x + a^2b^2 = 2a^2b^2 \Rightarrow b^2x = a^2b^2 \Rightarrow x = a^2$

Hence, the solution is $x = a^2$, $y = -b^2$

4. Solve the equations : $\frac{2x+1}{3} + \frac{3y+2}{5} = 2$ and $\frac{2(2x+1)}{3} - \frac{3(3y+2)}{5} = -1$

Sol. Given equations are: $\frac{2x+1}{3} + \frac{3y+2}{5} = 2$ ...(1)

and $\frac{2(2x+1)}{3} - \frac{3(3y+2)}{5} = -1$ ...(2)

Let $\frac{2x+1}{3} = u$ and $\frac{3y+2}{5} = v$

Then, the equations become

$u + v = 2$ ... (3)

$2u - 3v = -1$ ... (4)

Multiplying (3) by 3,

$3u + 3v = 6$ ... (5)

Adding (4) and (5), $5u = 5 \Rightarrow u = 1$

Substituting this value of u in (3), $1 + v = 2 \Rightarrow v = 2 - 1 = 1$

Then, $\frac{2x+1}{3} = u = 1$ and $\frac{3y+2}{5} = v = 1$

$\Rightarrow \quad 2x + 1 = 3$ and $3y + 2 = 5$

$\Rightarrow \quad 2x = 3 - 1 = 2$ and $3y = 5 - 2 = 3$

$\Rightarrow \quad x = 1$ and $y = 1$

5. Solve the following pair of equations by substitution method:

$7x - 15y = 2$ ...(1)

$x + 2y = 3$ ...(2)

Sol. Step 1 : We pick either of the equations and write one variable in terms of the other.

Let us consider the Equation (2) :

$x + 2y = 3$ and write it as $x = 3 - 2y$ ...(3)

Step 2 : Substitute the value of x in Equation (1). We get

$7(3 - 2y) - 15y = 2$

i.e., $21 - 14y - 15y = 2$ i.e., $-29y = -19$. Therefore, $y = \frac{19}{29}$

Step 3 : Substituting this value of y in Equation (3), we get

$$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$$

Therefore, the solution is $x = \frac{49}{29},\ y = \frac{19}{29}$

Verification : Substituting $x = \frac{49}{29},\ y = \frac{19}{29}$, you can verify that both the Equations (1) and (2) are satisfied.

6. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there ?

Sol. Let the ten's and the unit's digits in the first number be x and y, respectively.

So, the first number may be written as $10x + y$ in the expanded form (for example, $56 = 10(5) + 6$).

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit. This number, in the expanded notation is $10y + x$ (for example, when 56 is reversed, we get $65 = 10(6) + 5$).

According to the given condition.

$$(10x + y) + (10y + x) = 66$$

i.e., $11(x + y) = 66$

i.e., $x + y = 6$ .... (1)

We are also given that the digits differ by 2, therefore,

either $x - y = 2$ .... (2)

or $y - x = 2$ .... (3)

If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$.

In this case, we get the number 42.

If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$.

In this case, we get the number 24.

Thus, there are two such numbers 42 and 24.

Verification : Here, $42 + 24 = 66$ and $4 - 2 = 2$. Also $24 + 42 = 66$ and $4 - 2 = 2$.

7. Solve the following system of equations graphically:

$2x + y = 3$

$2x - 3y = 7$

Identify which of these lines are coincident or parallel. Also, find the co-ordinates of the point where any of the lines cut Y-axis.

Sol. From equation (1) :

$2x + y = 3 \Rightarrow y = 3 - 2x$

x	0	1	–1	2
y	3	1	5	–1

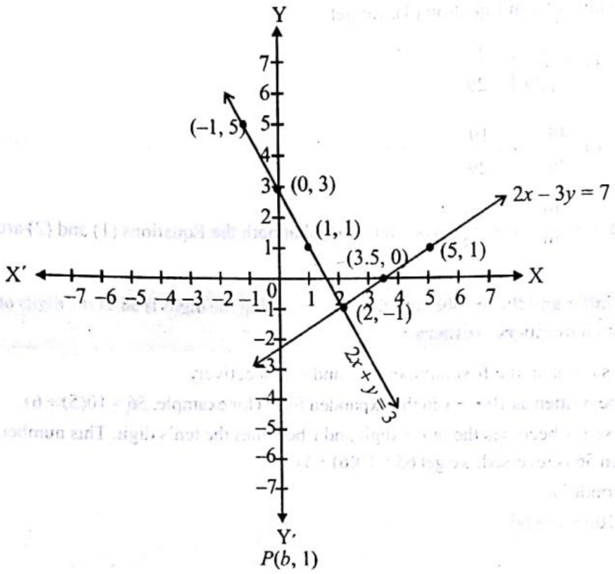
From equation (2) : $2x - 3y = 7 \Rightarrow 3y = 2x - 7$

$\therefore\ y = \frac{2x - 7}{3}$

x	2	3.5	5
y	–1	0	1

[**Note:** We choose such values for x that give integral values of y]

The graphs of the 3 lines are plotted as shown below:



P(b, 1)

Equations 1 and 2 represent coincident lines.
Equations 1 and 3, or 2 and 3 represent intersecting line with a unique solution (2, −1)
There are no parallel lines.
Line $2x+y=3$ or $4x+2y=6$ cuts Y-axis at (0, 3)

8. Solve $3x+2y+25=0$, $x+y+15=0$

Sol. $3x+2y+25=0$ ....(1)

$x+y+15=0$ ....(2)

Here, $a_1=3, b_1=2, c_1=25$

$a_2=1, b_2=1, c_2=15$

$$\begin{matrix} 2 & & 25 & & 3 & & 2 \\ & \times & & \times & & \times & \\ 1 & & 15 & & 1 & & 1 \end{matrix}$$

$\therefore$

$$\frac{x}{2\times15-25\times1}=\frac{y}{25\times1-15\times3}=\frac{1}{3\times1-2\times1}$$

$$\frac{x}{30-25}=\frac{y}{25-45}=\frac{1}{3-2}$$

$$\frac{x}{5}=\frac{y}{-20}=\frac{1}{1} \quad ....(1)$$

$$\frac{x}{5}=1,\ \frac{y}{-20}=1 \quad \Rightarrow \quad x=5, y=-20$$

9. For what values of k will the following pair of linear equations have infinitely many solutions?

$kx+3y-(k-3)=0$

$12x+ky-k=0$

Sol. Here, $\frac{a_1}{a_2} = \frac{k}{12}$, $\frac{b_1}{b_2} = \frac{3}{k}$, $\frac{c_1}{c_2} = \frac{k-3}{k}$

For a pair of linear equations to have finitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, we need $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$ or $\frac{k}{12} = \frac{3}{k}$ which gives $k^2 = 36$ i.e., $k = \pm 6$

Also, $\frac{3}{k} = \frac{k-3}{k}$ gives $3k = k^2 - 3k$, i.e., $6k = k^2$, which means $k = 0$ or $k = 6$.

Therefore, the value of k, that satisfies both the conditions, is $k = 6$. For this value, the pair of linear equations has infinitely many solutions.

10. Find the value of k for which the system of linear equation: $kx + 4y = k - 4$, $16x + ky = k$, has many solutions.

Sol. $kx + 4y = k - 4$ ....(1)

$16x + ky = k$ ....(2)

$a_1 = k$, $b_1 = 4$, $c_1 = -(k-4)$

$a_2 = 16$, $b_2 = k$, $c_2 = -k$

Here condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{16} = \frac{4}{k} = \frac{k-4}{k}$$

$$\frac{k}{16} = \frac{4}{k} \Rightarrow k^2 = 64 \quad x = \pm 8$$

Also, $\frac{4}{k} = \frac{k-4}{k} \Rightarrow 4k = k^2 - 4k$

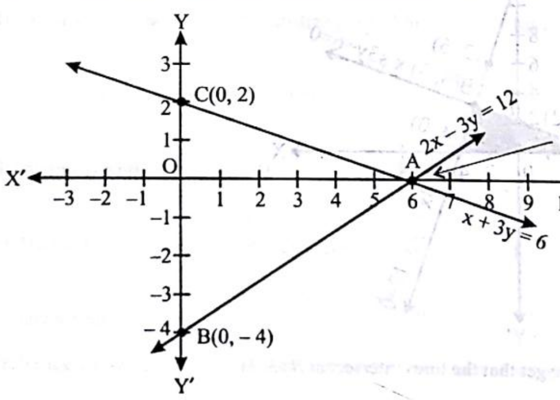
$\Rightarrow \quad k^2 - 8k = 0 \Rightarrow k(k-8) = 0$

$k = 0$ or $k = 8$ but $k = 0$ is not possible otherwise equation will be one variable.

$\therefore$ $k = 8$ is correct value for many solution.

11. Draw the graph of the following pair of linear equations : $x + 3y = 6$ and $2x - 3y = 12$ Hence, find the area of the region bounded by $x = 0, y = 0$ and $2x - 3y = 12$

Sol.



Consider both the equations, separately.

$x+3y=6$
$x=6-3y$

$2x-3y=12$
$2x=3y+12$
$x=\frac{3y+12}{2}$

We make the tables by giving the values to x for the both equations separately.

$x=6 \$ 3y$

x	0	6	3
y	2	0	1

$x=\frac{3y-12}{2}$

x	0	6	3
y	–4	0	–2

By plotting the points on the graph and joining them we get that the lines intersect at A (6, 0)
By joining the lines and points we get a ΔABC with vertices
A (6, 0), B (0, $4), C (0, 2). But $x=0, y=0$ and $2x \$ 3y=12$ gives us ΔOAB.

$\therefore$ Area of $\Delta OAB=\frac{1}{2}\times OA\times OB$ [$\because$ Area of $\Delta=\frac{1}{2}$ ñ base ñ corresponding altitude]

$=\frac{1}{2}\times 6\times 4=12$ sq. units

12. Solve the following system of linear equations graphically :
$3x+y-12=0$ and $x-3y+6=0$
Shade the region bounded by these lines and the x-axis. Also find the ratio of areas of triangles formed by the given lines with x-axis and y-axis.

Sol. Consider both the equations, separately

$3x+y=12$
$y=12-3x$

$x-3y=-6$
$3y=x+6$
$y=\frac{x+6}{3}$

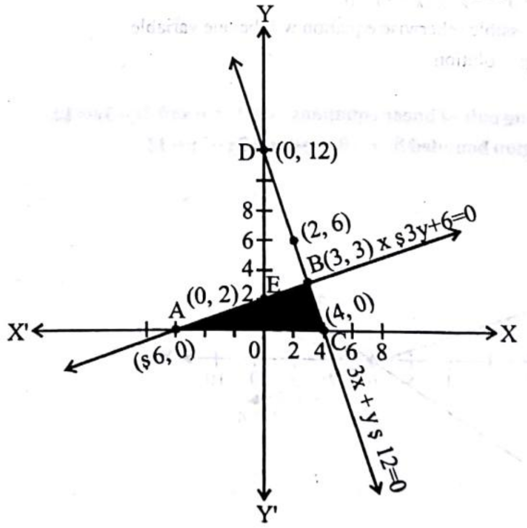
We make tables for both equations by giving the values to x.

$x=6-3y$

x	0	3	4
y	12	3	0

$x=\frac{3y-12}{2}$

x	0	3	–6
y	2	3	0



By plotting the points on the graph and joining them, we get that the lines intersect at B (3, 3)
$\therefore\ x=3, y=3$

Area of triangle ABC formed by the lines with x-axis $= \frac{1}{2} \times 10 \times 3 = 15$ sq. units

Area of triangle BED formed by lines with y-axis,

$= \frac{1}{2} \times 10 \times 3 = 15$ sq. units [$\because$ ar. of $\Delta = \frac{1}{2} \times$ base $\times$ corresponding altitude]

$\therefore$ Ratio of areas of triangles is given by $= \dfrac{ar(\Delta ABC)}{ar(\Delta BED)} = \dfrac{15}{15} = \dfrac{1}{1}$

Hence, required ratio = 1 : 1

13. Find the values of a and b for which the following system of linear equations has infinite solutions :
$2x + 3y = 7$ $(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$

Sol. $2x + 3y = 7$

$(a + b + 1)x + (a + 2b + 2)y = (4a + 4b + 1)$

In order that the two equations have infinite number of solutions, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$$\Rightarrow \underset{(A)}{\frac{2}{a+b+1}} = \underset{(B)}{\frac{3}{a+2b+2}} = \underset{(C)}{\frac{7}{4a+4b+1}}$$

Equating (A) and (C), we get $2(4a + 4b + 1) = 7(a + b + 1)$

$\Rightarrow 8a + 8b + 2 = 7a + 7b + 7$

$\Rightarrow a + b = 5$ ...(1)

Equating (B) and (C), we get $3(4a + 4b + 1) = 7(a + 2b + 2)$

$\Rightarrow 12a + 12b + 3 = 7a + 14b + 14$ or $5a - 2b = 11$ ...(2)

Multiplying (1) by (2) and adding to (2),

$\Rightarrow 7a = 10 + 11 \Rightarrow a = \dfrac{21}{7} = 3$

Now, $b = 5 - a = 5 - 3 = 2$

Hence for infinite solutions $a = 3, b = 2$

14. A boat goes 12 km. upstream and 40 km downstream in 8 hours. It can go 16 km, upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Sol. Let the speed of the boat in still water be x km/hour and the speed of the stream be y km/hr. then speed of boat in downstream is $(x + y)$ km/hr. and the speed of boat upstream is $(x - y)$ km/hr.

In Ist case : Distance covered in upstream = 12 km $\therefore$ time $= \dfrac{12}{x-y}$ $hr.$

distance covered in downstream = 40 km $\therefore$ time $= \dfrac{40}{x+y}$ $hr.$

Total time is 8 hrs. $\therefore \dfrac{12}{x-y} + \dfrac{40}{x+y} = 8$

In IInd case : Distance covered in upstream = 16 km $\therefore$ time $= \dfrac{16}{x-y}$ hr., downstream = 32 km $\therefore$ time $= \dfrac{32}{x+y}$ hr.

Total time taken = 8 hrs. $\therefore \dfrac{16}{x-y} + \dfrac{32}{x+y} = 8$

Solve them to get, x = Speed of boat = 6 km/hr, y = speed of stream = 2 km/hr.

15. It takes 12 hours to fill a swimming pool using 2 pipes. If the larger pipe is used for 4 hours and the smaller pipe for 9 hours, only half the pool is filled. How long would it take for each pipe alone to fill the pool ?

Sol. Let the time taken to fill the pool by the larger pipe = x hours and that by the smaller pipe = y hours.
Therefore, in 1 hour, volume of pool filled by the larger pipe = $1/x$
and by the smaller pipe = $1/y$
Given that both pipes can fill the pool in 12 hours.

$\therefore \quad \frac{12}{x}+\frac{12}{y}=1$ ...(1)

Given also that if the larger pipe is used for 4 hours and the smaller pipe for 9 hours, only half the pool is filled.

$\therefore \quad \frac{4}{x}+\frac{9}{y}=\frac{1}{2}$ ...(2)

Multiplying (2) by 3, $\frac{12}{x}+\frac{27}{y}=\frac{3}{2}$ ...(3)

Subtracting (3) from (1), $\frac{12}{y}-\frac{27}{y}=1-\frac{3}{2}\Rightarrow-\frac{15}{y}=-\frac{1}{2}\Rightarrow y=30$

Substituting the value of y in equation (1), $\frac{12}{x}+\frac{12}{30}=1$

$\Rightarrow \frac{12}{x}=1-\frac{12}{30}=1-\frac{2}{5}=\frac{3}{5} \quad \Rightarrow x=20$

Therefore, time taken by the larger pipe = 20 hr. and time taken by the smaller pipe = 30 hr.

16. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Sol. Let the initial speed, be u, and the distance is d; then time, $t=\frac{d}{u}$ or $d=ut$ ...(1)

According to problem : $d=(u+10)(t-2)$ [$\because$ increasing speed by 10 results in decrease in time by 2].

or $\quad ut=ut-2u+10t-20$

$\Rightarrow \quad 10t-2u=20$ ...(2)

Also $d=(u-10)(t+3)$ [$\because$ decreasing speed by 10 results in increase of time by 3]

$\Rightarrow \quad ut=ut+3u-10t-30$

$\Rightarrow \quad 3u-10t=30$ ...(3)

Adding equation (2) and (3)

$$\begin{array}{r} -2u+10t=20 \\ 3u-10t=30 \\ \hline u=50km/hr \end{array}$$

$\Rightarrow 3\times50-10t=30\Rightarrow t=12$ hrs.

From equation (1), the distance covered by the train, $d=ut=50\times12=600$ km.

17. *A* and *B* are friends and their ages differ by 2 years. A's father *D* is twice as old as *A* and *B* is twice as old as his sister *C*. The ages of *D* and *C* differ by 40 years. Find the ages of *A* and *B*.

Sol. Let the ages of A and B be x years and y years respectively.
Since, their ages differ by 2

$\therefore\ x-y=\pm 2$

Since, D is twice as old as A

$\therefore \quad$ D's age $=2x$ ...(1)

And B is twice as old as his sister C.

$\therefore$ C's age = $y/2$ years. ...(2)

From (1) and (2) it is clear that D is older than C

Since, ages of D and C differ by 40 years

$\therefore \quad 2x - \frac{y}{2} = 40 \Rightarrow 4x - y = 80$

Now, we have $x - y = 2$ and $4x - y = 80$ or $x - y = -2$ *and* $4x - y = 80$

on solving $x - y = 2$ and $4x - y = 80$, we get $x = 26$ and $y = 24$

on solving $x - y = -2$ and $4x - y = 80$, we get $x = 27\frac{1}{3}$ *and* $y = 29\frac{1}{3}$.

Hence, A's age = 26 years and B's age = 24 years or A's age = $27\frac{1}{3}$ years and B's age = $29\frac{1}{3}$ years.

18. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines (ii) parallel lines (iii) coincident lines

Sol. Let the linear equation be, $ax + by + c = 0$ ... (i)

(i) For lines to be intersecting with $2x + 3y - 8 = 0, \frac{a}{2} \neq \frac{b}{3} \Rightarrow b \neq \frac{3}{2}a$

There can be many such equations with the condition that $b \neq \frac{3}{2}a$

We can find the such linear equation which represent a line interesting the given line by putting $b = \frac{3}{4}a$ and $c = -3a$ in equation (i), then,

$ax + \frac{3}{4}ay - 3a = 0 \Rightarrow x + \frac{3}{4}y - 3 = 0 \Rightarrow 4x + 3y - 12 = 0$

(ii) For parallel lines, $\frac{a}{2} = \frac{b}{3} \neq \frac{c}{-8}$

$b = \frac{3}{2}a$, $c \neq -4a$ and there can be many such equation. We can find one such linear equation which represent a line parallel to the given line by putting $c = -3a$. Thus, required equation,

$ax + \frac{3}{2}ay - 3a = 0 \Rightarrow 2x + 3y - 6 = 0$

(iii) For co-incident lines, $\frac{a}{2} = \frac{b}{3} = \frac{c}{-8} = k$, say where k is a constant.

$\therefore a = 2k, b = 3k, c = -8k$

Then, required equation is $2kx + 3ky - 8k = 0$ or $k(2x + 3y - 8) = 0$

There can be many such equations for different values of k, showing co-incident lines. We can get one such example if $k = 2$, then, $4x + 6y - 16 = 0$.

19. Solve the following simultaneous equations by using the elimination method:
$2x + 3y = 15$; $4x - 3y = 3$

Sol. Label the equations as follows:

$2x+3y=15$ ....(1)

$4x-3y=3$ ....(2)

Notice that $3y$ appears on the left-hand side of both equations. Adding the left-hand side of (1) and (2), and then the right-hand sides, gives:

$$2x+3y+4x-3y=15+3$$
$$6x=18$$
$$\frac{6x}{6}=\frac{18}{6} \Rightarrow x=3$$

We have added equals to equals, and addition eliminates y.

Substituting $x=3$ in (1) gives :

$$2\times3+3y=15$$
$$6+3y=15$$
$$6x+3y-6=15-6$$
$$3y=9$$
$$\frac{3y}{3}=\frac{9}{3} \Rightarrow y=3$$

So, the solution is (3, 3)

20. Solve $9x-4y=8$; $13x+7y=101$

Sol. $9x-4y=8$ ....(1)

$13x+7y=101$ ....(2)

Multiply eq. (1) by 7 and eq. (2) by 4, we get

$63x-28y=56$

Add $52x+28y=404$

$115x=460$

$$\Rightarrow \quad x=\frac{460}{115} \Rightarrow x=4$$

Substitute $x=4$ in eq. (1)

$$9(4)-4y=8 \Rightarrow 36-8=4y \Rightarrow 28-4y \Rightarrow y=\frac{28}{4}=7$$

$\therefore\ x=4, y=7$

21. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $6x+3y=6xy$; $2x+4y=5xy$

(ii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$; $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Sol. (i) Given equations are $6x+3y=6xy$ and $2x+4y=5xy$

Dividing both the sides of both the equation by xy, we get $\frac{6}{y}+\frac{3}{x}=6$ and $\frac{2}{y}+\frac{4}{x}=5$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Equations become $3u+6v=6$ and $4u+2v=5$

Or $u+2v=2$ ...(1)

and $4u+2v=5$ ...(2)

Subtracting (1) from (2) we get, $3u=3 \Rightarrow u=1$ and $1+2v=2 \Rightarrow v=\frac{1}{2}$

$u=1=\frac{1}{x}, \Rightarrow x=1$ and $v=\frac{1}{2}=\frac{1}{y} \Rightarrow y=2$. So, $x=1$, and $y=2$.

(ii) Given equations are : $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Let us take $u=\frac{1}{3x+y}$ and $\frac{1}{3x-y}=v$ so , equation become, $u+v=\frac{3}{4}$ and $\frac{u}{2}-\frac{v}{2}=-\frac{1}{8}$

or $4u+4v=3$ and $4u-4v=-1$

Adding both we get $8u=2 \Rightarrow u=\frac{1}{4}$ and $4\times\frac{1}{4}+4v=3 \Rightarrow v=\frac{2}{4}=\frac{1}{2}$

$u=\frac{1}{3x+y}=\frac{1}{4} \Rightarrow 3x+y=4$ ...(1)

and $v=\frac{1}{3x-y}=\frac{1}{2} \Rightarrow 3x-y=2$ ...(2)

Adding (1) and (2) we get, $6x=6 \Rightarrow x=1$; and putting $x=1$ in any equation, say equation (1),
$3\times1+y=4 \Rightarrow y=1$. So, $x=1$, $y=1$

22. Solve the following system of linear equations for x and y.

$$a(x+y)+b(x-y)-\left(a^2-ab+b^2\right)=0 \text{ and } a(x+y)-b(x-y)-\left(a^2+ab+b^2\right)=0$$

Sol. The given system of equations is

$$a(x+y)+b(x-y)-\left(a^2-ab+b^2\right)=0 \text{ and } a(x+y)-b(x-y)-\left(a^2+ab+b^2\right)=0$$

This can be written as

$$(a+b)x+(a-b)y-\left(a^2-ab+b^2\right)=0 \text{ and } (a-b)x+(a+b)y-\left(a^2+ab+b^2\right)=0$$

Here $a_1=a+b$, $b_1=a-b$
$a_2=a-b$, $b_2=a+b$

and $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ i.e $-\frac{a+b}{a-b}\neq\frac{a-b}{a+b}$

Also, $a_1b_2-a_2b_1=(a+b)(a+b)-(a-b)(a-b)=(a+b)^2-(a-b)^2=4ab\neq0$

Therefore, the given system of equations has a unique solution.
Now, we can solve this system of equations by using cross-multiplication method which gives :

$$\Rightarrow \frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{x}{-(a-b)\left(a^2+ab+b^2\right)+(a+b)\left(a^2-ab+b^2\right)}=\frac{y}{-(a-b)\left(a^2-ab+b^2\right)+(a+b)\left(a^2+ab+b^2\right)}$$

$$=\frac{1}{(a+b)(a+b)-(a-b)(a-b)}$$

$$\Rightarrow \frac{x}{-\left(a^3-b^3\right)+\left(a^3+b^3\right)}=\frac{y}{2b\left(2a^2+b^2\right)}=\frac{1}{4ab} \Rightarrow \frac{x}{2b^3}=\frac{y}{2b\left(2a^2+b^2\right)}=\frac{1}{4ab}$$

$$\Rightarrow x=\frac{2b^3}{4ab}=\frac{b^2}{2a} \text{ and } y=\frac{2b\left(2a^2+b^2\right)}{4ab}=\frac{2a^2+b^2}{2a}$$

Hence, the solution of the system is $x=\frac{b^2}{2a}$ and $y=\frac{2a^2+b^2}{2a}$

23. Solve: $\frac{15}{x}+\frac{2}{y}=17, \frac{1}{x}+\frac{1}{y}=\frac{36}{5}$

Sol. We have the equations : $\frac{15}{x}+\frac{2}{y}=17$ ....(1)

and $\frac{1}{x}+\frac{1}{y}=\frac{36}{5}$ ....(2)

Multiplying eq. (2) by 2, we get, $\frac{2}{x}+\frac{2}{y}=\frac{72}{5}$ ....(3)

Subtracting (3) from (1), we get

$$\frac{15}{x}-\frac{2}{y}=17-\frac{72}{5} \Rightarrow \frac{13}{x}=\frac{85-72}{5} \Rightarrow \frac{13}{x}=\frac{13}{5}$$

$\Rightarrow$ $x \times 13=13 \times 5$ $x=\frac{13 \times 5}{13}=5$

Putting the value of x in (2)

$$\frac{1}{5}+\frac{1}{y}=\frac{36}{5} \Rightarrow \frac{1}{y}=\frac{36}{5}-\frac{1}{5}$$

$\Rightarrow$ $\frac{1}{y}=\frac{35}{5} \Rightarrow 35 \times y=5 \Rightarrow y=\frac{5}{35}$ $\therefore$ $y=\frac{1}{7}$ Hence, $x=5,\ y=\frac{1}{7}$

24. The numerator of a fraction is 4 less than the denominator if the numerator is decreased by 2 and the denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Sol. Let the numerator and denominator of the fraction be x and y respectively.

Then required fraction $=\frac{x}{y}$

$\therefore$ $y-x=4$ ....(1)

and $y+1=8(x-2)$

$\Rightarrow$ $y-8x=-17$ ....(2)

Subtracting (1) from (2),

$y-8x-(y-x)=-17-4$

$-7x=21$

$x=\frac{21}{7}=3$

$\therefore$ $y=4+3=7$

$\therefore$ Required fraction $=\frac{3}{7}$

25. The sum of two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.

Sol. Let unit digit be x and ten's digit be y. No. will be $10y+x$.

According to problem, $(10y+x)+(10x+y)=165$

$x+y=15$ .... (1)

and $x-y=3$ .... (2)

or $-(x-y)=3$ .... (3)

$x+y=15$	$x+y=15$
$x-y=3$	$-x+y=3$
$2x=18$	$2y=18$
$\therefore\ x=9$	$\therefore\ y=9$
$\therefore\ y=6$	$\therefore\ x=6$
$\therefore$ No. will be 6, 9	$\therefore$ No. will be 9, 6

26. Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hrs. and if they go in opposite directions, they meet in 9/7 hours. Find their speeds.

Sol. Let the speeds of the cars starting from A and B be x km/hr and y km/hr. respectively

Acc. to problem, $9x-90=9y$ .... (1)

and $\frac{9}{7}x+\frac{9}{7}y=90$ .... (2)

Solving we get $x=40$ km/hr, $y=30$ km/hr.

Speed of car $A=40$ km/hr speed of car $B=30$ km/hr.

27. A vessel contain's mixture of 24 ℓ milk and 6 ℓ water and a second vessel contains a mixture of 15 ℓ milk and 10 ℓ water. How much mixture of milk and water should be taken from the first and the second vessel separately and kept in a third vessel so that the third vessel may contain a mixture of 25 ℓ milk and 10 ℓ water ?

Sol. Let $x\ \ell$ of mixture be taken from lst vessel and $y\ \ell$ of the mixture be taken from 2nd vessel and kept in 3rd vessel so that $(x+y)\ \ell$ of the mixture in third vessel may contain 25 ℓ of milk and 10 ℓ of water.

A mixture of $x\ \ell$ from lst vessel contains $\frac{24}{30}x=\frac{4}{5}x\ell$ of mile and $\frac{x}{5}\ell$ of water. And a mixture of $y\ \ell$ from 2nd vessel contains $\frac{3y}{5}\ \ell$ of milk and $\frac{2y}{5}\ \ell$ of water.

$\therefore\ \frac{4}{5}x+\frac{3}{5}y=25$ .... (1) ; $\frac{x}{5}+\frac{2}{5}y=10$ .... (2)

Solve it to get x and y, i.e., $x=20\ell,\ y=15\ell$

28. Solve the systems of equations graphically :

(i) $2x+3y=10$

$3x-y=4$

(ii) $\frac{2x+1}{3}+\frac{3y-1}{2}=2$

$\frac{3x-1}{2}+\frac{2y+1}{3}=2$

Sol. (i) $2x+3y=10$

$\Rightarrow\ 3y=10-2x$

$\Rightarrow\ y=\frac{10-2x}{3}$

When $x=5$, $y=\frac{10-2(5)}{3}=0$

$3x-y=4$

$\Rightarrow\ -y=4-3x$

$\Rightarrow\ y=3x-4$

When $x=5$, $y=3(0)-4=-4$

When $x = 8$, $y = \frac{10-2(8)}{3} = -2$

When $x = 11$, $y = \frac{10-2(11)}{3} = -4$

$2x+3y=10$

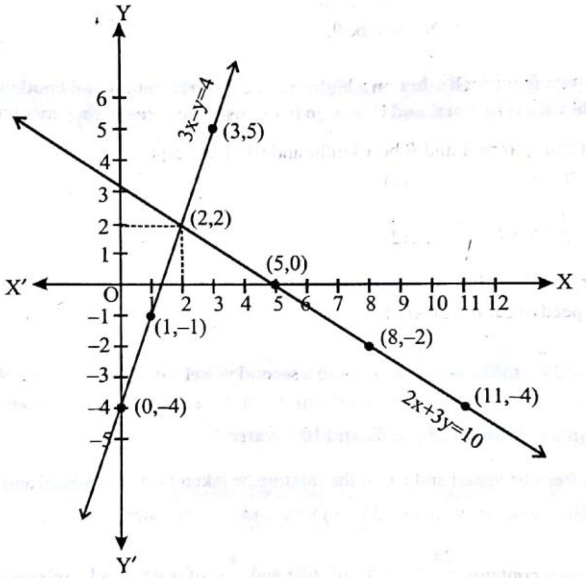
x	5	8	11
y	0	−2	−4

When $x = 1$, $y = 3(1) - 4 = -1$

When $x = 3$, $y = 3(3) - 4 = 5$

$3x - y = 4$

x	0	1	3
y	−4	−1	5



From the graph, the solution is the point of intersection of the lines, i.e., $x = 2, y = 2$.

(ii) $\frac{2x+1}{3} + \frac{3y-1}{2} = 2$; .... (1) $\frac{3x-1}{2} + \frac{2y+1}{3} = 2$ .... (2)

$\frac{2x+1}{3} + \frac{3y-1}{2} = 2$

Multiplying both sides by 6

$2(2x+1) + 3(3y-1) = 12$

$\Rightarrow 4x + 2 + 9y - 3 = 12$

$\Rightarrow 4x + 9y = 13$

$\Rightarrow 9y = 13 - 4x$

$\Rightarrow y = \frac{13-4x}{9}$

When $x = 1$, $y = \frac{13-4(1)}{9} = 1$

When $x = 10$, $y = \frac{13-4(10)}{9} = -3$

When $x = 19$, $y = \frac{13-4(19)}{9} = -7$

$\frac{3x-1}{2} + \frac{2y+1}{3} = 2$

Multiplying both sides by 6

$3(3x-1) + 2(2y+1) = 12$

$\Rightarrow 9x - 3 + 4y + 2 = 12$

$\Rightarrow 9x + 4y = 13$

$\Rightarrow 4y = 13 - 9x$

$\Rightarrow y = \frac{13-9x}{4}$

When $x = 1$, $y = \frac{13-9(1)}{4} = 1$

When $x = 5$, $y = \frac{13-9(5)}{4} = -8$

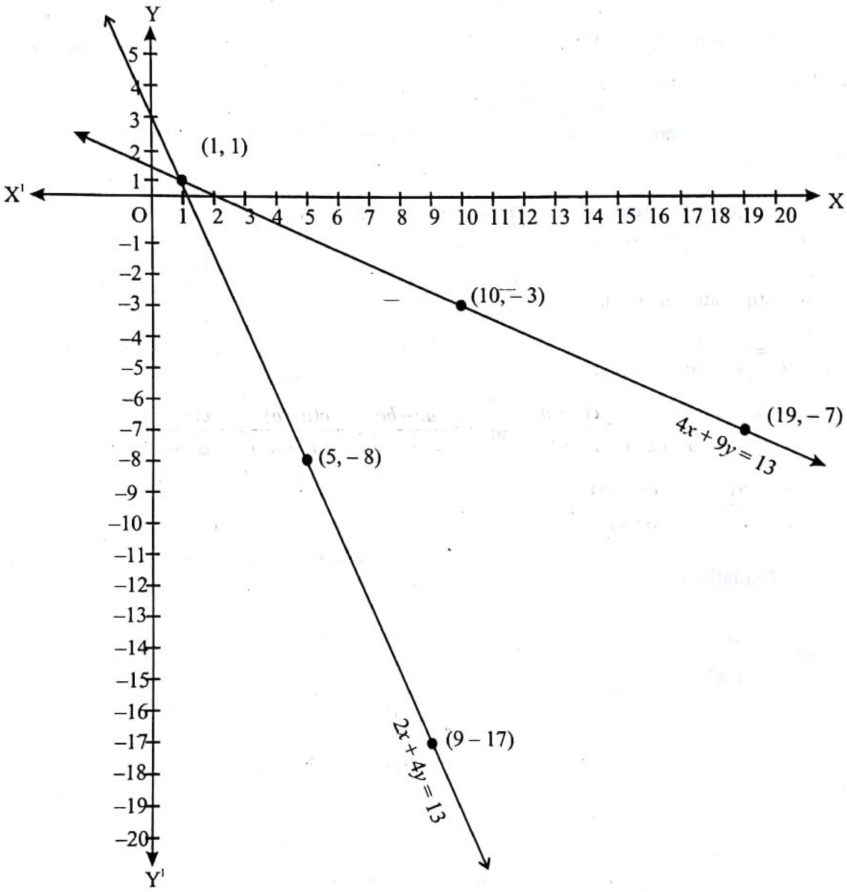
When $x = 9$, $y = \frac{13-9(9)}{4} = -17$

Equation (1)

x	1	10	19
y	1	–3	–7

Equation (2)

x	1	5	8
y	1	–8	–17



From the graph, the solution is the point of intersection of the lines i.e. $x = 1, y = 1$

29. Solve the equations : $\frac{2x+1}{3}+\frac{3y+2}{5}=2,\ \frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1$

Sol. $$\frac{2x+1}{3}+\frac{3y+2}{5}=2 \qquad ....(1)$$

and $$\frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1 \qquad ....(2)$$

Let $\frac{2x+1}{3}=u$ and $\frac{3y+2}{5}=v$

The, the equations become

$u+v=2$ .... (3)

$2u-3v=-1$ .... (4)

Multiplying (3) by 3,

$3u+3v=6.$ .... (5)

Adding (4) and (5)

$5u=5 \Rightarrow u=1$

Substituting this value of u in (3),

$1+v=2 \Rightarrow v=2-1=1$

Then $\frac{2x+1}{3}=u=1$ and $\frac{3y+2}{5}=v=1$

$\Rightarrow \quad 2x+1=3$ and $3y+2=5$

$\Rightarrow \quad 2x=3-1=2$ and $3y=5-2=3$

$\Rightarrow \quad x=1$ and $y=1$

Therefore, the solution is $x=1, y=1$

30. Solve the systems of equations by the cross-multiplication method.

$\boldsymbol{ax+by=c}$; $\quad \boldsymbol{bx-ay=c}$

Sol. $ax+by=c,\ bx-ay=c$

Using the cross-multiplication method,

$$\frac{x}{-ac-bc}=\frac{y}{ac-bc}=\frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad x=\frac{-ac-bc}{-a^2-b^2}=\frac{-c(a+b)}{-(a^2+b^2)}=\frac{c(a+b)}{a^2+b^2} \text{ and } y=\frac{ac-bc}{-a^2-b^2}=\frac{c(a-b)}{-(a^2+b^2)}=-\frac{c(a+b)}{a^2+b^2}$$

Therefore, $x=\frac{c(a+b)}{a^2+b^2},\ y=-\frac{c(a+b)}{a^2+b^2}$

31. Solve the system of equations :

$\boldsymbol{ax+by=1}$

$\boldsymbol{bx+ay=\frac{2ab}{a^2+b^2}}$

Sol. $ax+by=1$ .... (1)

$bx+ay=\frac{2ab}{a^2+b^2}$ .... (2)

$$(a+b)x+(a+b)y=1+\frac{2ab}{a^2+b^2}$$

$$(a+b)x+(a+b)y=\frac{a^2+b^2+2ab}{a^2+b^2} \Rightarrow (a+b)(x+y)=\frac{(a+b)^2}{a^2+b^2}$$

$$\Rightarrow \quad x+y=\frac{a+b}{a^2+b^2} \quad \text{......... (3)}$$

Subtracting (2) from (1)

$$(a-b)x+(b-a)y=1-\frac{2ab}{a^2+b^2}$$

$$\Rightarrow \quad (a-b)x-(a-b)y=\frac{a^2+b^2-2ab}{a^2+b^2}$$

$$\Rightarrow \quad (a-b)(x-y)=\frac{(a-b)^2}{a^2+b^2} \Rightarrow x-y=\frac{a-b}{a^2+b^2} \quad \text{............. (4)}$$

Adding (3) and (4),

$$2x = \frac{a+b}{a^2+b^2} + \frac{a-b}{a^2+b^2} = \frac{2a}{a^2+b^2} \Rightarrow x = \frac{a}{a^2+b^2}$$

Subtracting (4) from (3)

$$2y = \frac{a+b}{a^2+b^2} - \frac{a-b}{a^2+b^2} = \frac{2b}{a^2+b^2} \Rightarrow y = \frac{a}{a^2+b^2}$$

Therefore the solution is $x = \frac{a}{a^2+b^2}$, $y = \frac{b}{a^2+b^2}$

32. A part of the monthly expenses of a family is constant and the remaining varies with the price of wheat. When the price of wheat is ₹ 250 per quintal, the total monthly expenses are ₹ 1000 and when it is ₹ 240 per quintal, the total monthly expenses ₹ 980 per quintal. Find the total monthly expenses of the family when the cost of wheat is ₹ 350 per quintal.

Sol. Let the constant part of the expenditure = ₹ x
and the variable part = ₹ y × price of wheat.
Given that when the price of wheat is ₹ 250 per quintal, the total expenses are ₹ 1000.
$\therefore \quad x + 250y = 1000$ .... (1)
Given also that when the price of wheat is ₹ 240 per qunital, the total expenses are ₹ 980
$\therefore \quad x + 240y = 980$ .... (2)
Subtracting (2) from (1),
$10y = 20 \Rightarrow y = 2$
Substituting this value of y in (1)
$x + 250(2) = 1000$
$\Rightarrow \quad x = 1000 - 500 = 500$
Therefore, when the price of wheat is ₹ 350 per quintal,
total expenses $= x + 350y = 500 + 350(2) =$ ₹ 1200
$\therefore$ total expenses = ₹ 1200

33. The sum of the digits of a two-digit number is 8. If the digits are reversed, the number is decreased by 54. Find the original number.

Sol. Let the two-digit number be $10x + y$.
Then, we have : $x + y = 8$ .... (i)
and $10y + x = 10x + y$ 54 or, $x - y = 54/9 = 6$ .... (ii)
Solving equations (i) and (ii), we get.
$x = (8+6)/2 = 7$ and $y = 1$
$\therefore$ The required number $= 7 \times 10 + 1 = 71$
But the same question can be solved using this sample formula. The required number

$$= 5\left[\text{Sum of digits} + \frac{\text{Decrease}}{9}\right] + \frac{1}{2}\left[\text{Sum of digits} - \frac{\text{Decrease}}{9}\right]$$

$= 5(8+6) + 1/2\,(8-6) = 70 + 1 = 71.$

34. Solve the following simultaneous equations by using the elimination method:
$2x + 3y = 13$, $3x + 2y = 12$

Sol. Label the equations as follows:
$2x + 3y = 13$ ..... (1) and $3x + 2y = 12$ ..... (2)
Multiplying (1) by 2 and (2) by 3 gives:
$4x + 6y = 26$ ..... (3) and $9x + y = 12$ ..... (4)
Subtracting (3) from (4) gives:
$9x + 6y - 4x - 6y = 36 - 26 \Rightarrow 5x = 10$

$\frac{5x}{5}=\frac{10}{5} \Rightarrow x=2.$ So, the solution is (2, 3).

Substituting $x=2$ in (1) gives $2\times 2+3y=13 \Rightarrow 4+3y=13$

$4+3y-4=13-4 \Rightarrow 3y=9 \Rightarrow \frac{3y}{3}=\frac{9}{3} \Rightarrow y=3.$ So, the solution is (2, 3)

35. Solve the following simultaneous equations by using the substitution method : $x=2y+10$, $2x+y=5$

Sol. Label the equations as follows:

$x=2y+10$ ..... (1)

$2x+y=5$ ..... (2)

Sustituting $x=2y+10$ in (2) gives :

$2(2y+10)+y=5$ $4y+20+y=5 \Rightarrow 5y+20=5$

$\Rightarrow 5y+20-20=5-20 \Rightarrow 5y=-15$ $\frac{5y}{5}=\frac{-15}{2} \Rightarrow y=-3$

Substituting $y=-3$ in (1) gives : $x=2(-3)+10=4$, So, the solution is (4, – 3)

36. Vikas tells his son "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Represent this situation both algebraically and graphically.

Sol. Let the present age of Vikas = x years

and present age of his son = y years

Seven years ago father's age $(x-7)$ years

and son's age = $(y-7)$ years

According to given condition, we get

$(x-7)=7(y-7)$

$\Rightarrow x-7y=-42$ or $x=7y-42$

Points to be plotted (0, 6) and (7, 7)

x	0	7
y	6	7

According to the second condition :

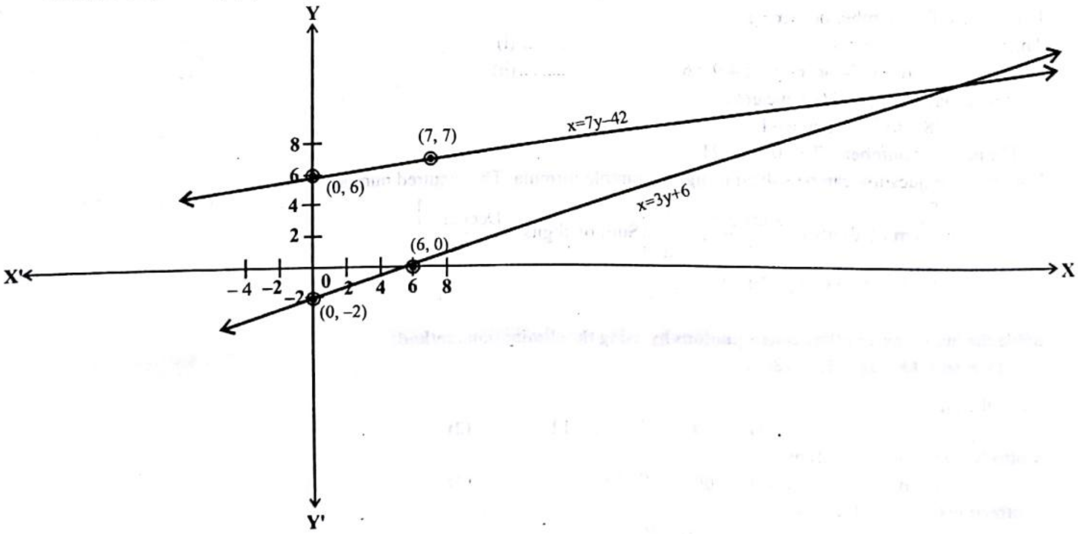
After three years

Father's age = $(x+3)$ years, Son's age = $(y+3)$ years

$\therefore (x+3)=3(y+3) \Rightarrow x-3y=6$ or $x=3y+6$

Points to be plotted (6, 0) and (0, –2)

x	6	0
y	0	–2



1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is ....
2. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is ....
3. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. Nuri age is ....
4. Two distinct natural numbers are such that the sum of one number and twice the other number is 6. The two numbers are ....
5. If $p+q=k, p-q=n$ and $k>n$, then q is .... (positive/ negative).
6. 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. The cost of one pencil is ....
7. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. The dimensions of the rectangle are ....
8. Sum of the ages of X and Y, 12 years, ago, was 48 years and sum of the ages of X and Y, 12 years hence will be 96 years. Present age of X is ....
9. The number of common solutions for the system of linear equations $5x+4y+6=0$ and $10x+8y=12$ is ....
10. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. The distance covered by the train is ....
11. If $2x+3y=5$ and $3x+2y=10$, then $x-y=$ ....
12. If $\frac{1}{x}+\frac{1}{y}=k$ and $\frac{1}{x}-\frac{1}{y}=k$, then the value of y ....

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. If a pair of linear equations is given by $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ and $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$. In this case, the pair of linear equations is consistent.
2. If a pair of linear equations is given by $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ and $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$. In this case, the pair of linear equations is consistent.
3. If a pair of linear equations is given by $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ and $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$. In this case, the pair of linear equations is consistent.
4. If the total cost of 3 chairs and 2 tables is ₹ 1200 and the total cost of 12 chairs and 8 tables is ₹ 4800, then the cost of each chair must be ₹ 200 and each table must be ₹ 300. (True/False)
5. $3x-y=3, 9x-3y=9$ has infinite solution.
6. $\sqrt{2}x+\sqrt{3}y=0$, $\sqrt{3}x-\sqrt{8}y=0$ has no solution.
7. 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz are 3 and 7, respectively.
8. $3x+2y=5, 2x-3y=7$ are consistent pair of equation.
9. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. ₹ 400 are fixed charges.
10. If the total cost of 2 apples and 3 mangoes is ₹ 22, then the cost of each apple and each mango must be ₹ 5 and ₹ 4, respectively, (where cost of each apple and mango is an integer). (True/False)
11. Every solution of the equation is a point on the line representing it.
12. In a ΔABC, $\angle C=3\angle B=2(\angle A+\angle B)$, then angles are 20°, 40°, 100°.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II give value of x and y for pair of equation given in column I, match them correctly.

Column I	Column II
(A) $2x+y=8, x+6y=15$	(p) (3,4)
(B) $5x+3y=35, 2x+4y=28$	(q) (1/14, 1/6)
(C) $\frac{1}{7x}+\frac{1}{6y}=3, \frac{1}{2x}-\frac{1}{3y}=5$	(r) (4,5)
(D) $15x+4y=61$, $4x+15y=72$	(s) (3,2)

2. Column II give pair of two number for solution to in column I, match them correctly.

Column I	Column II
(A) Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m	(p) (39, 13)
(B) The difference between two numbers is 26 and one number is three times the other.	(q) (40, 10)
(C) Five years hence, the age of Jacob will be three times that of his daughter. Five years ago, Jacob's age was seven times that of his daughter.	(r) (35, 71)
(D) If 1 is added to each of the given two numbers, then their ratio is 1 : 2 . If 5 is subtracted from each of the numbers, then their ratio is 5 : 11.	(s) (20, 16)

3. Match the column

Column I	Column II
(A) $5y-4=14, y-2x=1$	(p) Infinite solutions
(B) $6x-3y+10=0, 2x-y+9=0$	(q) Consistent
(C) $3x-2y=4, 9x-6y=12$	(r) No solution
(D) $2x-3y=8, 4x-6y=9$	(s) Inconsistent

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Solve the equations

$3x+2y=11$ ...(i)

$2x+3y=4$ ...(ii)

2. Check wehether the following given pair of equations has no solution, unique solution or infinite solutions.

$3x+4y=8$

$9x+12y=24$

3. For what value of k will the equations $x+5y-7=0$ and $4x+20y+k=0$ represent coincident lines?
4. Solve : $3x+2y+25=0,\ x+y+15=0$
5. Solve the following equations, algebraically

$x+2y=-1$ and $2x-3y=12$

6. Find the value of P for which the given system of equations has only solution (i.e., unique solution)

$Px-y=2;\ 6x-2y=3$

7. Six years hence, a man's age will be three times the age of his son and three years ago, he was nine times as old as his son. Fin their present ages.
8. Sanjay starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹ 4500 after four years of service and ₹ 5400 after 10 years, find his initial salary and annual increment.
9. Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hr, if he travels 160 km by train and the rest by car. He takes 12 minutes more, if he travels 240 km by train and the rest by car. Find the speed of train and the car.
10. The perimeter of a rectangle is 40 cm. The ratio of its side is 2 : 3. Find its length and breadth.
11. Solve the equations

$$\frac{x}{3}+\frac{y}{2}=4$$

$$\frac{2x}{3}-\frac{y}{4}=3$$

12. A can do a piece of work in 24 days. If B is 60% more efficient than A, then find the number of days required by B to do the twice as large as the earlier work.
13. A group of soldiers can completely destroy an enemy bunker in 7 days. However 12 soldiers fell ill. The remaining now can do the job in 10 days. Find the original group strength.
14. A laboratory technician has acid solution in two concentrations, 50% and 100%. He wants to mix the right amount of each to make 400 mL of 60% acid solution by volume. How many millilitres of each solution is needed?
15. Determine the value of c for which the following system of linear equations has no solution:

$cx+3y=3,\ 12x+cy=6.$

16. Solve for x and y :

$$\frac{2}{3}x+\frac{3}{y}=13,\ \frac{5}{x}-\frac{4}{y}=-2,\ x,y\neq 0$$

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the values of x and y in the system of equations.

$ax+by-a+b=0$

$bx-ay-a-b=0$

2. Determine the values of a and b for which the following system of linear equations has infinitely many solutions:

$3x-(a+1)y=2b-1,\ 5x+(1-2a)y=3b$

3. Solve : $x+4y=14;\ 7x-3y=5$
4. Find the value of a and b so that the following system of equations have infinitely many solution.

$2x-y=5;\ (a-2b)x-(a+b)y=15$

5. Solve the following system of linear equations :

$2(ax-by)+(a+4b)=0$

$2(bx+ay)+(b-4a)=0$

6. If $(x-4)$ is a factor of $x^3+ax^2+2bx-24$ and $a-b=8$, find the values of a and b.
7. Solve graphically for x and y :

$2x+3y=12,\ x-y=1$. Shade the region between the two lines and x-axis.

8. Draw the graph of $x-y+1=0$ and $2x+y-10=0$. Calculate the area bounded by these lines and x-axis.
9. For what value of p will the following system of equations represent coincident lines?

$x+5y=7$

$3x+15y=p$

10. Determine the value of k for which the following system of equations becomes consistent :
$7x-y=5, 21x-3y=k$.

11. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

12. One hundred men in 10 days do one third of a piece of work. The work is then required to be completed in another 13 days. On the next day (the eleventh day) 50 more men are employed, and on the day after that, another 50. How many men must be relieved at the end of the 18th day so that the rest of the men, working for the remaining time, will just complete the work ?

13. The set up cost of a machine that produces brass plates is ₹ 750. After set up, it costs ₹ 0.25 to produce each plate. Management is considering the purchase of a larger machine that can produce the same plate at a cost of ₹ 0.20 per plate. If the set up cost of the larger machine is ₹ 1,200, how many plates would the company have to produce so that total cost is same for both the machines?

14. Formulate the following problem as a pair of equations, and hence find their solutions:
Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

15. Solve graphically the system of linear equations:
$$4x-3y+4=0$$
$$4x+3y-20=0$$
Find the area of the region bounded by these lines and X-axis.

16. Points A and B are 90 km. apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hrs. and if they go in opposite directions, they meet in 9/7 hrs. Find their speeds.

17. In a cyclic quadrilateral $ABCD$, $\angle A=(2x+11)°$, $\angle B=(y+12)°$, $\angle C=(3y+6)°$ and $\angle D=(5x-25)°$, find the angles of the quadrilateral.

18. A lady has 25 p and 50 p coins in her purse. If in all she has 40 coins totalling ₹12.50, find the number of coins of each type she has.

19. The monthly incomes of A and B are in the ratio of 9 : 7 and their monthly expenditures are in the ratio of 4 : 3. If each saves ₹ 1600 per month, find the monthly income of each.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Solve graphically the following system of linear equations:
$2x+3y=9,\ x-y=2$

2. For what value of k will the system of linear equations have infinite number of solutions :
$kx+4y=k-4,\ 16x+ky=k$?

3. Places A and B are 80 km apart from each other on a highway. A car starts from A and another starts from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directons they meet in 1 hour and 20 minutes. Find the speed of the cars.

4. Determine the values of a and b for which the following system of linear equations has infinitely many solutions :
$3x-(a+1)y=2b-1$ and $5x+(1-2a)y=3b$

5. A car averages 12.5 L/ 100 km in city driving and 7.5 L/100 km on the highway. In a week of mixed driving, the car used 35 L of fuel and travelled 400 km. Determine the distance travelled in highway driving.

6. After covering a distance of 30 km with a uniform speed there is some defect in a train engine and therefore, its speed is reduced to 4/5 of its original speed. Consequently, the train reaches its destination late by 45 minutes. Had it happened after covering 18 kilometers more, the train would have reached 9 minutes earlier. Find the speed of the train and the distance of journey.

7. A man travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But if he travels 200 km by train and 30 minutes. But if he travels 200 km by train and rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

8. Solve the following system of linear equations graphically :
$x-y=1,\ 2x+y=8$.
Shade the area bounded by these two lines and theY-axis.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. A can do a piece of work in 24 days. If B is 60% more efficient than A, then the number of days required by B to do the twice as large as the earlier work is –

(a) 24 (b) 36
(c) 15 (d) 30

2. X's salary is half that of Y's. If X got a 50% rise in his salary and Y got 25% rise in his salary, then the percentage increase in combined salaries of both is –

(a) 30 (b) $33\frac{1}{3}$

(c) $37\frac{1}{2}$ (d) 75

3. The points (7, 2) and (–1, 0) lie on a line –

(a) $7y=3x-7$ (b) $4y=x+1$

(c) $y=7x+7$ (d) $x=4y+1$

4. At present ages of a father and his son are in the ratio 7 : 3, and they will be in the ratio 2 : 1 after 10 years. Then the present age of father (in years) is –

(a) 42 (b) 56

(c) 70 (d) 77

5. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is –

(a) 2 (b) 3

(c) 5 (d) 7

6. A motor boat takes 2 hours to travel a distance 9 km. down the current and it takes 6 hours to travel the same distance against the current. The speed of the boat in still water and that of the current (in km/hour) respectively are –

(a) 3, 1.5 (b) 3, 2

(c) 3.5, 2.5 (d) 3, 1

7. The 2 digit number which becomes (5/6)th of itself when its digits are reversed. The difference in the digits of the number being 1is

(a) 45 (b) 54

(c) 36 (d) None of these

8. x & y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x+y$ is

(a) 10 (b) 11

(c) 12 (d) 13

9. If $3x+4y : x+2y = 9 : 4$, then $3x+5y : 3x-y$ is equal to–

(a) $4:1$ (b) $1:4$

(c) $7:1$ (d) $1:7$

10. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is –

(a) 36 (b) 63

(c) 48 (d) 84

11. $a, b, c, (a > c)$ are the three digits, from left to right of a three digit number. If the number with these digits reversed is subtracted from the original number, the resulting number has the digit 4 in its unit's place. The other two digits from left to right are –

(a) 5 and 4 (b) 5 and 9

(c) 4 and 5 (d) 9 and 5

12. A man can row a boat in still water at the rate of 6 km per hour. If the stream flows at the rate of 2 km/hour, he takes half the time going downstream than going upstream the same distance. His average speed for upstream and down stream trip is –

(a) 6 km/hour

(b) 16/3 km/hour

(c) Insufficient data to arrive at the answer

(d) none of the above

13. A boat travels with a speed of 15 km/h in still water. In a river flowing at 5 km/hr, the boat travels some distance downstream and then returns. The ratio of average speed to the speed in still water is –

(a) $8:3$ (b) $3:8$

(c) $8:9$ (d) $9:8$

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. I. If $x-y=xy=1-x-y$, then $x+y$ is $\frac{5}{6}$

II. The system of equations $3x+2y=a$ and $5x+by=4$ has infinitely many solutions for x and y, then $a=4, b=3$

III. If $\frac{x}{a}+\frac{y}{b}=2$ and $ax-by=a^2-b^2$, then $x=a, y=b$

Which is true?

(a) I only (b) II only

(c) III only (d) None of these

2. I. If $3x-5y=-1$ and $x-y=-1$, then $x=-2, y=-1$

II. $2x+3y=9, 3x+4y=5 \Rightarrow x=-21, y=17$

III. $\frac{2x}{a}+\frac{y}{b}=2, \frac{x}{a}-\frac{y}{b}=4 \Rightarrow x=2a, y=2b$

Which is true?

(a) I (b) II

(c) III (d) None of these

3. Let $x=-y$ where $y>0$. Which of the following statements is/are correct?

(a) $x^2y>0$ (b) $x+y=0$

(c) $xy<0$ (d) $\frac{1}{x}-\frac{1}{y}=0$

4. If a pair of linear equations is consistent, then the lines will be

(a) parallel (b) always coincident

(c) inter secting (d) coincident

5. For what values of k, do the equations $3x-y+8=0$ and $6x-ky=-16$ represent coincident lines?

(a) solution of $3^k-9=0$

(b) solution of $2^k-8=0$

(c) 2

(d) 3

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

If we have two simultaneous equations

$ax + by = c$ ...(1)

and $bx + ay = d$, then in order to solve ...(2)

we find (1) + (2) and then (1) – (2), we shall get

$(a+b)x + (a+b)y = c + d$

i.e. $x + y = \dfrac{c+d}{a+b}$

and $(a-b)x - (a-b)y = c - d$

i.e. $x - y = \dfrac{c-d}{a-b}$

To find x, (3) + (4) gives, $2x = \dfrac{c+d}{a+b} + \dfrac{c-d}{a-b}$

$\Rightarrow \quad x = \dfrac{1}{2}\left(\dfrac{c+d}{a+b} + \dfrac{c-d}{a-b}\right)$

To find y, (3) + (4) gives, $y = \dfrac{1}{2}\left(\dfrac{c+d}{a+b} - \dfrac{c-d}{a-b}\right)$

Read the above passage carefully and mark the correct choice.

1. The solution of
   $217x + 131y = 913$
   $131x + 217y = 827$ is
   (a) $x = 2,\ y = 3$ (b) $x = 3,\ y = 2$
   (c) $x = 2,\ y = 2$ (d) $x = 3,\ y = 3$

2. The solution of
   $37x + 41y = 70$
   $41x + 37y = 86$ is
   (a) $x = 3,\ y = 1$ (b) $x = 3,\ y = -1$
   (c) $x = -3,\ y = 1$ (d) $x = 1,\ y = 3$

3. The solution of
   $x + 2y = \dfrac{3}{2}$
   $2x + y = \dfrac{3}{2}$ is
   (a) $x = 3,\ y = 1$ (b) $x = \dfrac{1}{2},\ y = \dfrac{1}{2}$
   (c) $x = \dfrac{1}{2},\ y = 0$ (d) $x = 0,\ y = \dfrac{1}{2}$

PASSAGE-II

A system of linear equations is given as follows :

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

1. Condition for two lines to have a unique solution is
   (a) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ (b) $\dfrac{a_1}{a_2} \neq \dfrac{c_1}{c_2}$
   (c) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (d) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

2. Condition for two lines to have infinitely many solutions is
   (a) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ (b) $\dfrac{a_1}{a_2} \neq \dfrac{c_1}{c_2}$
   (c) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (d) None of these

3. Both lines are parallel only if
   (a) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ (b) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
   (c) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (d) None of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** $3x + 4y + 5 = 0$ and $6x + ky + 9 = 0$ represent parallel lines if $k = 8$
   Reason : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represent parallel lines if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$
   Which is the correct answer

2. **Assertion :** $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has no solution if $k = 2$
   Reason : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are consistent if $\dfrac{a_1}{a_2} \neq \dfrac{k_1}{k_2}$

3. **Assertion :** If the system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then $2a - b = 0$
   Reason : The system of equations $3x - 5y = 9$ and $6x - 10y = 8$ has a unique solution.

4. **Assertion :** If the pair of lines are coincident, then we say that pair is consistent and it has a unique solution.
   Reason : If the pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations.
5. **Assertion :** If $kx - y - 2 = 0$ and $6x - 2y - 3 = 0$ are inconsistent, then $k = 3$
   Reason : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are inconsistent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
6. **Assertion :** $3x - 4y = 7$ and $6x - 8y = k$ have infinite number of solution if $k = 14$
   Reason : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
7. **Assertion :** The linear equations $x - 2y - 3 = 0$ and $3x + 4y - 20 = 0$ have exactly one solution
   Reason : The linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ have a unique solution
8. **Assertion :** $bx + 2y = 5$ and $3x + y = 1$ have a unique solution if $k = 6$
   Reason : $x + 2y = 3$ and $5x + ky + 7 = 0$ have a unique solution $k \neq 1$

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

	Column-I		Column-II
(A)	No solution	(p)	$5x - 15y = 8, 3x - 9y = \frac{24}{5}$
(B)	Infinitely many solutions	(q)	$2x + 4y = 10, 3x + 6y = 12$
(C)	Unique solution	(r)	$3x - 2y = 4, 6x - 4y = 8$
(D)	System is consistent	(s)	$2x + y = 6, 4x - 2y - 4 = 0$
		(t)	$3x - y = 8, x - \frac{y}{3} = 3$
		(u)	$x - y = 8, 3x - 3y = 16$

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. For what value of k will the following system of linear equations have no solutions?
   $3x + y = 1$ and $(2k-1)x + (k-1)y = 2k+1$
2. Solve the system of equations : $ax + by = 1$ and $bx + ay = \frac{2ab}{a^2 + b^2}$
3. A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number.
4. The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.
5. Find the values of a and b for which the following system of linear equations has infinite number of solutions:
   $$2x + 3y = 7$$
   $$(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$$
6. Three bodies move in the same straight line from point A to point B. The second body began moving 5 sec and the third body 8 sec later than the first one. The speed of the first body is less than that of the second by 6 m/s and the speed of the third body is equal to 30 m/s. If the distance $AB = x$ m and if it is known that all the three reach B at the same instant of time, then form equations connecting x and v, where v is the speed in m /sec of the first body.
7. Solve the following pairs of equations by reducing them to a pair of linear equations:
   (i) $6x + 3y = 6xy$; $2x + 4y = 5xy$
   (ii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$; $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$
8. Solve the following system of linear equations for x and y.
   $a(x + y) + b(x - y) - (a^2 - ab + b^2) = 0$ and
   $a(x + y) - b(x - y) - (a^2 + ab + b^2) = 0$
9. Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction they meet in 9 hours and if they go in opposite directions they meet in 9/7 hours. Find their speeds.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. consistent
2. inconsistent.
3. 50
4. 4 and 1
5. Positive
6. ₹3
7. (17,9)
8. Cannot be determined
9. zero
10. 600 km.
11. 5
12. Does not exist

TRUE / FALSE

1. True
2. False
3. True
4. False
5. True
6. False
7. True
8. True
9. True
10. False
11. True
12. False

MATCH THE FOLLOWING :

1. (A) $\to$ s (B) $\to$ r (C) $\to$ q (D) $\to$ p
2. (A) $\to$ s (B) $\to$ p (C) $\to$ q (D) $\to$ r
3. (A) $\to$ q (B) $\to$ s (C) $\to$ p (D) $\to$ r

VERY SHORT ANSWER QUESTIONS :

1. To eliminate y, we have to make the coefficients of y in (i) and (ii) equal.

Multiplying (i) by 3, we get

$$9x+6y=33 \quad \text{...(iii)}$$

Multiplying (ii) by 2, we get

$$4x+6y=8 \quad \text{...(iv)}$$

Subtracting (iv) from (iii), we have

$$5x=25$$

$\therefore$ $x=5$

From (i), $2y=11-3x=11-3(5)=-4$

i.e., $2y=-4$ or $y=-2$

$\therefore$ $x=5$; $y=-2$ is the solution.

2. For these two equations

$a_1=3, a_2=9, b_1=4, b_2=12, c_1=-8, c_2=-24,$

$$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Since, $\frac{3}{4}=\frac{4}{12}=\frac{-8}{-24}$

The above pair of equations will have infinite solutions.

3. Given equations are

$x+5y-7=0$ and $4x+20y+k=0$

Here, $a_1=1 \quad b_1=5 \quad c_1=-7$

$a_2=4 \quad b_2=20 \quad c_2=k$

We know that equations represent coincident lines if they are consistent with many solutions.

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{1}{4}=\frac{5}{20}=\frac{-7}{k} \Rightarrow \frac{1}{4}=\frac{-7}{k} \Rightarrow k=-28$$

Hence, for $k=-28$, given equations represent coincident lines.

4.

$$3x+2y+25=0 \quad \text{.............(1)}$$
$$x+y+15=0 \quad \text{.............(2)}$$

Here, $a_1=3,\ b_1=2,\ c_1=25$

$a_2=1,\ b_2=1,\ c_2=15$

$$\therefore \quad \begin{array}{cccc} 2 & 25 & 3 & 2 \\ 1 & 45 & 1 & 1 \end{array}$$

$$\frac{x}{2\times15-25\times1}=\frac{y}{25\times1-15\times3}=\frac{1}{3\times1-2\times1}$$

$$\frac{x}{30-25}=\frac{y}{25-45}=\frac{1}{3-2}$$

$$\frac{x}{5}=\frac{y}{-20}=\frac{1}{1} \Rightarrow x=5,\ y=-20$$

5.

$$x+2y=-1 \quad \text{.............(1)}$$

and $2x-3y=12$ ....(2)

Multiplying (1) by 2 and subtracting (2) from (1)

$$2x+4y=-2 \quad \text{...........(3)}$$
$$2x-3y=+12 \quad \text{...........(4)}$$
$$- \quad + \quad -$$

$$7y=-14 \Rightarrow y=-2$$

Putting the value of y = – 2 in (1), we get

$$x+2(-2)=-1 \Rightarrow x-4=-1$$
$$\Rightarrow x=-1+4 \Rightarrow x=3$$

6.

$$Px-y=2 \quad \text{.............(1)}$$
$$6x-2y=3 \quad \text{.............(2)}$$

$a_1=P,\ b_1=-1,\ c_1=-2$

$a_2=6,\ b_2=-2,\ c_2=-3$

Condition for unique solution is $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$$\Rightarrow \frac{P}{6}\neq\frac{+1}{+2} \Rightarrow P\neq\frac{6}{2} \Rightarrow P\neq3$$

$\therefore$ P can have all real values except 3.

7. Let man's present age by x years and his son's present age be y years.
According to problem,
$x+6=3(y+6)$(after 6 years)
$x-3=9(y-3)$(before 3 years)
On solving, $x=30, y=6$

8. Let the annual increment be ₹ y and initial salary be ₹ x
$\therefore\ x+4y=4500$ .... (1)
and $x+10y=5400$ .... (2)
Solving eqs (1) and (2), we get
$x=3900$ and $y=150$
$\therefore$ Initial salary = ₹ 3900 and increment = ₹ 150

9. Let the speed of train be x km/hr. and car be y km/hr respectively. According to problem, $\frac{160}{x}+\frac{600}{y}=8$

Also, $\frac{240}{x}+\frac{520}{y}=\frac{41}{5}$

Solving these equation, we get $x=80$ km/hr and $y=100$ km/hr.

10. Let length and breadth be x cm and y cm respectively.
According to problem,
$2(x+y)=40$ .... (1)
and $\frac{y}{x}=\frac{2}{3}$ .... (2)
on solving, $x=12, y=8$
$\therefore$ Length be 12 cm. and breadth be 8cm.

11. Clear the fractions in (i) by multiplying by 6. Then,
$2x+3y=24$ ... (iii)
Clear the fractions in (ii) by multiplying by 12. The
$8x-3y=36$ ... (iv)
Adding (iii) and (iv) $10x=60$ i.e., $x=6$ from (iii)
$3y=24-2x=24-2(6)=12$
i.e., $3y=12$ or $y=4$
$\therefore\ x=6;\ y=4$ is the solution.

12. A, can do a work in 24 days. So, A does $\frac{1}{24}$ th of work in 1 day. Since, B is 60% more efficient, he will do $\frac{1}{24}(1.60)$ work in 1day. So, B can do $\frac{1.6}{24}$ th of work in 1 day.

Let B take x days to do 2 unit of work then $\frac{1.6}{24}x=2$,

$\therefore x=\frac{24\times 2}{1.6}=30$

Hence, B will do twice as much as A in 30 days.

13. Here, first of all, let us see how WORK can be defined. It is obvious that work can be measured as "destruction of the enemy bunkers."
In the first case, let us say that there were S number of soldiers in the group. So they had to work for 7 days for the work which we call W.
$\Rightarrow\ S\times 7=W$ ...(1)
Now 12 fell ill and the remaining did the work in 10 days.
Hence the new equation is
$(S-12)\times 10=W$ ...(2)
Just compare the two equations to get the answer.
$S\times 7=(S-12)\times 10 \Rightarrow 7S=10S-120 \Rightarrow 120=3S \Rightarrow S=40$ soldiers.
Hence, there were 40 soldiers in the group initially.

14.

	Volume of Acid (mL)	Concentration	Volume of solution (mL)
50% solution	0.50x	0.50	x
100% solution	1.00y	1.00	y
60% solution	(0.60)(400)	0.60	400

The second and fourth columns give the equations:
$0.5x+y = 240$ ...(1)
$x+y = 400$ ...(2)
Subtract: (1) − (2). $-0.5x = -160$
$x = 320$
Substitute 320 for x in (2) : $y = 80$
The technician needs 320 mL of 50% solution and 80 mL of 100% solution.

15. $cx+3y=3;\ 12x+cy=6$

For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\Rightarrow\ \frac{c}{12}=\frac{3}{c}\neq\frac{3}{6}\ \Rightarrow\ \frac{c}{12}=\frac{3}{c}\Rightarrow c^2=36$
$\Rightarrow\ c=\pm 6$ .... (1)
$\frac{3}{c}\neq\frac{3}{6}$
$c\neq\frac{18}{3}=6\ \Rightarrow\ c\neq 6$ .... (2)
From (1) and (2), $c=-6$

16. Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$
$2u+3v=13$ .... (1)
$5u-4v=-2$ .... (2)
On solving these equation, we get $u=2$ and $v=3$
then, $x=\frac{1}{u}=\frac{1}{2}$ and $y=\frac{1}{v}=\frac{1}{2}$

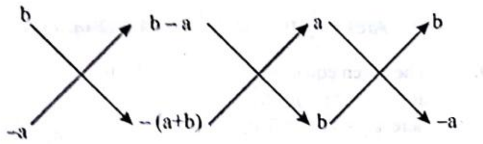
SHORT ANSWER QUESTIONS :

1. $ax+by-a+b=0$ .... (1)
$bx-ay-a-b=0$ .... (2)

Here $\frac{a_1}{a_2}=\frac{b}{a}$ and $\frac{b_1}{b_2}=\frac{b}{-a}$

$\Rightarrow\ \frac{a}{b}\neq-\frac{b}{a}$ $\therefore$ A unique solution exists.

Now, writing the coefficients of y, constant and x in the following array



We get,

$$\frac{x}{-b(a+b)-(-a)(b-a)} = \frac{y}{b(b-a)+a(a+b)} = \frac{1}{-a^2-b^2}$$

$$x = \frac{-ba-b^2+ab-a^2}{-a^2-b^2} = \frac{-b^2-a^2}{-a^2-b^2} = 1$$

$$y = \frac{b^2-ab+a^2+ab}{-a^2-b^2} = \frac{b^2+a^2}{-(a^2+b^2)} = -1$$

2. The equations $3x-(a+1)y = 2b-1$

$5x+(1-2a)y = 3b$

The system will have infinite number of solutions

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, $a_1 = 3,\ b_1 = -(a+1),\ c_1 = 2b-1$

$a_2 = 5,\ b_2 = 1-2a,\ c_2 = 3b$

$\therefore \frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b}$

Taking I and II

$\frac{3}{5} = \frac{-(a+1)}{1-2a}$

$-5a-5 = 3-6a$

$-5a+6a = 3+5$

$a = 8$

Taking I and III

$\frac{3}{5} = \frac{2b-1}{3b}$

$10b-5 = 9b$

$10b-9b = 5$

$b = 5$

$\therefore a = 8, b = 5$

3. $x+4y = 14$ ....(1)

$7x-3y = 5$ ....(2)

From eq. (1), $x = 14-4y$ ....(3)

Substitute the value of x in eq. (2)

$7(14-4y)-3y = 5$

$\Rightarrow 98-28y-3y = 5$

$\Rightarrow 98-31y = 5$

$\Rightarrow 93 = 31y$

$\Rightarrow y = \frac{93}{31} \Rightarrow y = 3$

Now substitute value of y in equa. (3)

$7x-3(3) = 5$

$\Rightarrow 7x = 14$

$\Rightarrow x = \frac{14}{7} = 2$

Hence, $x = 2,\ y = 3$

4. $2x-y = 5$ ....(1)

$(a-2b)x-(a+b)y = 15$ ....(2)

Here $a_1 = 2,\ b_1 = -1,\ c_1 = -5$

$a_2 = a-2b,\ b_2 = -(a+b),\ c_2 = -15$

for many solution, condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{2}{a-2b} = \frac{1}{(a+b)} = \frac{-5}{-15} \Rightarrow \frac{2}{a-2b} = \frac{1}{a+b} = \frac{1}{3}$$

Here, $\frac{2}{a-2b} = \frac{1}{3} \Rightarrow a-2b = 6$ ....(3)

also, $\frac{1}{a+b} = \frac{1}{3} \Rightarrow a+b = 3$ ....(4)

Now solve, $a-2b = 6$

$a+b = 3$

Multiply equation (2), by 2

$a-2b = 6$

$2a+2b = 6$

$3a = 12$

$\therefore a = 4,\ b = -1$

5. $2ax-2by+a+4b = 0$ ....(1)

and $2bx+2ay+b-4a = 0$ ....(2)

Multiplying eq. (1) with b and eq. (2) with a, we get

$2abx-2b^2y+ab+4b^2 = 0$ ....(3)

and $2abx+2a^2y+ab-4a^2 = 0$ ....(4)

Subtracting (4) from (3), we get

$-(2b^2+2a^2)y+4b^2+4a^2 = 0$

$\Rightarrow -(2b^2+2a^2)y = -4b^2-4a^2 \Rightarrow y = 2$

Substituting y = 2 in eq. (1), we get

$2ax-2b\times 2+a+4b = 0$

$\Rightarrow x = -1/2 \quad \therefore \quad x = -1/2,\ y = 2$

6. As $(x-4)$ is a factor of $p(x) = x^3+ax^2+2bx-24$, therefore, $p(4) = 0$

$\Rightarrow (4)^3+a(4)^2+2b(4)-24 = 0$

$\Rightarrow 64+16a+8b-24 = 0$

$\Rightarrow 16a+18b+40 = 0$

$\Rightarrow 2a+b+5 = 0$ ....(1)

Also given, $a-b = 8$ ....(2)

Adding (1) and (2), we get

$3a = 3 \Rightarrow a = 1$

Substituting a = 1 in eq. (2), we get

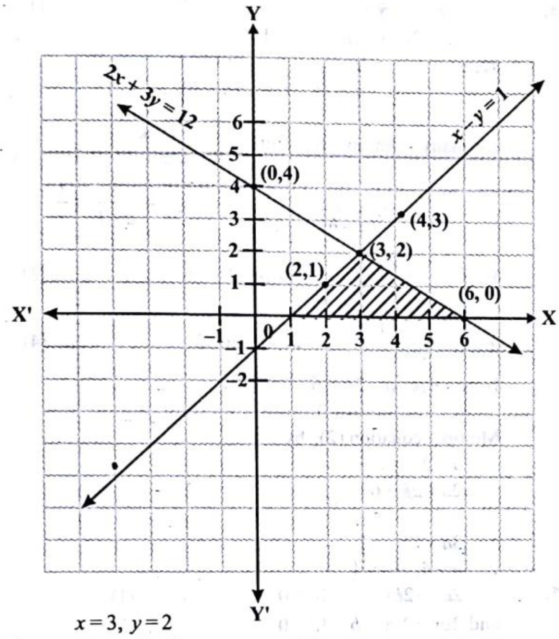
$1-b = 8 \Rightarrow b = -7$

7. $2x+3y = 12 \Rightarrow x = \frac{12-3y}{2}$

x	3	0	6
y	2	4	0

and $x-y = 1 \Rightarrow x = 1+y$

x	2	3	4
y	1	2	3



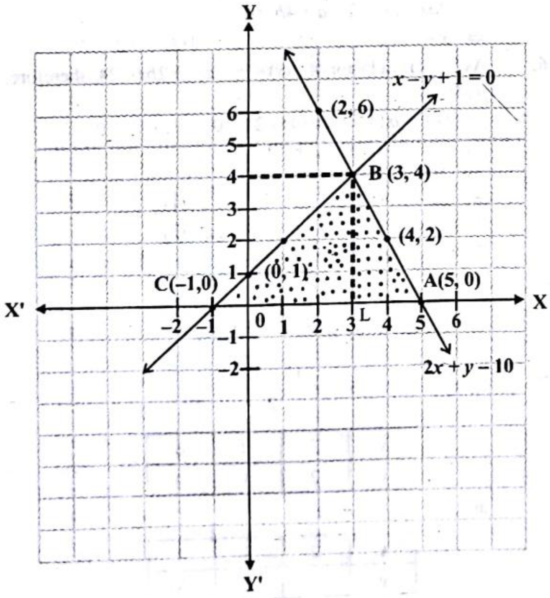
$x = 3,\ y = 2$

8. $x - y + 1 = 0 \Rightarrow \quad y = x + 1$

x	-1	1	0
y	0	2	1

and $2x + y - 10 = 0 \Rightarrow y = 10 - 2x$

x	2	4	5
y	6	2	0



Plotting these points on the graph, we get shaded portion is the area bounded by the lines and x-axis.

$$\text{Area} = \frac{1}{2} AC \times BL = \frac{1}{2} \times 6 \times 4 = 12\ sq.\, units$$

9. The given equations are $x + 5y - 7 = 0$
and $3x + 15y - p = 0$
here, $a_1 = 1, b_1 = 5, c_1 = -7$
$a_2 = 3, b_2 = 15, c_2 = -8$ for coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

thus, $\frac{1}{3} = \frac{5}{15} = \frac{-7}{-p}$ or $\frac{7}{p}$

$\frac{1}{3} = \frac{7}{p} \Rightarrow p - 3 \times 7 = 21$

i.e., for $p = 21$, the given system of equations will have infinite number of solutions or will represent coincident lines.

10. Given equations are : $7x - y = 5$ and $21x - 3y = k$

Here $a_1 = 7 \quad b_1 = -1 \quad c_1 = 5$

$a_2 = 21 \quad b_2 = -3 \quad c_2 = k$

We know that the equations are consistent with unique solution

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Also, the equations are consistent with many solutions

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \quad \frac{7}{21} = \frac{-1}{-3} = \frac{5}{k} \Rightarrow \frac{1}{3} = \frac{5}{k} \Rightarrow k = 15$

Hence, for $K = 15$, the system becomes consistent.

11. Let Yash make x correct and y wrong answers then $3x - y = 40$ and $4x - 2y = 50$.

So, equations are $3x - y - 40 = 0$ ...(1)
and $4x - 2y - 50 = 0$
or $2x - y - 25 = 0$ ...(2)

Subtracting (2) from (1), $3x - 2x - 40 + 25 = 0$ or $x - 15 = 0$

$\Rightarrow x = 15$ and putting $x = 15$ in eq. (1) $3 \times 15 - y - 40 = 0$

$\Rightarrow -y + 5 = 0$. So, $y = 5$

Number of total question in the text $= x + y = 15 + 5 = 20$.

12. 100 men do $\frac{1}{3}$rd of work in 10 days. So, 100 men do complete work in 30 days.

So man-days for complete work $= 100 \times 30$. Same work is completed by 100 men for 10 days + 150 men for 1 day + 200 men for 7 days + x men for 5 days.

where x is the number of men who work from 19th to 23rd day.

So, $100\times10+150\times1+200\times7+x\times5=100\times30$

$\Rightarrow 5x=2000-1400-150 \Rightarrow 5x=450 \Rightarrow x=90$

Hence, $200-90=110$ men should be relieved.

13. Let C be the cost of producing p number of brass plates.

So, for old machine, $C=750+0.25p$

For new machine, $C=1200+0.20p$

Since, total cost of production is same for both the machines,

$750+0.25p=1200+0.20p \Rightarrow 0.05p=1200-700=450$

$$\Rightarrow p=\frac{450}{0.05}=\frac{450}{5}\times100=9000$$

So, the company has to produce 900 plates.

14. Let the speed of train be u and speed of bus $= v$. Roohi travels 60 km by train and 240 km by bus, so, $\frac{60}{u}+\frac{240}{v}=4$.

When Roohi travels 100 km by train and 200 km by bus,

$$\frac{100}{u}+\frac{200}{v}=4\frac{1}{6}$$

so, equations are: $\frac{15}{u}+\frac{60}{v}=1$ and $\frac{100}{u}+\frac{200}{v}=\frac{25}{6}$

Let $\frac{1}{u}=x$ and $\frac{1}{v}=y$ so, equation becomes

$15x+60y=1$ ...(1)

and $100x+200y=\frac{-25}{6}$

$\Rightarrow 4x+8y=\frac{1}{6} \Rightarrow 24x+48y=1$ ...(2)

Equations are

$15x+60y-1=0$ and $24x+48y-1=0$

Solving by cross multiplication method:

$$\frac{x}{-60+48}=\frac{-y}{-15+24}=\frac{1}{15\times48-24\times60}$$

$$\Rightarrow \frac{x}{-12}=\frac{y}{-9}=\frac{1}{-30\times24}$$

$$\Rightarrow x=\frac{1}{60} \text{ and } y=\frac{1}{80}$$

so, $u=60$ and $v=80$

Speed of train = 60 km/hr and speed of bus = 80 km/hr

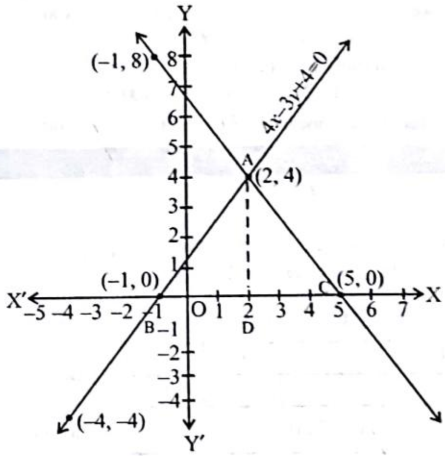
15. Consider both the linear equations separately

$4x-3y+4=0$	$4x+3y-20=0$
$4x=3y-4$	$4x=20-3y$
$x=\frac{3y-4}{4}$	$x=\frac{20-3y}{4}$

We make the tables for both the equations by giving the different values to x.

x	-1	2	-4
y	0	4	-4

x	5	2	-1
y	0	4	8



By plotting the points on graph and joining them we get that the two lines intersect at A(2, 4)

$\therefore x=2, y=4$ is the solution.

Now, Area of region bounded by lines and x-axis

$$=\text{ar}(\Delta ABC)=\frac{1}{2}.BC.AD=\frac{1}{2}\times6\times4=12 \text{ sq.units.}$$

$$\left(\because \text{area of } \Delta=\frac{1}{2}\times\text{base}\times\text{altitude}\right)$$

16. Let the speeds of the cars starting than A and B be x km/hr and y km/hr. respectively

According to problem,

$9x-90=9y$ ....(1)

$\frac{9}{7}x+\frac{9}{7}y=90$ ....(2)

Solving we get $x=40$ km/hr., $y=30$ km/hr.,

speed of car $A=40$ km/hr & speed of car $B=30$ km/hr.

17. According to problem $(2x+11)^\circ+(3y+6)^\circ=180^\circ$

$(y+12)^\circ+(5x-25)^\circ=180^\circ$

Solving we get, $x=\frac{416}{13}$ and $y=\frac{429}{13}$

$x=32, y=33$

$\therefore \angle A=75^\circ, \angle B=45^\circ, \angle C=105^\circ, \angle D=135^\circ$

18. Let the lady has x coins of 25 p and y coins of 50 p.

Then, according to problem

$x+y=40$ ....(1)

$25x+50y=1250$ ....(2)

Solving for x & y we get

$x=30$ (25 p coins) & $y=10$ (50 p coins)

19. Let monthly incomes of A and B be ₹ $9x$ and ₹ $7x$, and their expenditure be ₹ $4y$ and ₹ $3y$ respectively.
According to the given condition.

$9x-4y=1600$ .... (1)

and $7x-3y=1600$ .... (2)

Multiplying (1) by 3 and (2) by 4 and subtracting,
we get, $27x-12y-28x+12y=4800-6400$

$\Rightarrow -x=-1600 \Rightarrow x=1600$

$\therefore$ Monthly income of
$A=₹(9\times 1600)=₹\,14{,}400$

Monthly income of $B=₹(7\times 1600)=₹\,11{,}200$.

LONG ANSWER QUESTIONS :

1. $2x+3y=9$

$x-y=2$

$\Rightarrow 2x=9-3y \Rightarrow x=2+y \Rightarrow x=\dfrac{9-3y}{2}$

(3, 1) (6, –1) (–3, 5)

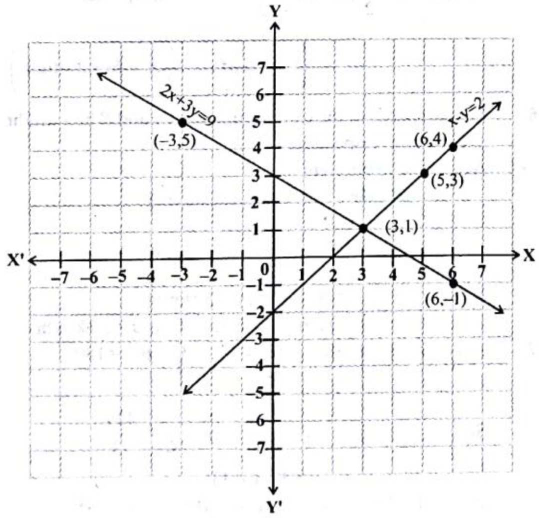
x	3	6	–3
y	1	–	5

(3, 1), (5, 3), (6, 4)

x	3	5	6
y	1	3	4

By plotting the points and joining them, the lines intersect at (3, 1).

$\therefore x=3, y=1$



2. $kx+4y=k-4$

$16x+ky=k$

For infinite number of solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \frac{a_1}{a_2}=\frac{k}{16},\ \frac{b_1}{b_2}=\frac{4}{k},\ \frac{c_1}{c_2}=\frac{k-4}{k}$$

$$\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{k}{16}=\frac{4}{k}$$

$\Rightarrow k^2=64 \Rightarrow k=\sqrt{64}$

$\Rightarrow k=\pm 8$ .... (1)

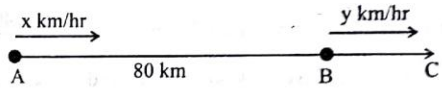
$$\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{4}{k}=\frac{k-4}{k}$$

$\Rightarrow 4k=k^2-4k$ .... (2)

$8k=k^2 \Rightarrow k=0$ or $k=8$

From (1) and (2), we get, $k=8$

3.

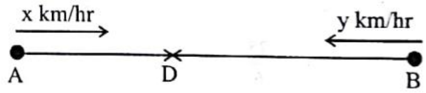


Let speed of the car from A be x km/hr.
Speed of the car from B be y km/hr.
If they move in the same direction they meet in 8 hours.
Let they meet at C after 8 hours.

$\therefore$ Distance $AC=8x$ km, Distance $BC=8y$ km.

$\therefore 8x=8y+80 \Rightarrow x-y=10$ .... (1)

If they move in opposite directions they meet at D (say) after 1 hour 20 minutes.



$\therefore$ Distance $AD=\dfrac{4}{3}x\ km$

$\left[\because 1 \text{ hr. } 20 \text{ minutes}=1\dfrac{1}{3}\text{hr}=\dfrac{4}{3}\text{hr.}\right]$

Distance $BD=\dfrac{4}{3}y\ km.$

$\therefore \dfrac{4}{3}x+\dfrac{4}{3}y=80 \Rightarrow 4x+4y=240$

$\Rightarrow x+y=60$ .... (2)

From (1) and (2), we get

$\therefore$ Speed of car starting from $A=35$ km/hr.
Speed of car starting from $B=25$ km/hr.

4. The equations are:

$3x-(a+1)y=2b-1$ and $5x+(1-2a)y=3b$

In a system of equation $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ the system will have infinite number of solutions

If $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$

Here, $a_1 = 3,\ b_1 = -(a+1),\ c_1 = (-)(2b-1)$

$a_2 = 5,\ b_2 = 1-2a,\ c_2 = -3b$

$$\therefore \quad \underset{\text{I}}{\frac{3}{5}} = \underset{\text{II}}{\frac{-(a+1)}{1-2a}} = \underset{\text{III}}{\frac{-(2b-1)}{-3b}}$$

Taking I and II, we get a. Taking I and III, we get b.

$$\frac{3}{5} = \frac{-(a+1)}{1-2a} \qquad \frac{3}{5} = \frac{2b-1}{3b}$$

$$-5a - 5 = 3 - 6a \qquad 10b - 5 = 9b$$

$$-5a + 6a = 3 + 5 \qquad 10b - 9b = 5$$

$$a = 8 \qquad b = 5$$

$\therefore \quad a = 8, b = 5$

5. The calculation is simplified if the fuel consumption is expressed in litres per kilometre :

12.5 L/100 km = 0.125 L/km and 7.5 L/100 km = 0.075 L/km

Type of Driving	Fuel Consumed (L)	Fuel Consumption (L/km)	Distance (km)
City	0.125x	0.125	x
Highway	0.075y	0.075	y
Mixture	35		400

The second and fourth column give the equations :

$$0.125x + 0.075y = 35 \quad ...(1)$$

$$x + y = 400 \quad ...(2)$$

Multiply (1) by 1000 : $125x + 75y = 35{,}000$

Multiply (2) by 75 : $75x + 75y = 30{,}000$

Subtract : (1) – (2). $50x = 5000$

$x = 100$

Substitute 100 for x in (2) : $y = 300$

Thus, the car travelled 300 km in highway driving.

6. Let the original speed of the train be x km/hr and the length of the journey be y km. Then,

Time taken = (y/x) hrs.

When defect in the engine occurs after covering a distance of 30 km.

We have,

Let speed for a distance of first 30 km = x km/hr

And speed for the remaining $(y-30)$km = $\frac{4}{5}x$ km/hr

$\therefore$ Time taken to cover 30 km = $\frac{30}{x}$ hrs

Time taken to cover $(y-30)$ km

$$= \frac{y-30}{(4x/5)} \text{ hrs} = \frac{5}{4x}(y-30) \text{ hrs}$$

According to the given condition, we have,

$$\frac{30}{x} + \frac{5}{4x}(y-30) = \frac{y}{x} + \frac{45}{60}$$

$$\Rightarrow \frac{30}{x} + \frac{5y-150}{4x} = \frac{y}{x} + \frac{3}{4}$$

$$\Rightarrow 120 + 5y - 150 = 4y + 3x$$

$$\Rightarrow 3x - y + 30 = 0 \quad ... (1)$$

When defect in the engine occurs after covering a distance of 48 km.

Speed for a distance of first 48 km = x km/hr.

And speed for the remaining $(y-48)$ km = $\frac{4x}{5} km/hr$

$\therefore$ Time taken to cover 48 km = $\frac{48}{x}$ hrs.

Time taken to cover

$$(y-48) \text{ km} = \left(\frac{y-48}{4x/5}\right) hr = \left\{\frac{5(y-48)}{4x}\right\} hr$$

According to the given condition, the train now reaches 9 minutes earlier i.e., 36 minutes later.

$$\frac{48}{x} + \frac{5(y-48)}{4x} = \frac{y}{x} + \frac{36}{60} \Rightarrow \frac{48}{x} + \frac{5y-240}{4x} = \frac{y}{x} + \frac{3}{5}$$

$$\Rightarrow \quad 25y - 240 = 20y + 12x$$

$$\Rightarrow \quad 12x - 5y + 240 = 0 \quad ...(2)$$

Solving the equations (1) and (2), we get (Using cross - multiplication)

$$\Rightarrow \frac{x}{-240+150} = \frac{-y}{720-360} = \frac{1}{-15+12}$$

$$\Rightarrow \frac{x}{-90} = \frac{-y}{360} = \frac{1}{-3}$$

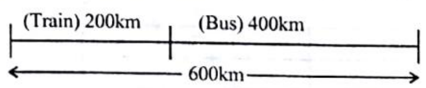
$$\Rightarrow x = \frac{-90}{-3} = 30 \quad and \quad y = \frac{-360}{-3} = 120$$

Hence, the original speed of the train is 30 km/hr and the length of the journey is 120 km.

7. Total distance = 600 km.

Let speed of train = x km/h and speed of car = y km/h

Case I : When 400 km covers by train and the rest by car.



$$\frac{400}{x} + \frac{200}{y} = \frac{13}{2} \quad ....(1)$$

Case II : $\frac{200}{x}+\frac{400}{y}=7$ ....(2)

(Train) 400km (Car) 200km

600km

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in (1) and (2), we get

$$400p+200q=\frac{13}{2} \quad ....(3)$$

$$200p+400q=7 \quad ....(4)$$

$$- \quad - \quad -$$

$$200p-200q=-\frac{1}{2}$$

$$p-q=-\frac{1}{400} \quad \therefore p=q-\frac{1}{400} \quad ....(5)$$

Putting $p=q-\frac{1}{400}$ in (3), we get

$$400\left(q-\frac{1}{400}\right)+200q=\frac{13}{2}$$

$\therefore$ $400q-1+200q=\frac{13}{2}$

or $600q=\frac{13}{2}+1=\frac{15}{2}$

$\therefore$ $q=\frac{15}{2}\times\frac{1}{60\times10}=\frac{1}{80}$

Putting $q=\frac{1}{80}$ in (5), we get

$$p=\frac{1}{80}-\frac{1}{400}=\frac{5-1}{400}=\frac{4}{400} \quad \text{or} \quad p=\frac{1}{100}$$

$\therefore$ $x=\frac{1}{p}=100\ km/h$ and $y=\frac{1}{q}=80\ km/h$

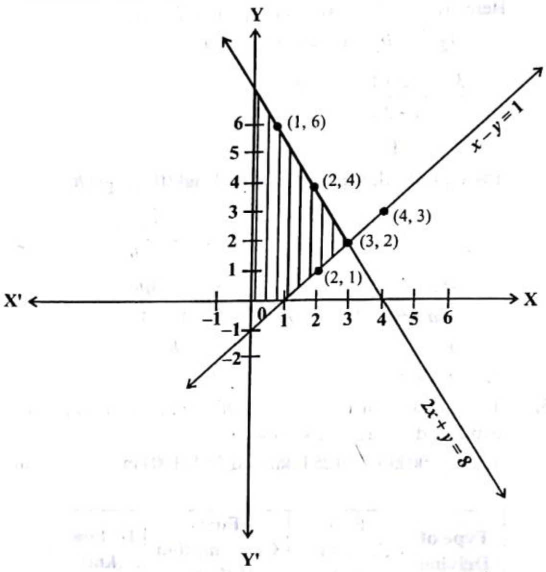
$\therefore$ speed of train = 100 km/h and speed of car = 80 km/h

8. $x-y=1$ $\qquad$ $2x+y=8.$

$x=1+y$ $\qquad$ $x=\frac{8-y}{2}$

x	2	3	4
y	1	2	3

x	3	2	1
y	2	4	6



Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (d) Work ratio of $A:B=100:160$ or $5:8$

$\therefore$ time ratio $=8:5$ or $24:15$

If A takes 24 days, B takes 15 days. Hence, B takes 30 days to do double the work.

2. (b) 96% of C.P. = ₹ 240

$\therefore$ 110% of C.P. = ₹ $\frac{240}{960}\times1100$ = Rs. 275

3. (b) The point satisfy the line $4y=x+1$

4. (c) Let the ages of father and son be $7x$, $3x$

$\therefore (7x+10):(3x+10)=2:1$ or $x=10$

$\therefore$ Age of the father is 70 years.

5. (d) Let the fraction be $\frac{x}{y}$.

$$\frac{x+1}{y+1}=4 \quad ....(1)$$

and $\frac{x-1}{y-1}=7$ ....(2)

Solving (1) and (2), we have $x=15$, $y=3$ i.e. $x=15$

6. (a) Downrate $=9\div2=4.5$ km/hr

Uprate $=9\div6=1.5$ km/hr

Speed of the boat $=(4.5+1.5)\div2=3$ km/hr

Speed of the current $=(4.5-1.5)\div2=1.5$ km/hr

7. (b) If the two digits are x and y, then the number is $10x+y$.

Now $\frac{5}{6}(10x+y)=10y+x$. Solving,

we get $44x+55y \Rightarrow \frac{x}{y}=\frac{5}{4}$.

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

8. (b) The numbers that can be formed are xy and yx. Hence $(10x+y)+(10y+x)=11(x+y)$. If this is a perfect square then $x+y=11$.

9. (c) $\frac{3x+4y}{x+2y}=\frac{9}{4}$

Hence, $12x+6y=9x+18y$ or $3x=2y$

$\therefore x=\frac{2}{3}y$. Substitute $x=\frac{2}{3}y$ in the required expression.

10. (c) Let unit's digit : x, tens digit : y

then $x=2y$, number $=10y+x$

Also $10y+x+36=10x+y$

$\therefore\ 9x-9y=36$ or $x-y=4$

Solve, $x=2y,\ x-y=4$

11. (b) $a>c$ hence $10+c-a=4$

Middle digit is reduced by 1, hence $10+(b-1)-b=9$

Hundred's digits now give $(a-1-c)$

From $10+c-a=4,\ a-c=6$

$\therefore\ a-c-1=5$

12. (b) Upstream speed = 4 km/hr and time = x hrs.

Downstream = 8 km/hr and time taken = $x/2$ hrs.

Hence average speed $=\frac{4x+8\times x/2}{x+x/2}=\frac{16}{3}$ km/hr.

13. (c) Let distance = d,

Time taken upstream $=\frac{d}{15-5}=\frac{d}{10}$

Time taken downstream $=\frac{d}{15+5}=\frac{d}{20}$

Hence average speed

$$=\frac{2d}{\frac{d}{10}+\frac{d}{20}}=\frac{2d\times 20}{3d}=\frac{40}{3}\,km/hr$$

Ratio $=\frac{40}{3}:15=40:45=8:9$

MORE THAN ONE CORRECT :

1. (a, c) **2.** (a, c)

3. (a, b, c)

(a) $x^2y>0\ [\because x^2>0, y>0]$ is true

(b) $x=-y \Rightarrow x+y=0 \therefore$ (b) is true

(c) $xy=(-y)(y)=-y^2<0 \therefore$ (c) is true

(d) $\frac{1}{x}-\frac{1}{y}=\frac{1}{x}+\frac{1}{-y}=\frac{1}{x}+\frac{1}{x}=\frac{2}{x}\neq 0$

$\therefore$ (d) is wrong.

4. (c, d)

5. (a, c)

For coincident lines, $\frac{3}{6}=\frac{-1}{-k}=\frac{8}{16}$

$\frac{1}{2}=\frac{1}{k}$

$k=2$

$3^k=9=3^2$

$k=2$

PASSAGE BASED QUESTIONS :

Passage-I

1. (b) We have

$217x+131y=913$

$131x+217y=827$

$$\therefore\ x=\frac{1}{2}\left(\frac{913+827}{217+131}+\frac{913-827}{217-131}\right)=\frac{1}{2}\left(\frac{1740}{348}-\frac{86}{86}\right)$$

$$=\frac{1}{2}(5+1)=3$$

$$y=\frac{1}{2}\left(\frac{913+827}{217+131}-\frac{913-827}{217-131}\right)=\frac{1}{2}(5-1)=2$$

$\therefore\ x=3,\ y=2$

2. (b) We have,

$37x+41y=70$

$41x+37y=86$

$$\therefore\ x=\frac{1}{2}\left(\frac{70+86}{37+41}+\frac{70-86}{37-41}\right)=\frac{1}{2}\left(\frac{156}{78}+\frac{-16}{-4}\right)$$

$$=\frac{1}{2}(2+4)=3$$

$$y=\frac{1}{2}\left(\frac{156}{78}-\frac{-16}{-4}\right)=\frac{1}{2}(2-4)=-1$$

Thus $x=1, y=-1$

3. (b) We have

$$\left.\begin{array}{l}2+2y=\frac{3}{2}\\ 2x+y=\frac{3}{2}\end{array}\right]\Rightarrow x=\frac{1}{2}\left[\frac{\frac{3}{2}+\frac{3}{2}}{1+2}=\frac{\frac{3}{2}-\frac{3}{2}}{1-2}\right]=\frac{1}{2}\left[\frac{3}{3}\right]=\frac{1}{2}$$

$$y=\frac{1}{2}\left[\frac{3}{3}-0\right]=\frac{1}{2}\ \therefore x=\frac{1}{2}, y=\frac{1}{2}$$

Passage-II

1. (b)

2. (a)

3. (b)

ASSERTION & REASON :

1. (a) Reason is true.
In Assertion, given lines represent parallel lines if

$\frac{3}{6}=\frac{4}{k}\neq\frac{5}{9}$

$\Rightarrow k=\frac{6\times4}{3}=8$ $\therefore$ Reason is also true

Since reason is the correct explanation for assertion

2. (b) Reason is true.
For assertion, given equation has no solution if

$\frac{1}{2}=\frac{1}{k}\neq\frac{-4}{-3}$ i.e. $\frac{4}{3}\Rightarrow k=2\left[\frac{1}{2}\neq\frac{4}{3}\text{holds}\right]$

$\therefore$ Assertion is true.
Since reason does not give result of assertion.

3. (c) Assertion, given system of equations has infinitely many solutions if

$\frac{2}{2a}=\frac{3}{a+b}=\frac{-7}{-28}$

i.e. $\frac{1}{4}\Rightarrow\frac{1}{a}=\frac{3}{a+b}=\frac{1}{4}\Rightarrow 3a=a+b\Rightarrow 2a-b=0$

Also clearly $a=4$, and $a+b=12\Rightarrow b=8$
$\therefore 2a-b=8-8=0$ $\therefore$ Assertion is true

But reason is false $\because \frac{3}{6}=\frac{-5}{-10}$

$[\because 3(-10)=(-5)(6)=-30]$

For unique solution if $a_1x+b_2y+c_2=0, \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

4. (d) Assertion is clearly false.
[$\because$ If the lines are coincident, then it has infinite number of solutions]
Reason is clearly true.

5. (a) 6. (b)
7. (c) 8. (d)

MULTIPLE MATCHING QUESTIONS :

1. (A) $\rightarrow$ q, t, u; (b) $\rightarrow$ p, r; (C) $\rightarrow$ s; (D) $\rightarrow$ p, r, s;

HOTS SUBJECTIVE QUESTIONS :

1. Comparing given equations with $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$

The system has no solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

We get, $a_1=3, b_1=1, c_1=1$
$a_2=2k-1, b_2=k-1, c_2=2k+1$

$\Rightarrow\frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$

From $\frac{3}{2k-1}=\frac{1}{k-1}\Rightarrow 3k-3=2k-1\Rightarrow k=3-1=2$

Now for $k=2$, we have $\frac{1}{k-1}=\frac{1}{2-1}=1$

and $\frac{1}{2k+1}=\frac{1}{2(2)+1}=\frac{1}{5}$

$\therefore$ For $k=2$, we have $\frac{1}{k-1}\neq\frac{1}{2k+1}$

Hence, the given system of linear equations have no solution if $k=2$

2. Given equations are: $ax+by=1$ .....(1)

and $bx+ay=\frac{2ab}{a^2+b^2}$ .....(2)

Adding equation (1) and (2)

$(a+b)x+(a+b)y=1+\frac{2ab}{a^2+b^2}$

$\Rightarrow (a+b)x+(a+b)y=\frac{a^2+b^2+2ab}{a^2+b^2}$

$\Rightarrow (a+b)(x+y)=\frac{(a+b)^2}{a^2+b^2}$

$\Rightarrow x+y=\frac{a+b}{a^2+b^2}$ ... (3)

Subtracting (2) from (1), $(a-b)x+(b-a)y=1-\frac{2ab}{a^2+b^2}$

$\Rightarrow (a-b)x-(a-b)y=\frac{a^2+b^2-2ab}{a^2+b^2}$

$\Rightarrow (a-b)(x-y)=\frac{(a-b)^2}{a^2+b^2}$

$\Rightarrow x-y=\frac{a-b}{a^2+b^2}$ ... (4)

Adding (3) and (4),

$2x=\frac{a+b}{a^2+b^2}+\frac{a-b}{a^2+b^2}=\frac{2a}{a^2+b^2}\Rightarrow x=\frac{a}{a^2+b^2}$

Subtracting (4) from (3),

$2y=\frac{a+b}{a^2+b^2}-\frac{a-b}{a^2+b^2}=\frac{2b}{a^2+b^2}\Rightarrow y=\frac{b}{a^2+b^2}$

$\therefore x=\frac{a}{a^2+b^2}, y=\frac{b}{a^2+b^2}$

3. Let x be the digit in the ten's place and y be the digit in the unit's place of the number
Then the number $=10x+y$
According to the given conditions,

$10x+y=8(x+y)+1 \Rightarrow 2x-7y=1$

Also, $10x+y=13(x-y)+2 \Rightarrow -3x+14y=2$

OR

($\because$ x can be greater than y or less than y)

$10x+y=13(y-x)+2 \Rightarrow 23x-12y=2$

Thus, we have either $\left.\begin{array}{r}2x-7y=1\\ -3x+14y=2\end{array}\right\}$

OR

$\left.\begin{array}{r}2x-7y=1\\ 23x-12y=2\end{array}\right\}$

On solving $2x-7y=1$ and $-3x+14y=2$, we get $x=4$ and $y=1$

$\therefore$ The number $=10x+y=10\times4+1=41$

OR

We have

$\left.\begin{array}{r}2x-7y=1\\ 23x-12y=2\end{array}\right\}$

Solving these equations, we get

$y=-\dfrac{19}{137}$ and $x=\dfrac{2}{137}$

But this is not possible since x, y are digits

Hence the number is 41.

4. Let the unit's place digit be x and the ten's place digit be y.

Original number $=x+10y$

Reversed number $=10x+y$ (By reversing the digits x and y.)

According to the question

$x+10y+10x+y=99$

$11x+11y=99$

$x+y=9$ (Dividing by 9)

$x=9-y$ .... (i)

Given, $x-y=\pm3$

When $x-y=3$	When $x-y=-3$
$9-y-y=3$ [From (i)]	$9-y-y=-3$ [From (i)]
$-2y=-6$	$-2y=-12$
$y=3$	$y=6$
Using $y=3$ (i), we get	Using $y=6$ in (i), we get
$x=9-3=6$	$x=9-6=3$
$\therefore$ Original number	$\therefore$ Original number
$=6+30=36$	$=3+60=63$

5. Compare both the given equations with $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$

Here $a_1=2, b_1=3, c_1=7, a_2=a+b+1, b_2=a+2b+2,$ $c_2=4(a+b)+1$

$\dfrac{a_1}{a_2}=\dfrac{2}{a+b+1};\ \dfrac{b_1}{b_2}=\dfrac{3}{a+2b+2};\ \dfrac{c_1}{c_2}=\dfrac{7}{4(a+b)+1}$

For [Infinite number of solutions]

$\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$

$\dfrac{2}{a+b+1}=\dfrac{3}{a+2b+2}=\dfrac{7}{4(a+b)+1}$

(I) (II) (III)

Taking I and II	Taking II and III
$\dfrac{2}{a+b+1}=\dfrac{3}{a+2b+2}$	$\dfrac{3}{a+2b+2}=\dfrac{7}{4(a+b)+1}$
$3a+3b+3=2a+4b+4$	$12a+12b+3=7a+14b+14$
$a-b=1$ .... (i)	$5a-2b=11$ .... (ii)

Multiplying (i) by 2 and subtracting (ii) from (i)

$2a-2b=2$

$5a-2b=11$

$-3a\quad=-9 \qquad \Rightarrow a=3$

Putting the value of a in (i), we get

$a-b=1 \qquad \Rightarrow 3-b=1$

$\Rightarrow\ -b=1-3=-2 \qquad \Rightarrow b=2$

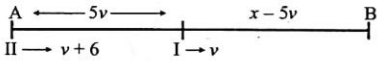
6. Speed of body I $=v$ m/s

Speed of body II $=(v+6)$ m/s

Speed of body III $=30$ m/s

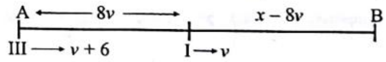
Distance $AB=x$ m

First consider bodies I and II only.



$\dfrac{x-5v}{v}=\dfrac{x}{v+6}$ ....(1)

Now consider bodies I and III only



$\dfrac{x-8v}{v}=\dfrac{x}{30}$

Hence, required equations are

$\dfrac{x}{v}-\dfrac{x}{v+6}=5,\ \dfrac{x}{v}-\dfrac{x}{30}=8$

7. (i) Given equations are $6x+3y=6xy$ and $2x+4y=5xy$

Dividing both the sides of both the equation by xy,

we get $\dfrac{6}{y}+\dfrac{3}{x}=6$ and $\dfrac{2}{y}+\dfrac{4}{x}=5$

Let $\dfrac{1}{x}=u$ and $\dfrac{1}{y}=v$. Equations become $3u+6v=6$

and $4u+2v=5$

or, $u+2v=2$ ...(1)

and $4u+2v=5$ ...(2)

Subtracting (1) from (2) we get, $3u=3 \Rightarrow u=1$ and

$1+2v=2 \Rightarrow v=\dfrac{1}{2}$

$u=1=\dfrac{1}{x}, \Rightarrow x=1$ and $v=\dfrac{1}{2}=\dfrac{1}{y} \Rightarrow y=2$.

So, $x=1$, and $y=2$.

(ii) Given equations are : $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Let us take $u=\frac{1}{3x+y}$ and $\frac{1}{3x-y}=v$ so , equation

become, $u+v=\frac{3}{4}$ and $\frac{u}{2}-\frac{v}{2}=-\frac{1}{8}$

or $4u+4v=3$ and $4u-4v=-1$

Adding both we get $8u=2 \Rightarrow u=\frac{1}{4}$ and

$4\times\frac{1}{4}+4v=3 \Rightarrow v=\frac{2}{4}=\frac{1}{2}$

$u=\frac{1}{3x+y}=\frac{1}{4} \Rightarrow 3x+y=4$ ...(1)

and $v=\frac{1}{3x-y}=\frac{1}{2} \Rightarrow 3x-y=2$ ...(2)

Adding (1) and (2) we get, $6x=6 \Rightarrow x=1$; and putting $x=1$ in any equation, say equation (1), $3\times1+y=4 \Rightarrow y=1$. So, $x=1$, $y=1$

8. The given system of equations is

$a(x+y)+b(x-y)-\left(a^2-ab+b^2\right)=0$ and

$a(x+y)-b(x-y)-\left(a^2+ab+b^2\right)=0$

This can be written as

$(a+b)x+(a-b)y-\left(a^2-ab+b^2\right)=0$ and

$(a-b)x+(a+b)y-\left(a^2+ab+b^2\right)=0$

Here $a_1=a+b$, $b_1=a-b$

$a_2=a-b$, $b_2=a+b$

and $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ i.e $-\frac{a+b}{a-b}\neq\frac{a-b}{a+b}$

Also, $a_1b_2-a_2b_1=(a+b)(a+b)-(a-b)(a-b)$

$=(a+b)^2-(a-b)^2=4ab\neq0$

Therefore, the given system of equations has a unique solution.

Now, we can solve this system of equations by using cross-multiplication method which gives :

$$\Rightarrow \frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{x}{-(a-b)\left(a^2+ab+b^2\right)+(a+b)\left(a^2-ab+b^2\right)}$$

$$=\frac{y}{-(a-b)\left(a^2-ab+b^2\right)+(a+b)\left(a^2+ab+b^2\right)}$$

$$=\frac{1}{(a+b)(a+b)-(a-b)(a-b)}$$

$$\Rightarrow \frac{x}{-\left(a^3-b^3\right)+\left(a^3+b^3\right)}=\frac{y}{2b\left(2a^2+b^2\right)}=\frac{1}{4ab}$$

$$\Rightarrow \frac{x}{2b^3}=\frac{y}{2b\left(2a^2+b^2\right)}=\frac{1}{4ab}$$

$$\Rightarrow x=\frac{2b^3}{4ab}=\frac{b^2}{2a} \text{ and } y=\frac{2b\left(2a^2+b^2\right)}{4ab}=\frac{2a^2+b^2}{2a}$$

Hence, the solution of the system is

$x=\frac{b^2}{2a}$ and $y=\frac{2a^2+b^2}{2a}$

9. Let Car X start from point A and Y from point B. Let the speed of car X be x km/hr and that of car Y be y km/hr. There are two cases :

Case I: When two cars move in the same directions:

Suppose two cars meet at point Q. Then,

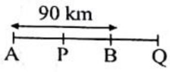
Distance travelled by car $X=AQ$, Distance travelled by car $Y=BQ$.

It is given that two cars meet in 9 hours.

$\therefore$ Distance travelled by car X in 9 hours = $9x$ km.

$\Rightarrow AQ=9x$

Distance travelled by car y in 9 hours = $9y$ km. $\Rightarrow BQ=9y$



Clearly, $AQ-BQ=AB \Rightarrow 9x-9y=90$

[$\because$ AB = 90 km]

$\Rightarrow x-y=10$ ... (1)

Case II When two cars move in opposite directions:

Let them meet at point P. Then,

Distance travelled by car $X=AP$, Distance travelled by car $Y=BP$.

In this case, two cars meet in 9/7 hours.

$\therefore$ Distance travelled by car X in $\frac{9}{7}$ hours = $\frac{9}{7}x$ km

$\Rightarrow AP=\frac{9}{7}x$, and that of car $Y=\frac{9}{7}y \Rightarrow BP=\frac{9}{7}y$

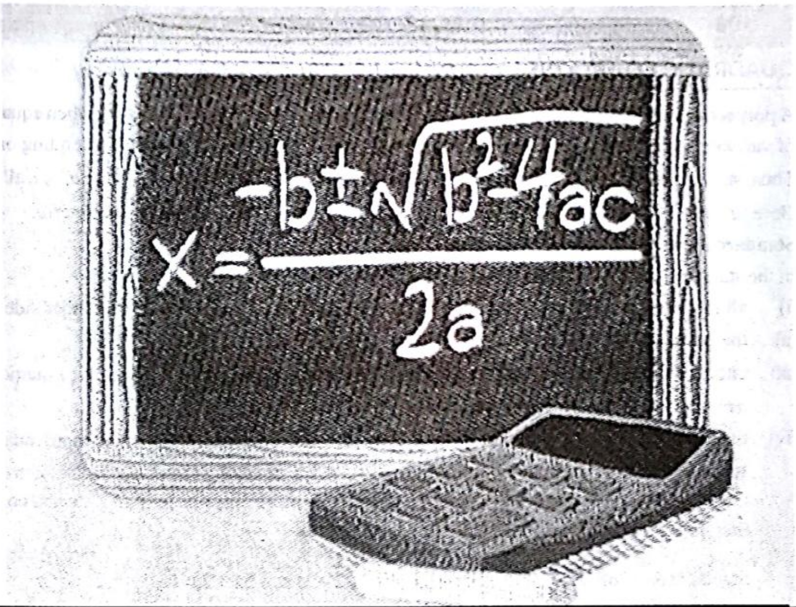
Clearly, $AP+BP=AB \Rightarrow \frac{9}{7}x+\frac{9}{7}y=90 \Rightarrow \frac{9}{7}(x+y)=90$

$\Rightarrow x+y=70$ ...(2)

Solving equations (1) and (2), we get $x=40$ and $y=30$.

Hence, speed of car X is 40 km/hr and speed of car Y is 30 km/hr.

chapter 4



QUADRATIC EQUATIONS & QUADRATIC INEQUALITIES

Introduction

In Chapter–2, you have studied the Polynomials in one variable and degree of polynomials. On the basic of the degree of Polynomials, polynomials are categorised as Linear, Quadratic and Cubic polynomial. The Polynomial of degree two in the standard form is $ax^2 + bx + c$ $(a \neq 0)$, when equated to zero is called a Quadratic Equation in standard form.

Thus, $ax^2 + bx + c = 0$, where a, b, c, all are real number but $a = 0$ is the standard form of at a Quadratic Equation.

The Babylonians, as early as 1800 BC. could solve a pair of simultaneous equations of the form : $x + y = p$, $xy = q$, which is equivalent to :

$$x^2 - px + q = 0.$$

In the Sulbha Sutras in India 8th century BC., quadratic equation of the form $ax^2 = c$ and $ax^2 + bx = c$ were explored using geometric methods. Chinese mathematician in 200 B.C., used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula.

Brahmagupta (AD. 598 – 665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx = c$. 'Liber embadorum' published in Europe in A.D. 1145 gave complete solutions of different quadratic equations.

In this chapter, you will study quadratic equations and their solutions (or roots). We also study the application of quadratic equation in solving real life problems.

When we put >, <, ≥ or ≤ at the place of '=' in the standard form of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$, we get the Quadratic Inequation as

$$ax^2 + bx + c > 0,\ ax^2 + bx + c < 0,\ ax^2 + bx + c \geq 0 \text{ or, } ax^2 + bx + c \leq 0.$$

QUADRATIC EQUATIONS:

A polynomial of any variable x of degree two in standard form $ax^2 + bx + c\ (a \neq 0)$, when equated to zero is called a Quadratic Equation of variable x. [In standard form of quadratic polynomial, the power of x are in descending order.]

Thus, standard form of a Quadratic Equation of variable x is $ax^2 + bx + c = 0$; where a, b, c all are real numbers but $a \neq 0$.

Here 'a' and 'b' are called coefficient of x^2 and x respectively. 'c' is called constant term.

Standard Form of a Quadratic Equation:

In the standard form of a quadratic equation $ax^2 + bx + c = 0$.

(i) all the terms (except zero) are in one side of equal sign (=) and zero (0) is in other side of equal sign.

(ii) the power of x are in descending order.

(iii) either or both 'b' and 'c' can be equal to zero but $a \neq 0$. Therefore a quadratic equation may be in the form $ax^2 + bx = 0$, $ax^2 + c = 0$, $ax^2 = 0$ or $x^2 = 0$.

(iv) the term of x^2 (i.e. ax^2), term of x (i.e. bx) or constant term (i.e. c) can not be more than one.

If not so, then by adding/subtracting the like terms or taking the common factor from like terms, we convert the quadratic equation in the standard form, in which there is no more than one term of x^2, x and constant term. For example:

Standard form of $x^2 - 4x + x + 2 = 0$ is $x^2 - 3x + 2 = 0$

Standard form of $\sqrt{2}x^2 - x^2 + 5x + 7 = 0$ is $\left(\sqrt{2}-1\right)x^2 + 5x + 7 = 0$.

Some times there are two or more constant terms which can not be added/subtracted or have no any common factor, then both the constant terms are put in to a bracket so that all constant terms become a single constant term for example:

Standard form of $\sqrt{5}x^2 + 7x - 3 + 2\sqrt{7} = 0$ is $\sqrt{5}x^2 + 7x - \left(3 - 2\sqrt{7}\right) = 0$

SOLUTION OF QUADRATIC EQUATIONS :

The values of x which satisfy the given quadratic equation are called solutions/roots of the given quadratic equation. For examples if $x = \alpha$ is one of solutions of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

To find the solutions (or to find the roots) of a quadratic equation, first we check wheather the given quadratic equation is in standard form or not. If not, then first convert the given quadratic equation in standard form, then by using any of the following given three methods, you can solve the given quadratic equation.

A quadratic equation has always two solutions/roots (i.e., two value of the variable 'x' always satisfy the quadratic equation $ax^2 + bx + c = 0$)

Method First – Method of Factorisation :

Quadratic Equation : $ax^2 + bx + c = 0$

By spliting the middle term 'bx' of L.H.S., factories the L.H.S. $(ax^2 + bx + c)$ in to linear factors. Then after equating each factor to zero, we find the values of the variable 'x' of the quadratic equation $ax^2 + bx + c = 0$.

These value of x are the solutions/roots of the given quadratic equation.

Special cases :

(i) If $b, c = 0$, then $ax^2 = 0, \Rightarrow x^2 = \dfrac{0}{a} = 0, \Rightarrow x = 0, 0$

(ii) If $b = 0$, then $ax^2 + c = 0, \Rightarrow x^2 = \dfrac{-c}{a}, \Rightarrow x = \pm\sqrt{\dfrac{-c}{a}}$

(iii) If $c = 0$, then $ax^2 + bx = 0, \Rightarrow x(ax + b) = 0, \Rightarrow x = 0, -\dfrac{b}{a}$

For examples:

(a) If $6x^2 - 5x - 4 = 0$, then $6x^2 - 8x + 3x - 4 = 0$ [split $-5x$ as $-8x + 3x$ such that $-5x = -8x + 3x$ and $(-8x) \times (3x) = (6x^2) \times (-4)$]
$\Rightarrow 2x(3x - 4) + 1(3x - 4) = 0 \Rightarrow (3x - 4)(2x + 1) = 0$

Now, equating each linear factor to 0, we get

$3x-4=0 \Rightarrow x=\frac{4}{3}$ and $2x+1=0 \Rightarrow x=-\frac{1}{2}$

Hence, solutions: $x=\frac{4}{3},-\frac{1}{2}$

(b) If $5x^2=0$, then $x^2=0$
Hence, solutions: $x=0,0$

(c) If $3x^2-15=0$, then solution : $x=\pm\sqrt{\frac{15}{3}}=\pm 5$

(d) If $2x^2-5x=0$, then $x(2x-5)=0$,

$\Rightarrow x=0, 2x-5=0 \Rightarrow x=0,\ x=\frac{5}{2}$

Hence solution : $x=0,\ \frac{5}{2}$

Method Second – Method of Completing the Square :

Consider the quadratic equations, $ax^2+bx+c=0$; $2x^2+5x+2=0$

(i) Shift constant term 'c' to the R.H.S.
$ax^2+bx=-c$; $2x^2+5x=-2$

(ii) Divide both sides by 'a'.

$$\frac{ax^2+bx}{a}=-\frac{c}{a}\ ;\ \frac{2x^2+5x}{2}=\frac{-2}{2} \Rightarrow x^2+\frac{b}{a}x=-\frac{c}{a}\ ;\ x^2+\frac{5}{2}x=-1$$

(iii) Write coefficient $\frac{'b'}{a}$ of x as $2\left(\frac{b}{2a}\right) \Rightarrow x^2+2\left(\frac{b}{2a}\right)x=-\frac{c}{a}\ ;x^2+2\left(\frac{5}{4}\right)x=-1$

(iv) Add $\left(\frac{b}{2a}\right)^2$ on both sides of equal sign (=) and completing the whole square on L.H.S.

$$\Rightarrow x^2+2\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}\ ;x^2+2\left(\frac{5}{4}\right)x+\left(\frac{5}{4}\right)^2=\left(\frac{5}{4}\right)^2-1$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}\ ;\left(x+\frac{5}{4}\right)^2=\frac{25-16}{16}$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}\ ;\left(x+\frac{5}{4}\right)^2=\frac{9}{16}$$

(v) Taking square root on both sides of equal sign (=).

$$\Rightarrow x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}\ ;\ x+\frac{5}{4}=\pm\frac{3}{4}$$

(vi) Now, shift $\frac{b}{2a}$ from L.H.S. to R.H.S. to get the value of x.

$$\Rightarrow x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}\ ;x=\pm\frac{3}{4}-\frac{5}{4}$$

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\ ;\ x=\frac{\pm 3-5}{4}=-\frac{1}{2},-2$$

Method Third – Using the Formula: In the method second, we see that solutions/roots of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ by completing the square,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{................(i)}$$

We can use this equation (i) as a formula to find the solutions/roots of the quadratic equation $ax^2 + bx + c = 0$.

For Example: $x^2 - 3x + 1 = 0$

$$\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1}, \text{ using the formula}$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

$$\Rightarrow \quad x = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$$

NATURE OF ROOTS :

Quadratic Equation: $ax^2 + bx + c = 0$ $(a \neq 0)$ value of $(b^2 - 4ac)$ is called discriminant of the quadratic equation. The value of $(b^2 - 4ac)$ is denoted by D.

$\therefore \quad D = b^2 - 4ac$

The discriminant plays an important role in finding the nature of the roots of the quadratic equation.

(i) If $D = 0$, then roots are real and equal.

(ii) If $D > 0$, then roots are real and unequal.

(iii) If $D < 0$, then roots are not real [Actually the roots are imaginary and unequal, which are also called complex conjugate like

$$\left[\frac{5+\sqrt{-3}}{4}, \frac{5-\sqrt{-3}}{4}\right]$$

SUM AND PRODUCT OF ROOTS :

If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then

(i) Sum of Roots, $\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

(ii) Product of Roots, $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

For example in equation $3x^2 + 4x - 5 = 0$

Sum of Roots $= -4/3$, Product of Roots $= -5/3$

RELATION BETWEEN ROOTS AND COEFFICIENTS :

If roots of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are α and β then:

(i) $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \frac{\sqrt{b^2 - 4ac}}{a} = \frac{\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\frac{b\sqrt{b^2 - 4ac}}{a^2} = \frac{-b\sqrt{D}}{a^2}$

(iv) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-\dfrac{b(b^2-3ac)}{a^3}$

(v) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)=(\alpha-\beta)[(\alpha-\beta)^2+3\alpha\beta]=\sqrt{(\alpha+\beta)^2-4\alpha\beta}\,\left\{(\alpha+\beta)^2-\alpha\beta\right\}=\dfrac{(b^2-ac)\sqrt{b^2-4ac}}{a^3}$

(vi) $\alpha^2+\alpha\beta+\beta^2=(\alpha+\beta)^2-\alpha\beta=\dfrac{b^2-ac}{a^2}$

(vii) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\dfrac{b^2-2ac}{ca}$

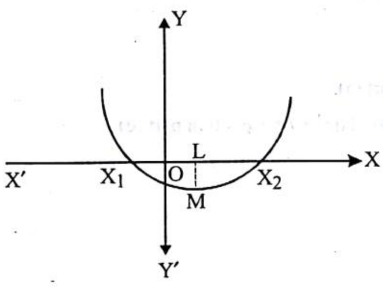
(viii) $\alpha^2\beta+\beta^2\alpha=\alpha\beta(\alpha+\beta)=\dfrac{c}{a}\cdot\left(-\dfrac{b}{a}\right)=-\dfrac{bc}{a^2}$

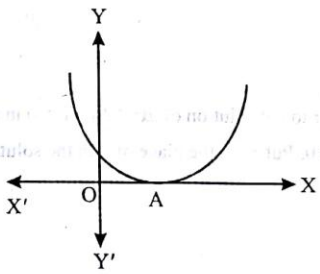
GRAPH OF QUADRATIC EXPRESSIONS :

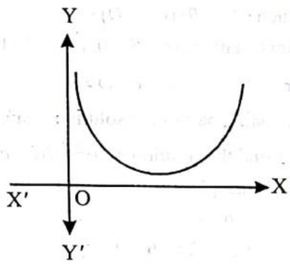
Consider the expression $y=ax^2+bx+c$ $(a\neq 0)$ and $a, b, c \in R$ then the graph between x, y is always a parabola if $a>0$ then the shape of the parabola is concave upward and if $a<0$ then the shape of the parabola is concave downwards. There is only 6 possible graph of a quadratic expression as given below:

Case -I When $a>0$

(i) **If $D>0$** (ii) **If $D=0$** (iii) **If $D<0$**

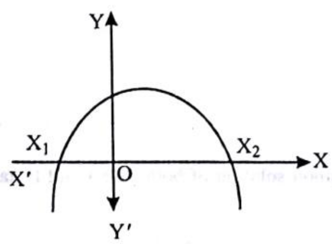


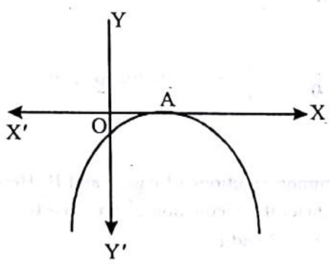


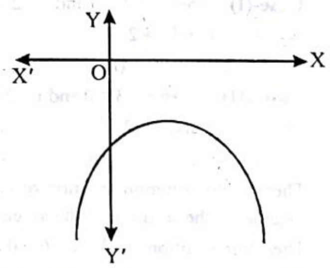


Case -II When $a<0$

(i) **If $D>0$** (ii) **When $D=0$** (iii) **When $D<0$**







QUADRATIC INEQUATION IN STANDARD FORMS :

$ax^2+bx+c>0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c<0$, $ax^2+bx+c\leq 0$; where a, b, c all are real but $a\neq 0$

SOLUTION OF QUADRATIC INEQUATIONS :

First factorise the L.H.S. of the given Quadratic Inequation in to Linear Factors as $(Ax+B)(Cx+D)$,
Where A, B, C and D are real numbers.

(a) When $ax^2+bx+c>0$, then $(Ax+B)(Cx+D)>0$

Hence, either $Ax+B>0$, $Cx+D>0$ or, $Ax+B<0$, $Cx+D<0$

Case-(I) : $Ax+B>0$ and $Cx+D>0$

$\Rightarrow \; x>-\frac{B}{A}$ and $x>-\frac{D}{C}$ ....(i)

Find the common solution of (i), if any.

Case -(II) : $Ax+B<0$ and $Cx+D<0$

$\Rightarrow \; x<-\frac{B}{A}$ and $x<-\frac{D}{C}$ ....(ii)

Find the common solution of (ii) if any

If there is no common solution in any case, then that case will be rejected and the common solution of the other case is the solution of $ax^2+bx+c>0$.

If there are common solutions in both the case (I) and (II). And there is a common solution in both the common solutions obtained in case - (I) and (II) also.

Then the common solution of the common solutions obtained in cases (I) and (II) is the solution of $ax^2+bx+c>0$. Otherwise both common solutions of case (I) and (II) taken together is the solution of $ax^2+bx+c>0$

(b) To find the solution of $ax^2+bx+c \geq = 0$, put $\geq$ at the place of $>$ in the solution of $ax^2+bx+c>0$ in part (a).

(c) When $ax^2+bx+c<0$

Then $(Ax+B)(Cx+D)<0$

Hence either $Ax+B>0$, $Cx+D<0$

or, $Ax+B<0$, $Cx+D>0$

Remaing part of the solution is similar to the solution of $ax^2+bx+c>0$ in part (a).

(d) To find the solution of $ax^2+bx+c \leq 0$, Put $\leq$ at the place of $<$ in the solution of $ax^2+bx+c<0$ in part (c)

For examples :

(a) $x^2+x-6>0$

$\Rightarrow x^2+3x-2x-6>0$

$\Rightarrow x(x+3)-2(x+3)>0$

$\Rightarrow (x+3)(x-2)>0$

Hence, either $x+3>0$, $x-2>0$

or $x+3<0$, $x-2<0$

Case-(I): When $x+3>0$ and $x-2>0$

$\Rightarrow \; x>-3$ and $x>2$

$\therefore \; x>2$ ....(i)

Case-(II): When $x+3<0$ and $x-2<0$

$\Rightarrow \; x<-3$ and $x<2$

$\therefore \; x<-3$ .... (ii)

There is no common solution of common solutions of case-I and II. Hence, common solution of both case-I and II, taken together is the solution of the given Quadratic Inequation $x^2+x-6>0$.

Therefore solution of $x^2+x-6>0$ is $x<-3$ and $x>2$

Which is repersented on number line as follows :

(b): $x^2+8x+15 \le 0$

$\Rightarrow \; x^2+3x+5x+15 \le 0$

$\Rightarrow \; x(x+3)+5(x+3) \le 0$

$\Rightarrow \; (x+3)(x+5) \le 0$

Hence, either $x+3 \ge 0, x+5 \le 0$

or, $x+3 \le 0, x+5 \ge 0$

Case-(I) : When $x+3 \ge 0, x+5 \le 0$

$\Rightarrow \; x \ge -3, x \le -5$

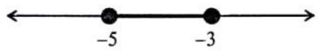
There is no common solution.

Case-(II) : When $x+3 \le 0, x+5 \ge 0$

$\Rightarrow \; x \le -3, x \ge -5$

$\therefore \; -5 \le x \le -3$

Since, there is no common solution in case-I, therefore, solution of the Quadratic Inequation $x^2+8x+15 \le 0$ is $-5 \le x \le -3$ only. Which is represented on the number line as follows:



Solution of Quadratic Inequations in Special Case:

If $b = 0$, then quadratic inequations $ax^2+bx+c>0$, $ax^2+bx+c \ge 0$, $ax^2+bx+c<0$, $ax^2+bx+c \le 0$ become $ax^2+c>0$, $ax^2+c \ge 0$, $ax^2+c<0$, $ax^2+c \le 0$, respectively.

(a) $ax^2+c>0 \Rightarrow x^2 > -\dfrac{c}{a}$

$\therefore x <$ Negative square root of $\left(-\dfrac{c}{a}\right)$ and $x >$ Positive square root of $\left(-\dfrac{c}{a}\right)$

(b) $ax^2+c \ge 0 \Rightarrow x^2 \ge -\dfrac{c}{a}$

$\therefore x \le$ Negative square root of $\left(-\dfrac{c}{a}\right)$ and $x \ge$ Positive square root of $\left(-\dfrac{c}{a}\right)$

(c) $ax^2+c<0 \Rightarrow x^2 < -\dfrac{c}{a}$

$\therefore x >$ Negative square root of $\left(-\dfrac{c}{a}\right)$ and $x <$ Positive square root of $\left(-\dfrac{c}{a}\right)$

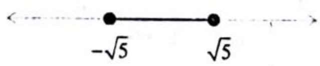
(d) $ax^2+c \le 0 \Rightarrow x^2 \le -\dfrac{c}{a}$

$\therefore x \ge$ Negative square root of $\left(-\dfrac{c}{a}\right)$; $x \le$ Positive square root of $\left(-\dfrac{c}{a}\right)$

For examples:

(a) $2x^2-8 \ge 0 \Rightarrow x^2 \ge 4$; $\therefore x \le -2$ and $x \ge 2$

(b) $x^2-5<0 \Rightarrow x^2<5$; $\therefore -\sqrt{5} < x < \sqrt{5}$



MISCELLANEOUS SOLVED EXAMPLES

1. A man walks a distance of 48 km in a given time. If he walks 2 km an hour faster, he will perform the journey 4 hours before. Find his normal rate of walking.

Sol. Let the normal rate of walking of the man be x km/hour.

Time taken to walk 48 km = $\frac{48}{x}$ hours

At 2 km an hour faster he will take = $\frac{48}{x+2}$ hours.

This time is 4 hour less than the usual time.

$\therefore \quad \frac{48}{x} = \frac{48}{x+2} + 4$

$48(x+2) = 48x + 4x(x+2)$

$\therefore \quad x^2 + 2x - 24 = 0$

$(x-4)(x+6) = 0$

$\therefore \quad x = 4$ or -6

$\therefore$ Rate of walking is 4 km/hour as ($x = -6$ is not admissible)

2. Evaluate $20 + \cfrac{1}{20 + \cfrac{1}{20...}}$

Sol. Let $x = 20 + \cfrac{1}{20 + \cfrac{1}{20....}}$ or $x = 20 + \frac{1}{x}$

Therefore, $x^2 - 20x - 1 = 0$

This gives $x = \frac{20+\sqrt{404}}{2} = 10 + \sqrt{101}$ or $x = \frac{20-\sqrt{404}}{2} = 10 - \sqrt{101}$

Since, the given expression can not be negative therefore, we neglect the negative value $10 - \sqrt{101}$. Hence, the desired value of the expression is $10 + \sqrt{101}$.

3. Find the condition that the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ may have a common root.

Sol. Let α be *a* common root of the given equations.
Then $\alpha^2 + a\alpha + b = 0$ and $\alpha^2 + b\alpha + a = 0$

By the method of cross-multiplication, we get $\frac{\alpha^2}{a^2-b^2} = \frac{\alpha}{b-a} = \frac{1}{b-a}$

This gives $\alpha^2 = \frac{a^2-b^2}{b-a} = -(a+b)$ and $\alpha = 1$

$\Rightarrow (1)^2 = -(a+b) \Rightarrow 1 = -a-b$

$\Rightarrow a + b + 1 = 0$ is the required condition.

4. Solve the equation $9x^2 - 12x + 20 = 0$ by factorization method only.

Sol. We have, $9x^2 \$ 12x + 20 = 0$

$\Rightarrow \quad 9x^2 \$ 12x + 4 + 16 = 0$

$\Rightarrow \quad (3x \$ 2)^2 + 16 = 0 \Rightarrow (3x \$ 2)^2 \$ 16i^2 = 0$

$\Rightarrow \quad \{(3x \$ 2) + 4i\}\{(3x \$ 2) \$ 4i\} = 0$

$\Rightarrow \quad (3x \$ 2 + 4i)(3x \$ 2 \$ 4i) = 0$

$\Rightarrow \quad 3x \$ 2 + 4i = 0, \text{ or } 3x \$ 2 \$ 4i = 0$

$\Rightarrow \quad 3x = 2 \$ 4i, \text{ or } 3x = 2 + 4i$

$\Rightarrow \quad x = \frac{2}{3} - \frac{4}{3}i \text{ or } x = \frac{2}{3} + \frac{4}{3}i$

Hence, the roots of the given equation are $\frac{2}{3} - \frac{4}{3}$i and $\frac{2}{3} + \frac{4}{3}$i.

5. Given that α, β are the roots of $lx^2 + mx + n = 0$, find the equation with roots $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$.

Sol. $\alpha + \beta = \frac{-m}{l}, \alpha\beta = \frac{n}{l}$

$(\alpha + \beta)^2 = \frac{m^2}{l^2}; (\alpha - \beta)^2 = (\alpha + \beta)^2 \$ 4\alpha\beta = \frac{m^2}{l^2} - 4\frac{n}{l} = \frac{m^2 - 4nl}{l^2}$

is $x^2 \$ x\ \{(\alpha + \beta)^2 + (\alpha \$ \beta)^2\} + (\alpha + \beta)^2 (\alpha \$ \beta)^2 = 0$

$x^2 \$ x \left\{\frac{m^2}{l^2} + \frac{m^2 - 4nl}{l^2}\right\} + \left(\frac{m^2}{l^2}\right)\left(\frac{m^2 - 4nl}{l^2}\right) = 0$

$\Rightarrow l^4x^2 \$ x\ \{m^2 + (m^2 \$ 4nl)\}\ l^2 + m^2 (m^2 \$ 4nl) = 0$

$\Rightarrow l^4x^2 \$ x\ \{2m^2 \$ 4nl) + m^2 (m^2 \$ 4nl) = 0$

6. If α, β are the roots of $x^2 + ax + b = 0$, find the equation for which $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}$ are the roots.

Sol. $\alpha + \beta = \$\alpha, \alpha\beta = b$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 \$ 2\alpha\beta = a^2 \$ 2b$

$\alpha^{\$2} + \beta^{\$2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{a^2 - 2b}{b^2}$

Required equation, $x^2 \$ x \left\{(a^2 - 2b) + \frac{a^2 - 2b}{b^2}\right\} + (a^2 - 2b)\frac{(a^2 - 2b)}{b^2} = 0$

$\Rightarrow x^2 \$ x \left\{\frac{b^2(a^2 - 2b) + a^2 - 2b}{b^2}\right\} + \frac{(a^2 - 2b)(a^2 - 2b)}{b^2} = 0$

$\Rightarrow b^2x^2 \$ x\{b^2 (a^2 \$ 2b) + a^2 \$ 2b\} + (a^2 \$ 2b)^2 = 0$

7. Given that α, β are the roots of $x^2 + bx + c = 0$, find the value of $(\alpha + b)^{-2} + (\beta + b)^{-2}$.

Sol. $\alpha + \beta = \$b, \alpha\beta = c$

Now, $\alpha^2 + b\alpha + c = 0 \Rightarrow \alpha(\alpha + b) = \$ c \Rightarrow (\alpha + b) = \frac{-c}{\alpha}$ and $\beta^2 + \beta b + c = 0 \Rightarrow \beta(\beta + b) + c = 0 \Rightarrow (\beta + b) = \frac{-c}{\beta}$

So, $(\alpha + b)^{\$2} = \frac{1}{(\alpha + b)^2} = \frac{1}{\left(\frac{-c}{\alpha}\right)^2} = \frac{\alpha^2}{c^2}$; $(\beta + b)^{\$2} = \frac{1}{(\beta + b)^2} = \frac{1}{\left(\frac{-c}{\beta}\right)^2} = \frac{\beta^2}{c^2}$

Thus, $(\alpha+b)^{-2}+(\beta+b)^{-2}=\frac{\alpha^2}{c^2}+\frac{\beta^2}{c^2}=\frac{\alpha^2+\beta^2}{c^2}$

Again, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-b)^2-2c=b^2-2c$

Thus, $\frac{\alpha^2+\beta^2}{c^2}=\frac{b^2-2c}{c^2}$

8. Solve $\sqrt{4x^2+4x+1}<3-x$

Sol. $\sqrt{4x^2+4x+1}<3-x \Rightarrow \sqrt{(2x+1)^2}<3-x$

$\Rightarrow \pm(2x+1)<3-x$

$\Rightarrow 2x+1<3-x$ or $-(2x+1)<3-x$

$\Rightarrow 3x<2$ or $2x+1>x-3$

$\Rightarrow x<\frac{2}{3}$ or $x>-4$

Hence, $-4<x<\frac{2}{3}$

9. If the roots of $(q^2+r^2)x^2-2r(p+q)x+(r^2+p^2)=0$ are equal, show that p, q and r are in G.P.

Sol. Since, the roots are equal,

Discriminant $=\{2r(p+q)\}^2-4(q^2+r^2)(r^2+p^2)=0$

$\Rightarrow 4r^2(p^2+2pq+q^2)-4(q^2r^2+q^2p^2+r^4+r^2p^2)=0$

$\Rightarrow (p^2r^2+2pqr^2+q^2r^2)-(q^2r^2+q^2p^2+r^4+r^2p^2)=0$

$\Rightarrow 2pqr^2-q^2p^2-r^4=0$

$\Rightarrow r^4-2pqr^2+q^2p^2=0$

$\Rightarrow (r^2-pq)^2=0 \Rightarrow (r^2=pq) \Rightarrow p, q$ and r are in G.P.

10. If the roots of $ax^2+bx+b=0$ are in the ratio $m:n$, then find the value of $\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}$.

Sol. Let the roots be $m\alpha$, $n\alpha$.

$m\alpha+n\alpha=\frac{-b}{a}$ and $m\alpha\times n\alpha=\frac{b}{a}$

We have to eliminate α.

$\alpha(m+n)=\frac{-b}{a} \Rightarrow \alpha=-\frac{b}{a(m+n)}$

Now, $mn\ \alpha^2=\frac{b}{a} \Rightarrow mn\ \frac{b^2}{a^2(m+n)^2}=\frac{b}{a} \Rightarrow \frac{mnb}{a(m+n)^2}=1$

$\Rightarrow \frac{mn}{(m+n)^2}=\frac{a}{b} \Rightarrow \frac{mn}{m^2+2mn+n^2}=\frac{a}{b}$

$\Rightarrow \frac{1}{\frac{m^2}{mn}+\frac{2mn}{mn}+\frac{n^2}{mn}}=\frac{a}{b} \Rightarrow \frac{1}{\frac{m}{n}+2+\frac{n}{m}}=\frac{a}{b} \Rightarrow \frac{m}{n}+2+\frac{n}{m}=\frac{b}{a}$

$\Rightarrow \left(\sqrt{\frac{m}{n}}\right)^2+2\sqrt{\frac{m}{n}}\sqrt{\frac{n}{m}}+\left(\sqrt{\frac{n}{m}}\right)^2=\frac{b}{a} \Rightarrow \left(\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}\right)^2=\frac{b}{a} \Rightarrow \sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}=\sqrt{\frac{b}{a}}$

11. Solve for $x : 3^{x+1}+3^{2x+1}=270$

Sol. $3^{x+1}+3^{2x+1}=270$

$\Rightarrow \quad 3.3^x+3^{2x}.3=270$

$\Rightarrow \quad 3^x+3^{2x}=90$

Substituting $3^x=a$, we get,

$a+a^2=90$

$\Rightarrow a^2+a-90=0$

$\Rightarrow a^2+10a-9a-90=0$

$\Rightarrow (a+10)(a-9)=0$

$\Rightarrow a=9$ or $a=-10$

If $3^x=9$, then $x=2$.

If $3^x=-10$, which is not possible.

$\therefore \ x=2$

12. Find the solution set of the equation $x+5-\dfrac{8}{x+5}=7$.

Sol. $x+5-\dfrac{8}{x+5}=7$

Multiply both sides by $(x+5)$, we get,

$(x+5)^2-8=7(x+5)$

i.e., $(x+5)^2-7(x+5)-8=0$

Put $u=x+5$

The equation reduces to $u^2-7u-8=0$

i.e., $(u-8)(u+1)=0$

$\therefore u=8$ or $u=-1$

$u=x+5$

i.e., $x+5=8 \Rightarrow x=8-5=3$

$x+5=-1 \Rightarrow x=-1-5=-6$

$\therefore$ roots are $x=3$ and $x=-6$.

The solution set $=\{-6, 3\}$.

13. A length of 60 cm is divided into equal parts. What is the number of these parts if, when this number is increased by unity, the length of each part is decreased by 1 mm?

Sol. The length = 60 cm or 600 mm. Let n be the number of parts.

Hence, length of each part $=\dfrac{600}{n}$ mm

When length of each part is reduced by 1 mm, the new length of each part $=\dfrac{600}{n}-1$ mm

When number of parts is increased by unity, the resulting number of parts $=(n+1)$

$\therefore \left(\dfrac{600}{n}-1\right)(n+1)=600 \Rightarrow \dfrac{600}{n}(n+1)-(n+1)=600$

Multiplying by n, $600(n+1)-n(n+1)=600n$

i.e., $600n+600-n^2-n=600n$

i.e., $n^2+n-600=0$

$\Rightarrow n^2+25n-24n-600=0$

i.e., $n(n+25)-24(n+25)=0$

i.e., $(n-24)(n+25)=0$

$\therefore \ n=24$ or $n=-25$ (inadmissible since n cannot be negative)

$\therefore$ number of parts = 24

14. If α and β are the roots of the quadratic equation $ax^2+bx+c=0$, find the value of $\alpha^3+\beta^3$

Sol. α and β be the roots of the equation $ax^2+bx+c=0$

$\therefore \quad \alpha+\beta=-\frac{b}{a}$ .... (i) and $\alpha\beta=\frac{c}{a}$ .... (ii)

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=\left(-\frac{b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)=-\frac{b^3}{a^3}+\frac{3bc}{a^2}=\frac{-b^3+3abc}{a^3}$$

15. An aeroplane travelled a distance of 400 km at an average speed of x km/h. On the return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for:
(i) the onward journey. (ii) the return journey.
If the return journey took 30 minutes less than the onward journey, write down an equation in x and find its value.

Sol. (i) Time taken for onward journey, $t=\frac{\text{Distance travelled}}{\text{Speed}}=\frac{400}{x}$ ....(i)

(ii) Time taken for return journey $t=\frac{400}{x+40}$ ......(ii)

Now, according to question, we have; $\frac{400}{x}=\frac{400}{x+40}+\frac{1}{2}$ $\quad\therefore\left[30\text{ min}=\frac{1}{2}h\right]$

$\Rightarrow \frac{400}{x}=\frac{800+x+40}{2(x+40)}$

$\Rightarrow 800x+32000=800x+x^2+40x$

$\Rightarrow x^2+40x-32000=0 \Rightarrow x^2+200x-160x-32000=0$

$\Rightarrow x(x+200)-160(x+200)=0 \Rightarrow (x-160)(x+200)=0$

Either $x-160=0$ or $x+200=0$

$\Rightarrow x=160$ or $x=-200$

Since, speed cannot be negative, so $x=-200$ is neglected.

Hence, $x=160$ km/hr.

16. Out of a group of swans, $\frac{7}{2}$ times the square root of the number are playing on the shore of a tank. The two remaining ones are playing, with amorous fight, in the water. What is the total number of swans?

Sol. Let us denote the number of swans by x.

There are two remaining swans.

Therefore, $x=\frac{7}{2}\sqrt{x}+2$

or $(x-2)^2=\left(\frac{7}{2}\right)^2$ or $4(x^2-4x+4)=49x$ or $4x^2-65x+16=0$ or $4x^2-64x-x+16=0$

or $4x(x-16)-1(x-16)=0$ or $(x-16)(4x-1)=0$

This gives $x=16$ or $x=\frac{1}{4}$

We reject $x=\frac{1}{4}$ (Why? Fractional swans cannot exist) and take $x=16$.

Hence, the total number of swans is 16.

Check : $\frac{7}{2}\times$ square root of $16=14$.

i.e., 14 swans are playing on the shore of the tank. The remaining are $16-14=2$, in accordance with the problem.

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word/ term to be filled in the blank space(s).*

1. A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and a ......
2. A quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots, if $b^2 - 4ac$ ....
3. Two numbers whose sum is 27 and product is 182 are ....
4. Two consecutive positive integers, sum of whose squares is 365 are ....
5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, the other two sides are ....
6. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. The speed of the stream ....
7. The equation $ax^2 + bx + c = 0, a \neq 0$ has no real roots, if ....
8. The values of k for which the equation $2x^2 + kx + x + 8 = 0$ will have real and equal roots are ....
9. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $a\alpha + b$ and $a\beta + b$ is ....
10. If r, s are roots of $ax^2 + bx + c = 0$, then $\frac{1}{r^2} + \frac{1}{s^2}$ is ....
11. If α is one of the roots of a quadratic equation $x^2 - 2px + p = 0$, then the other root is ....
12. The quadratic equation whose roots are the sum and difference of the squares of roots of the equation $x^2 - 3x + 2 = 0$ is....
13. If a, b are the roots of $x^2 + x + 1 = 0$ then $a^2 + b^2 =$ ....
14. If the sum of the squares of the roots of $x^2 + px - 3 = 0$ is 10, then the values of $P =$ ....
15. If α, β are the roots of $x^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $x^2 + qx + r = 0$, then $h =$ ....
16. In a homogeneous expression, all the terms will have the ....
17. Inter-changing the variables does not change the expression, in a ....
18. If k, l are the roots of $(x - \alpha)(x - \beta) = c$, where $c \neq 0$, then the roots of $(x - k)(x - l) + c = 0$ are ....
19. A quadratic equation cannot have more than .... roots.
20. Let $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$, be a quadratic equation, then this equation has no real roots if and only if ....

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. A quadratic equation cannot be solved by the method of completing the square.
2. If we can factorise $ax^2 + bx + c, a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
3. $(x - 2)(x + 1) = (x - 1)(x + 3)$ is a quadratic equation.
4. $(x^2 + 3x + 1) = (x - 2)^2$ is not a quadratic equation.
5. $x^2 + x - 306 = 0$ represent quadratic equation where product of two consecutive positive integer is 306.
6. The roots of the equation $(x - 3)^2 = 3$ are $3 \pm \sqrt{3}$
7. If sum of the roots is 2 and product is 5, then the quadratic equation is $x^2 - 2x + 5 = 0$
8. The value of x satisfying the equation $x^2 + p^2 = (q - x)^2$ is $\frac{p^2 - q^2}{2}$
9. Sum of the reciprocals of the roots of the equation $x^2 + px + q = 0$ is $1/p$.
10. The nature of roots of equation $x^2 + 2x\sqrt{3} + 3 = 0$ are real and equal.
11. If x is real, then the value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between – 5 and 4.
12. For the expression $ax^2 + 7x + 2$ to be quadratic, the possible values of a are non zero real numbers.
13. The polynomial, $\sqrt{3}x^2 + 2x + 1$ is a linear equation.
14. $x = 2$ is a root of the equation $x^2 - 5x + 6 = 0$.
15. If the roots of a quadratic equation are less than $ax^2 + bx + c$ are complex, then $b^2 = 4ac$.
16. If $a^2 - b^2 > 0$, then $a > b$ or $a < -b$.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II give roots of quadratic equations given in column I, match them correctly.

	Column I		Column II
(A)	$6x^2 + x - 12 = 0$	(p)	$(-6, 4)$
(B)	$8x^2 + 16x + 10 = 202$	(q)	$(9, 36)$
(C)	$x^2 - 45x + 324 = 0$	(r)	$(3, -1/2)$
(D)	$2x^2 - 5x - 3 = 0$	(s)	$(-3/2, 4/3)$

2. Match the column

Column I	Column II
(A) $(x-3)(x+4)+1=0$	(p) Forth degree polynomial
(B) $(x+2)^3=2x(x^2-1)$	(q) Quadratic equation
(C) $(2x-2)^2=4x^2$	(r) Non-quadratic equation
(D) $(2x^2-2)^2=3$	(s) linear equation

3. Column II give pair of two numbers for solution to problems given in column I, match them correctly.

Column I	Column II
(A) The sum of the squares two positive integers is 208. If the square of the larger number is 18 times the smaller.	(p) $(7,49)$
(B) A year ago, the father was eight times as old as his son. Now his age is the square of his son's age.	(q) $(5,29)$
(C) The age of father is equal to the square of the age of his son. The sum of the age of father and five times the age of the son is 66 years.	(r) $(36,6)$
(D) Two years ago, Jacob's age was three times the square of John's age. In three years' time. John's age will be one-fourth of Jacob's age.	(s) $(8,12)$

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. A polynomial function of the 2nd degree has what form?
2. What do we mean by a root of a quadratic?
3. What are the three methods for solving a quadratic equation, that is, for finding the roots?
4. If α & β $(\alpha>\beta)$ are the roots of equation $3x^2-2x-1=0$, find the value of $3\alpha+2\beta$
5. Form a quadratic equation whose roots α & β satisfy the system of equations $2\alpha-3\beta=7$ & $3\alpha-2\beta=8$
6. If α & β are the roots of the equation $x^2-3x+p=0$, find p such that $\alpha=2\beta$
7. If the sum of the roots of the equation is 2 & sum of their cubes is 98, then find the equation.
8. Solve for x : $\sqrt{(3^{x+1}+6)}-\sqrt{(3^x+3)}=1$
9. If(cos 30 + sin 30) is a root of the quadratic equation then, find the quadratic equation.
10. Without solving, find the sum and the product of the roots of the equations : $4x^2-3x+5=0$
11. Construct the quadratic equation whose roots are $3+\sqrt{3}, 3-\sqrt{3}$
12. Solve $x+\dfrac{5}{x}-6=0$
13. Solve the following equations for factorisation.
    (a) $-2x^2+3x+2=0,\ p,q\in R$
    (b) $8x^2-22x-21=0$
14. For what value of p will the equations have real roots ?
    $px^2+3x-4=0$
15. Find the real roots of the equation, if possible (by using quadratic formula)
    $2x^2-5\sqrt{3}x+6=0$
16. Form the quadratic equations for the roots given.
    $\dfrac{3+\sqrt{5}}{4}, \dfrac{3-\sqrt{5}}{4}$
17. Find the inqequality, $\dfrac{3x^2-7x+8}{x^2+1}\le 2$
18. Solve : $\sqrt{9x^2+6x+1}<5-x$
19. If the roots of $x^2-x+l^2=0$ are not real, find l.
20. Solve the following quadratic equations by factorization method:
    $x^2+10x-21=0$
21. Solve for x : $a^2b^2x^2+b^2x-a^2x-1=0$
22. Construct a quadratic equation whose roots are
    $3+\sqrt{7}-3-\sqrt{7}$
23. Find the values of a and b such that $x=1, x=-2$ are solutions of the quadratic equation $x^2+ax+b=0$.
24. Solve the following:
    $100x^2-20x+1=0$.
25. $2x^2+x+4=0$
26. Find the value(s) of k for which each of the following quadratic equation has equal roots:
    $2x^2+kx+3=0$
27. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the square of the other two by 60, find the numbers.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. If I had walked 1 *km/h* faster, I would have taken 10 min less to walk 2 *km*. Find the rate of my walking.
2. Solve : $9^{x+2}-6\times 3^{x+1}+1=0$
3. Evaluate $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\infty}}}$
4. Solve : $\dfrac{1}{x+5}+\dfrac{1}{x+4}=\dfrac{1}{x+2}+\dfrac{1}{x+7}$
5. Solve : $3^{x+2}+3^{-x}=10$
6. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their age (in years) was 124. Determine their present ages.

7. A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost him ₹ 1 less. How many books did he buy?
8. A segment AB of 2m length is divided at C into two parts such that $AC^2 = AB \cdot CB$. Find the length of the part CB.
9. Two persons while solving a quadratic equation, committed the following mistakes :
One of them made a mistake in the constant term and got the roots as 5 and 9.
Another one committed an error in the coefficient of x and he got the roots as 12 and 4.
But in the meantime, they realised that they are wrong and they managed to get it right jointly. Find the quadratic equation.
10. Solve the equation : $(x+1)(x+2)(x+3)(x+4)-8=0$.
11. The angry Arjun carried some arrows for fighting with Bheeshma. With half the arrows, he cut down the arrow thrown by Bheeshma on him and with six other arrows he killed the rath driver of Bheeshma. With one arrow each he knocked down respectively the rath, flag and the bow of Bheeshma. Finally, with one more than four times the square root of arrows he laid Bheeshma unconscious on an arrow bed. Find the total number of arrows Arjun had.
12. Find the number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$
13. Find the condition such that the expression $x^2 + 2(a+b+c)x + 3(bc+ca+ab)$ will be a perfect square.
14. Find the solution set of $(x+1)(x-1)^2(x-2) \geq 0$
15. Find the solution set of inequality $\dfrac{2x}{x^2-9} \leq \dfrac{1}{x+2}$
16. Determine the value(s) of p for which the quadratic equation $4x^2 - 3px + 9 = 0$ has real roots.
17. If the roots of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.
18. Solve for x : $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}, a \neq 0, b \neq 0, x \neq 0$.
19. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.
20. Find the value of k if $x^2 + x - k = 0$ and $x^2 - 10x + (2k-3) = 0$ have 3 as a common root.
21. Solve for x : $(x+2)(2x+1)(3x+5)(6x+1)+2=0$
22. Solve the equation $x^2 + px + 45 = 0$, it is being given that the squared difference of its roots is equal to 144.
23. A number consists of two digits whose product is 18. If 27 is added to the number, the number formed will have the digits in reverse order, when compared to the original number. Find the number.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. A group of girls planned a picnic. The budget for food was ₹ 2400. Due to illness, 10 girls could not go to the picnic and cost of food for each girl increased by ₹ 8. How many girls had planned the picnic?
2. A plane left 40 minutes late due to bad weather and in order to reach the destination 1600 km away in time, it had to increase its speed by 400 km/hour from its usual speed. Find its usual speed.
3. Some students planned to go for a picnic. The budget for food was ₹ 240. As four students failed to go, the cost of food for each student increased by ₹ 10. How many students had gone for the picnic?
4. A takes 12 days less than B to finish a piece of work. If A and B together can finish the work in 8 days, find the time taken by B to finish the work.
5. Two trains leave New Delhi station at the same time. The first train travels due west and the second, due north. The speed of the second train is 5 km/hr greater than that of the first train. If, after two hours, they are 50 km apart, find the average speed of each train.
6. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are $\alpha + \dfrac{1}{\beta}$ and $\beta + \dfrac{1}{\alpha}$.
7. Find the solution of equation
$$\frac{p+q-x}{r} + \frac{q+r-x}{p} + \frac{r+p-x}{q} + \frac{4x}{p+q+r} = 0$$
8. If α and β are roots of the equation $A(x^2+m^2) + Amx + cm^2x^2 = 0$, then find the value of $A(\alpha^2+\beta^2) + A\alpha\beta + c\alpha^2\beta^2$
9. Find the solution of the equation $\sqrt{x-2} + \sqrt{4-x} = \sqrt{6-x}$
10. Determine the values of x which satisfy the simultaneous inequations $x^2 + 5x + 4 > 0$ and $-x^2 - x + 42 > 0$.
11. Given that α, β are the roots of $ax^2 + bx + c = 0$ and $\dfrac{\alpha}{1-\alpha}, \dfrac{\beta}{1-\beta}$ are the of roots $px^2 + qx + r = 0$, find the relation among p, q and r in terms of a, b and c.
12. One side of a square is increased by x% while next side is reduced by x% to form a rectangle. The area of the rectangle is 4% less than the area of the original square. Find x.

13. On a windy day a boy rides to a place 24 km away and returns by the same road. The wind adds 2 km per hour to his speed on his outward journey and retards him by the same amount on his way home. He takes one hour more for his return journey than for the onward journey. At what rate does he ride when there is no wind?

14. B takes 16 days less than A to do a piece of work. If both working together can do it in 15 days, in how many days will B alone complete the work?

15. Solve the equation : $6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0$

EXERCISE 2

MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The corrects roots are
   (a) $3, -2$ (b) $-3, 2$
   (c) $-6, -1$ (d) $6, -1$
2. What is the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be twice the other
   (a) $b^2 = 4ac$ (b) $2b^2 = 9ac$
   (c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$
3. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{\alpha\beta+b}+\frac{\beta}{a\alpha+b}=?$
   (a) $2/a$ (b) $2/b$
   (c) $2/c$ (d) $-2/a$
4. If $\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$, then $x = ?$
   (a) 3 (b) 2
   (c) 4 (d) None of these
5. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2+\beta^2+\gamma^2$ is equal to
   (a) $-15/4$ (b) $15/4$
   (c) $9/4$ (d) 4
6. If the equation $2x^2 + x + k = 0$ and $x^2 + x/2 - 1 = 0$ have 2 common roots then the value of k is
   (a) 1 (b) 3
   (c) -1 (d) -2
7. If $x^2 + y^2 = 25$, $xy = 12$, then $x =$
   (a) $\{3, 4\}$ (b) $\{3, -3\}$
   (c) $\{3, 4, -3, -4\}$ (d) $\{-3, -3\}$
8. If $x=\sqrt{7+4\sqrt{3}}$, then $x+\frac{1}{x}=$
   (a) 4 (b) 6
   (c) 3 (d) 2
9. If the roots of the given equation $2x^2 + 3(\lambda - 2)x + \lambda + 4 = 0$ be equal in magnitude but opposite in sign, then $\lambda =$
   (a) 1 (b) 2
   (c) 3 (d) 2/3
10. If the roots of the equation $px^2 + 2qx + r = 0$ and $qx^2-2\sqrt{pr}x+q=0$ be real, then
    (a) $p = q$ (b) $q^2 = pr$
    (c) $p^2 = qr$ (d) $r^2 = pq$
11. The value of m for which the equation $\frac{a}{x+a+m}+\frac{b}{x+b+m}=1$ has roots equal in magnitude but opposite in sign is
    (a) $\frac{a+b}{a-b}$ (b) 0
    (c) $\frac{a-b}{a+b}$ (d) $\frac{2(a-b)}{a+b}$
12. The equation $2x^2 + 2(p + 1)+x + p = 0$, where p is real, always has roots that are
    (a) Equal
    (b) Equal in magnitude but opposite in sign
    (c) Irrational
    (d) Real
13. If the ratio of the roots of the equation $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$, then
    (a) $r^2b = qc^2$ (b) $r^2c = qb^2$
    (c) $c^2r = q^2b$ (d) $b^2r = q^2c$
14. If $a + b + c = 0$ and a, b, c are rational, then the roots of the equation : $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are
    (a) rational (b) irrational
    (c) imaginary (d) equal

15. The ratio of the roots of $bx^2 + nx + n = 0$ is $p : q$, then

(a) $\sqrt{\frac{q}{p}}+\sqrt{\frac{p}{q}}+\sqrt{\frac{\ell}{n}}=0$ (b) $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}}=0$

(c) $\sqrt{\frac{q}{p}}+\sqrt{\frac{p}{q}}+\sqrt{\frac{\ell}{n}}=0$ (d) $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}}=0$

16. If α,β are the roots of $x^2-2px+q=0$ and γ,δ are roots of $x^2-2rx+s=0$ and $\alpha,\beta,\gamma,\delta$ are in A.P. then

(a) $p-q=r^2-s^2$ (b) $s-q=r^2-p^2$
(c) $r-s=p^2-q^2$ (d) None of these

17. The real roots of the equation $x^{2/3}+x^{1/3}-2=0$ are

(a) 1, 8 (b) –1, –8
(c) –1, 8 (d) 1, –8

18. $\frac{8x^2+16x-51}{(2x-3)(x+4)}>3$, if x satisfies

(a) $x<-4$ (b) $-3<x<3/2$
(c) $x>5/2$ (d) all the above

19. Which of the following is not a quadratic equation?

(a) $x^2-2x+2(3-x)=0$
(b) $x(x+1)+1=(x-2)(x-5)$
(c) $(2x-1)(x-3)=(x+5)(x-1)$
(d) $x^3-4x^2-x+1=(x-2)^3$

20. If one root of the quadratic equation $ax^2+bx+c=0$ is the reciprocal of the other, then

(a) $b=c$ (b) $a=b$
(c) $ac=1$ (d) $a=c$

21. The roots of the equation $x+\frac{1}{x}=3\frac{1}{3}$, $x\neq 0$, are

(a) 3, 1 (b) $3,\frac{1}{3}$
(c) $3,-\frac{1}{3}$ (d) $-3,-\frac{1}{3}$

22. If the equation $2x^2-6x+p=0$ has real and different roots, then the values of p are given by

(a) $p<\frac{9}{2}$ (b) $p\leq\frac{9}{2}$
(c) $p>\frac{9}{2}$ (d) $p\geq\frac{9}{2}$

23. Which of the following equations have no real roots?

(a) $x^2-2\sqrt{3}x+5=0$ (b) $2x^2+6\sqrt{2}x+9=0$
(c) $x^2-2\sqrt{3}x-5=0$ (d) $2x^2-6\sqrt{2}x-9=0$

24. If the equation $(m^2+n^2)x^2-2(mp+nq)x+p^2+q^2=0$ has equal roots, then

(a) $mp=nq$ (b) $mq=np$
(c) $mn=pq$ (d) $mq=\sqrt{np}$

25. Given that $(x+1)$ is a common factor of x^2+ax+b and x^2+cx-d, then

(a) $a+b=c+d$ (b) $a=b+c+d$
(c) $a+c=b+d$ (d) $d=a-b+c$

26. If one root of $x^2-px+q=0$ is the n[th] power of the other root, then $\frac{1}{q^{n+1}}+\frac{n}{q^{n+1}}$ is equal to

(a) $-p$ (b) q
(c) $-q$ (d) p

27. The roots of $x^2-bx+c=0$ are each decreased by 2. The resulting equation is $x^2-2x+1=0$. Then

(a) $b=6,c=9$ (b) $b=3,c=5$
(c) $b=2,c=-1$ (d) $b=-4,c=3$

28. If one root of $ax^2+bx+c=0$ is equal to n^{th} power of the other, then $(ac^n)^{1/(n+1)}+(a^nc)^{1/(n+1)}$

(a) 1 (b) 1
(c) b (d) $-b$

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. If α,β are roots of the equation $x^2-5x+6=0$ then the equation whose roots are $\alpha+3$ and $\beta+3$ is

(a) $2x^2-11x+30=0$ (b) $-x^2+11x=0$
(c) $x^2-11x+30=0$ (d) $2x^2-22x+60=0$

2. If equation $x^2-(2+m)x+1(m^2-4m+4)=0$ has coincident roots, then:

(a) $m=0$ (b) $m=6$
(c) $m=2$ (d) $m=\frac{2}{3}$

3. Which of the following equations have no real roots?

(a) $x^2-2\sqrt{3}x+5=0$ (b) $-2x^2+6\sqrt{2}+11=0$
(c) $x^2-2\sqrt{3}x-5=0$ (d) $2x^2-6\sqrt{2}x-9=0$

4. Two numbers whose sum is 8 and the absolute value of whose difference is 10 are roots of the equation

(a) $x^2-8x+9=0$ (b) $x^2-8x-9=0$
(c) $x^2+8x-9=0$ (d) $-(-x^2+8x+9)=0$

5. Zeroes of polynomial $p(x)=x^2-3x+2$ are

(a) 3 (b) 1
(c) 4 (d) 2

6. If the given expression is a complete square, then which of the following formulae we use to factorise it?

(a) $a^2+2ab+b^2=(a+b)^2$
(b) $a^2-2ab+b^2=(a-b)^2$
(c) $(a-b)(a+b)=(a^2-b^2)$
(d) $(x+a)(x+b)=x^2+(a+b)x+ab$

7. The equality $b^2+5>9b+12$ is satisfied if

(a) $b>9$ (b) $b<1$
(c) $b>0$ (d) $b<0$

8. Let α and β be the roots of a quadratic equation $ax^2 + bx + c = 0$, then

(a) $\alpha+\beta=\frac{-b}{a}$ (b) $\alpha.\beta=\frac{c}{a}$

(c) $\alpha.\beta=\frac{b}{a}$ (d) $\alpha.\beta=\frac{-c}{a}$

9. If α and β are the roots of a quadratic equation then the product of roots is

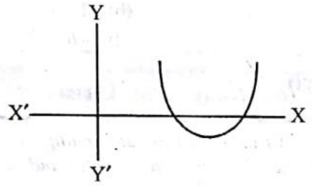
(a) Numerator = Coeff of x^2

(b) Denominator = Constant term

(c) Numerator = Constant term

(d) Denominator = Coeff of x^2

10. For the below figure of $ax^2 + bx + c = 0$



(a) $a<0$ (b) $b>0$

(c) $D>0$ (d) $a>0$

11. The value of m so that the equation $3x^2 - 2mx - 4 = 0$ and $x(x-4m)+2=0$ may have a common root is –

(a) $1/\sqrt{2}$ (b) $-1/\sqrt{2}$

(c) 1/2 (d) – 1/2

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage I

Let us consider a quadratic equation

$$x^2 + 3ax + 2a^2 = 0$$

If the above equation has roots α, β and it is given that $\alpha^2 + \beta^2 = 5$

(i) Value of a is

(a) 1 (b) – 1

(c) ± 1 (d) none of these

(ii) Value of D for the above quadratic equation is

(a) $D>0$ (b) $D<0$

(c) $D=0$ (d) none of these

(iii) Product of roots is

(a) 2 (b) 1

(c) –3 (d) 3

Passage-II

Let us consider a quadratic equation $x^2 + \lambda x + \lambda + 1.25 = 0$, where λ is a constant. The value of λ such that the above quadratic equation has

(i) two distinct roots

(a) $\lambda<5$ (b) $\lambda>-1$

(c) $\lambda>5$ or $\lambda<-1$ (d) none of these

(ii) two coincident roots

(a) $\lambda<5$ or $\lambda=-1$ (b) $\lambda=1$ or $\lambda=5$

(c) $\lambda=-5$ or $\lambda=1$ (d) none of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** The equation $(x-p)(x-r)+\lambda(x-q)(x-s) = 0, p<q<r<s$, has non-real roots if $\lambda>0$.

Reason: The equation $ax^2 + bx + c = 0$, $a, b, c \in R$, has non-real roots if $b^2 - 4ac < 0$.

2. **Assertion :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Reason : If a, b, c are odd integer then the roots of the equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

3. **Assertion :** If $1 \le a \le 2$ then

$$\sqrt{a+2\sqrt{a-1}}+\sqrt{a-2\sqrt{a-1}}=2$$

Reason : If $1 \le a \le 2$ then $(a-1)>1$.

4. **Assertion :** If one root is $\sqrt{3}-\sqrt{2}$, then the equation of lowest degree with rational coefficients $x^4-10x^2+1=0$.

Reason : For a polynomial equation with rational coefficient irrational roots occurs in pairs.

5. **Assertion :** Degree of the polynomial $5x^2 + 3x + 4$ is 2.

Reason : The degree of a polynomial of one variable is the highest value of the exponent of the variable.

6. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin (-1, 0, 1)$

Assertion : $(p^2 - q)(b^2 - ac) \ge 0$

Reason : $b \ne pa$ or $c \ne qa$

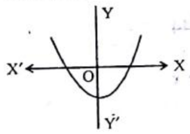
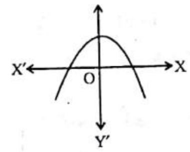
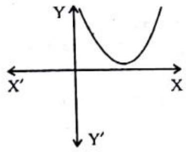
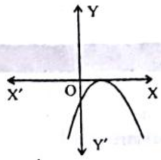
MMQ Muliple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column-I	Column-II
(A) If α, β are roots of $ax^2+bx+c=0$, then one of the roots of the equation $ax^2-bx(x-1)+c(x-1)^2=0$	(p) $a<0, b>0$
(B) If the roots of $ax^2+b=0$ are real, then	(q) real and equal
(C) Roots of $4x^2-4x+1=0$	(r) $\dfrac{\beta}{1+\beta}$
(D) Roots of $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are always	(s) $a>0, b<0$
	(t) Real
	(u) $\dfrac{\alpha}{1+\alpha}$

2. D be the discriminant of the quadratic equation $ax^2+bx+c=0$

Column-I	Column-II
(A) 	(p) $a<0$
(B) 	(q) $a>0$
(C) 	(r) $D<0$
(D) 	(s) $D>0$
	(t) $D=0$

HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Solve the equation : $12x^4-56x^3+89x^2-56x+12=0$
2. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?
3. If the ratio of the roots of the equation $x^2-2ax+b=0$ is equal to that of the roots $x^2-2cx+d=0$, then prove that $\dfrac{a^2}{c^2}=\dfrac{b}{d}$.
4. A scenery costs ₹ R_1. A shopkeeper gives a discount of $x\%$ and reduces its price to R_2. He gives a further discount of $x\%$ on the reduced price R_2 to reduce it further to R_3, which reduces it by ₹ 415. A customer bargains with him and takes an $x\%$ discount on R_3 and buys the scenery for ₹ 3362.8. Find the original price R_1 of the scenery.
5. A businessman bought some items for ₹ 600, keeping 10 items for himself, then sold the remaining items at a profit of ₹ 5 per item. From the amount received in this deal he could buy 15 more items. Find the original price of each item
6. A swimming pool is filled by three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.
7. Out of a certain number of Saras birds one-fourth the number are moving about in lotus plants, $\dfrac{1}{9}$th coupled with $\dfrac{1}{4}$th as well as 7 times the square root of the number move on a hill, 56 birds remain in Vacula tree, what is the total number of birds?
8. If $k \notin [0, 8]$, find the value of x for which the inequality $\dfrac{x^2+k^2}{k(6+x)} \geq 1$ is satisfied.
9. Find two numbers, whose difference multiplied by the difference of their squares = 160 and whose sum multiplied by the sum of their squares gives the number 580.
10. Find the value of p if $\alpha^2+\beta^2-\alpha\beta=3\dfrac{1}{4}$ where α and β are roots of $x^2+px+1=0$
11. $3x^4-20x^3-94x^2-20x+3=0$

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1 $\neq 0$ 2. >0 3. 13, 14
4. 13, 14 5. 5 cm, 12 cm. 6. 6 km/hr.
7. $b^2 < 4ac$ 8. 7 and –9
9. $x^2 - bx + ca = 0$ 10. $\dfrac{b^2 - 2ac}{c^2}$ 11. $\dfrac{\alpha}{2\alpha - 1}$
12. $x^2 - 8x + 15 = 0$ 13. –1 14. ±2
15. $\frac{1}{2}(b-q)$ 16. Same degree
17. Symmetric expression
18. α, β $[(x-\alpha)(x-\beta) - c \equiv (x-k)(x-l)$ because the roots are k, l.
$\Rightarrow a\ (x-\alpha)(x-\beta) \equiv (x-k)(x-l) + c$, etc.]
19. two 20. $b^2 < 4ac$

TRUE / FALSE

1. False 2. True 3. False
4. True 5. True 6. True
7. True 8. False 9. False
10. True
11. True;

Let $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 3} = y$

$x^2 + 14x + 9 = x^2y + 2xy + 3y$
$(1-y)x^2 + (14-2y)x + (9-3y) = 0$

Since, x to be real, so $D \geq 0$

$\Rightarrow (14-2y)^2 - 4(1-y)(9-3y) \geq 0$.
$\Rightarrow (7-y)^2 - (1-y)(9-3y) \geq 0$
$\Rightarrow -2y^2 - 2y + 40 \geq 0$
$\Rightarrow -2(y^2 + y - 20) \geq 0$
$y^2 + y - 20 \leq 0$

($\therefore$ sign of inequality changes, if we multiply both sides by a negative number).

$(y-4)(y+5) \leq 0$
$\therefore -5 \leq y \leq 4$

12. True 13. False 14. True
15. False 16. True

MATCH THE FOLLOWING :

1. (A) $\to$ s; (B) $\to$ p; (C) $\to$ q; (D) $\to$ r

(A) $6x^2 + x - 12 = 0$
$6x^2 + 9x - 8x - 12 = 0$
$3x(2x+3) - 4(2x+3) = 0$
$(3x-4)(2x+3x) = 0$
$x = \frac{4}{3}, \frac{-3}{2}$

(B) $8x^2 + 16x + 10 = 20$
$8x^2 + 16x - 192 = 0$
$8x^2 + 48x - 32x - 192 = 0$
$8x(x+6) - 32(x+6) = 0$
$x = 4, -6$

(C) $x^2 - 45x + 324 = 0$
$x^2 - 36x - 9x + 324 = 0$
$x(x-36) - 9(x-36) = 0$
$x = 9, 36.$

(D) $2x^2 - 5x - 3 = 0$
$2x^2 - 6x + x - 3 = 0$
$2x(x-3) + 1(x-3) = 0$
$x = \frac{-1}{2}, 3$

2. (A) $\to$ q; (B) $\to$ r; (C) $\to$ s; (D) $\to$ p
3. (A) $\to$ s, (B) $\to$ p, (C) $\to$ r; (D) $\to$ q

(A) Let the smaller number be x.
Then, $18x + x^2 = 208$
$x^2 + 18x - 208 = 0.$
$x^2 + 26x - 8x - 208 = 0$
$x(x+26) - 8(x+26) = 0$
$x = 8, -26$
$\therefore$ (larger number)$^2 = 18(8) = 144$
larger number = 12 present

(B) Let the son's present age be x.
Then, father's age $= x^2$
Now, $x^2 - 1 = 8(x-1)$
$x^2 - 8x + 7 = 0$
$x^2 - 7x - x + 7 = 0$
$(5-7)(5-1) = 0$
$5 = 1, 7$
$\therefore$ Father's age $= (7)^2 = 49$

(C) Let the son's age be x
Father's age $= x^2$
$x^2 + 5x = 66$
$x^2 + 5x - 66 = 0$
$x^2 + 11x - 6x - 66 = 0$
$x(x+11) - 6(x+11) = 0$
$x = 6, 11$
$\therefore$ Father's age $= (6)^2 = 36$

VERY SHORT ANSWER QUESTIONS :

1. $y = ax^2 + bx + c$
2. A solution to the quadratic equation.
3. 1. Factoring. 2. Completing the square.
3. The quadratic formula
4. 7/3

5. $x^2 - x - 2 = 0$

6. 2

7. $x^2 - 2x - 15 = 0$

8. $x = 0$

9. $2x^2 - 2x - 1 = 0$

10. $4x^2 - 3x + 5 = 0,\ a = 4, b = -3, c = 5$

Sum of roots $= -\dfrac{b}{a} = \dfrac{3}{4}$.

Product of roots $= \dfrac{c}{a} = \dfrac{5}{4}$

11. The sum of the roots is 6. Their product is $9 - 3 = 6$.
Therefore, the quadratic is $x^2 - 6x + 6 = 0$

12. $x + \dfrac{5}{x} - 6 = 0 \Rightarrow \dfrac{x^2 + 5 - 6x}{x} = 0$

$\Rightarrow x^2 - 6x + 5 = 0 \Rightarrow x^2 - 5x - x + 5 = 0$

$\Rightarrow x(x-5) - 1(x-5) = 0$

$\Rightarrow (x-5)(x-1) = 0 \Rightarrow x = 1$ or $x = 5$

Hence, the roots are 1 and 5.

13. (a) $-2x^2 + 3x + 2 = 0 \Rightarrow -2x^2 + 4x - x + 2 = 0$

$\Rightarrow -2x(x-2) - (x-2) = 0 \Rightarrow (-2x-1)(x-2) = 0$

$\Rightarrow -2x - 1 = 0$ or $x - 2 = 0 \Rightarrow x = -1/2$ or $x = 2$

(b) $8x^2 - 22x - 21 = 0$

$\Rightarrow 8x^2 - 28x + 6x - 21 = 0$

$\Rightarrow 4x(2x-7) + 3(2x-7) = 0$

$\Rightarrow (4x+3)(2x-7) = 0$

$\therefore$ either $4x + 3 = 0$ or $2x - 7 = 0$

$\Rightarrow x = -3/4$ or $x = 7/2$

14. $px^2 + 3x - 4 = 0 \Rightarrow a = p, b = 3, c = -4$

$\therefore D = b^2 - 4ac = 9 - 4p \times (-4) = 9 + 16p$

$\Rightarrow 9 + 16p \geq 0 \quad (\because$ for real roots, $D \geq 0)$

$\Rightarrow 16p \geq 0 - 9 \Rightarrow p \geq -\dfrac{9}{16}$

15. $2x^2 - 5\sqrt{3}x + 6 = 0,\ a = 2,\ b = 5\sqrt{3}, c = 6$

$D = (5\sqrt{3})^2 - 4 \times 2 \times 6 = 75 - 48 = 27 > 0$

$\therefore$ Roots of equation (1) are $x = \dfrac{-5\sqrt{3} \pm \sqrt{27}}{2 \times 2}$

Either $x = \dfrac{-(5\sqrt{3}) + 3\sqrt{3}}{4}$ or $x = \dfrac{-5\sqrt{3} - 3\sqrt{3}}{4}$

$\Rightarrow x = \sqrt{3}/2$ or $x = -2\sqrt{3}$

16. Here, $S = \dfrac{3+\sqrt{5}}{4} + \dfrac{3-\sqrt{5}}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

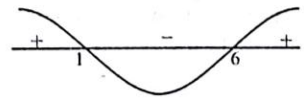
and $P = \left(\dfrac{3+\sqrt{5}}{4}\right)\left(\dfrac{3-\sqrt{5}}{4}\right) = \dfrac{9-5}{16} = \dfrac{1}{4}$

$\therefore$ The required equation is $x^2 - Sx + P = 0$

i.e., $x^2 - \dfrac{3}{2}x + \dfrac{1}{4} = 0 \Rightarrow 4x^2 - 6x + P = 0$

17. (d) Domain : $x \in R$
given inequality is equivalent to

$$\frac{3x^2 - 7x + 8}{x^2 + 1} - 2 \leq 0$$



$$\Rightarrow \frac{3x^2 - 7x + 8 - 2x^2 - 2}{x^2 + 1} \leq 0$$

$$\Rightarrow \frac{3x^2 - 7x + 6}{x^2 + 1} \leq 0 \Rightarrow \frac{(x-1)(x-6)}{x^2 + 1} \leq 0$$

$\Rightarrow x \in [1, 6]$

18. $x < 1$ or $x > -3$ $[\pm(3x+1) < 5 - x$, etc.$]$

19. $l > \dfrac{1}{2}$ and $l < \dfrac{-1}{2}$ [Discriminant $\Delta < 0$, etc.]

20. $-3i, -7i$

21. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$\Rightarrow b^2 \times (a^2x + 1) - (a^2x + 1) = 0$ i.e., $(b^2x - 1)(a^2x + 1) = 0$

$\Rightarrow b^2x - 1 = 0$ or $a^2x + 1 = 0$

$\therefore x = \dfrac{1}{b^2}$ or $x = \dfrac{-1}{a^2}$

22. $x^2 - 6x + 2 = 0$

23. $a = 1, b = -2$

24. $\dfrac{1}{10}, \dfrac{1}{10}$

25. No real roots

26. $2\sqrt{6}, -2\sqrt{6}$

27. Let the middle number be x, then the other two numbers are $x - 1$ and $x + 1$. According to the given condition, $x^2 - [(x+1)^2 - (x-1)^2] = 60]$

SHORT ANSWER QUESTIONS :

1. Let the rate of walking be v km/hr and time taken be t hr. Now,

$$(v+1)\left(t - \frac{10}{60}\right) = 2 = vt$$

$$vt = 2$$

$$t = \frac{2}{v} \qquad \text{..............(i)}$$

Putting the value of (i) in $(v+1)\left(t - \dfrac{10}{60}\right) = 2$

$$(v+1)\left(t - \frac{2}{v} - \frac{1}{6}\right) = 2$$

$2-\frac{v}{6}+\frac{2}{v}-\frac{1}{6}=2$

$v^2+v-12=0$

$(v-3)(v+4)=0$

neglecting $v=-4$, we get, $v=3$ km/hr.

2. $3^{2(x+2)}-2\times 3\times 3^{x+1}+1=0$

$3^{2(x+2)}-2\times 3^{x+2}+1=0$

let $3^{x+2}=a$

$a^2-2a+1=0$

$a^2-a-a+1=0$

$(a-1)^2=0$ $a(a-1)-1(a-1)=0$

$a=1$

$3^{x+2}=1=3^0$

$x=-2.$

3. Let $x=\sqrt{6+\sqrt{6+\sqrt{6+......\infty}}}$ Then, $x=\sqrt{6+x}$

$\Rightarrow x^2=6+x \Rightarrow x^2-x-6=0$

$\Rightarrow (x-3)(x+2)=0 \Rightarrow x=3$ or $x=-2.$

4. Given : $\frac{1}{x+5}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+7}$

$\Rightarrow \frac{1}{x+5}-\frac{1}{x+2}=\frac{1}{x+7}-\frac{1}{x+4}$

$\Rightarrow \frac{x+2-x-5}{(x+5)(x+2)}=\frac{x+4-x-7}{(x+7)(x+4)}$

$\Rightarrow (x+7)(x+4)=(x+5)(x+2)$

$\Rightarrow x^2+11x+28=x^2+7x+10$

$\Rightarrow 4x=-18 \Rightarrow x=-\frac{9}{2}$

5. $3^{x+2}+3^{-x}=10 \Rightarrow 9(3^x)+\frac{1}{3^x}=10$

Substituting $3^x=y$, we get, $9y+\frac{1}{y}=10$

$\Rightarrow 9y^2-10y+1=0$

$\Rightarrow 9y^2-9y-y+1=0$

$\Rightarrow (9y-1)(y-1)=0$

$\Rightarrow 9y-1=0$ or $y-1=0$

$\Rightarrow y=1/9$ $y=1$

i.e. $3^x=3^{-2}$ $3^x=3^0$

$\Rightarrow x=-2$ $x=0$

Hence, the required solutions are -2 and 0.

6. Present ages (in years)

Father $=x$, Son $=y$

5 years ago, Father $=x-5$, Son $=y-5$

According to the given conditions

$x+y=45$ ....(1)

$(x-5)(y-5)=124$ ....(2)

$(x-5)[45-x-5]=124$

Son's age = 9 year and father's age 36 year.

7. Let the number of books in I condition $=x$

Books bought in II condition $=(x+4)$

Amount paid for books = ₹ 80

According to the question

$\frac{80}{x}-\frac{80}{x+4}=₹1 \Rightarrow \frac{80x+320-80x}{x(x+4)}=\frac{1}{1}$

$\Rightarrow x^2+4x=320 \Rightarrow x^2+4x-320=0$

$\Rightarrow x^2+20x-16x-320=0$

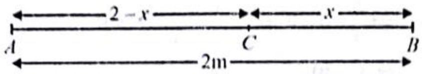
$\Rightarrow x(x+20)-16(x+20)=0$

$\Rightarrow (x+20)(x-16)=0$

$\Rightarrow x=-20$ or $x=16$ $\because$ Quantity cannot be -ve

$\therefore$ Number of books bought = 16

8.



In the figure $AB=2$m

Let $CB=x$ m. Then $AC=(2-x)$m

Now, it is given that $AC^2=AB.CB$

$\therefore (2-x)^2=2x$ or $4+x^2-4x=2x$ or $x^2-6x+4=0$

$x=\frac{6\pm\sqrt{36-4\times1\times4}}{2}=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm2\sqrt{5}}{2}$

$=3\pm\sqrt{5}$

But $3+\sqrt{5}$ is not possible as it is more than the total length, and shows external division.

Hence, $CB=3-\sqrt{5}$ m.

9. When mistake was committed by first one in writing constant term, he got roots as 5 and 9. But co-efficient of x was written correctly so, sum of roots $=5+9=14$. The other person did mistake in writing co-efficient of x, and got roots as 12 and 4. But constant term was written correctly so, product of roots $=12\times4=48$. So, the correct quadratic equation is where sum of roots $=14$ and product of roots $=48$.

The equation is x^2-(sum of roots) $x+$ product of roots $=0$

i.e., $x^2-14x+48=0$

10. The given equation is : $(x+1)(x+2)(x+3)(x+4)-8=0$

In such type of equations we combine the factors in such a way that the product of two factors together gives some common polynomial. Rewriting the equation, we have

$(x+1)(x+4)(x+2)(x+3)-8=0$

or $(x^2+5x+4)(x^2+5x+6)-8=0$

Let $x^2+5x=y$

$\therefore (y+4)(y+6)-8=0$ or $y^2+10y+24-8=0$

or $y^2+10y+16=0$ or $(y+8)(y+2)=0$

$\therefore$ $y=-8$ or -2

But $y=x^2+5x$

$\therefore$ $x^2+5x=-8$ | or $x^2+5x=-2$

or $x^2+5x+8=0$ | or $x^2+5x+2=0$

$\therefore$ $x=\frac{-5\pm\sqrt{25-32}}{2}$ | $\therefore x=\frac{-5\pm\sqrt{25-8}}{2}=\frac{-5\pm\sqrt{17}}{2}$

Discriminant <0

So, there is no real solution.

11. Let Arjun has x arrows. So, he spent $\frac{x}{2}$ for cutting the arrows thrown by Bheeshma, and 6 arrows to kill his rath driver; and 3 more for rath, flag and bow of Bheeshma. He laid the Bheeshma unconscious by $4\sqrt{x}+1$ arrows.

So, $\frac{x}{2}+6+3+4\sqrt{x}+1=x$

$\Rightarrow\ 10+4\sqrt{x}=\frac{x}{2}\Rightarrow 20+8\sqrt{x}=x$ or $8\sqrt{x}=x-20$,

Squaring both sides, we get

$(8\sqrt{x})^2=(x-20)^2\Rightarrow 64x=x^2-40x+400$

or $x^2-104x+400=0\Rightarrow(x-100)(x-4)=0$

So, $x=100$ or $x=4$

$x=4$ is not possible, hence, $x=100$.

Hence, total number of arrows that Arjun had is 100.

12. Given $|x|^2-3|x|+2=0=0$

Here, we consider two cases viz,. $x<0$ and $x>0$

Case-I : $x<0$, This gives $x^2+3x+2=0$

$\Rightarrow(x+2)(x+1)=0\ \Rightarrow\ x=-2,-1$

Also, $x=-1,-2$ satisfy $x<0$, so $x=-1,-2$ is solution in this case.

Case-II : $x>0$, This gives $x^2-3x+2=0$

$\Rightarrow(x-2)(x-1)=0\ \Rightarrow\ x=2,1$

so $x=2$, 1 is solution in this case.

Hence, the number of solutions are four i.e.,

$x=-1,1,2,-2$

13. The expression will be a perfect square if the roots of the corresponding quadratic equation are equal.

Condition for that is D = 0

$\Rightarrow\ 4(a+b+c)^2-4\,.\,1\,.\,3\,(bc+ca+ab)=0$

$\Rightarrow\ a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow\ \frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}=0$

$\therefore\ a=b=c$

14. $(x+1)(x-1)^2(x-2)\geq 0$

$\Rightarrow\ (x+1)(x-2)\geq 0$

and $x=1$ (As $(x-1)^2\geq 0$)

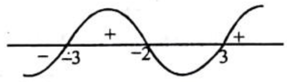
$\Rightarrow\ x\leq -1$ or $x\geq 2$ and $x=1$

Solution is $(-\infty,-1]\cup\{1\}\cup[2,\infty)$

15. We have $x^2-9\neq 0$ and $x+2\neq 0$ and

$\frac{2x}{x^2-9}-\frac{1}{x+2}\leq 0\Rightarrow\frac{2x^2+4x-x^2+9}{(x+2)(x^2-9)}\leq 0$

$\Rightarrow\frac{x^2+4x+9}{(x+2)(x^2-9)}\leq 0\ \Rightarrow(x+2)(x+3)(x-3)<0$



$(\because\ x^2+4x+9>0\ \forall\, x\in R)$

From the wavy curve shown , we have

$x\in(-\infty,-3)\cup(-2,3)$

16. $p\geq 4$ or $p\leq -4$

17. **Hint :** Discriminant $=0\Rightarrow(b-c)^2-4(a-b)(c-a)=0$

$\Rightarrow 4a^2+b^2+c^2-4ab+2bc-4ca=0$

$\Rightarrow(-2a+b+c)^2=0\Rightarrow -2a+b+c=0.$

18. $(-a,-b)$

Hint : $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}\Rightarrow\ \frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$\Rightarrow\ \frac{x-(a+b+x)}{(a+b+x)x}=\frac{b+a}{ab}\Rightarrow\frac{-(a+b)}{(a+b+x)x}=\frac{a+b}{ab}$

$\Rightarrow\ \frac{-1}{(a+b+x)x}=\frac{1}{ab}\Rightarrow x^2+(a+b)x+ab=0$

$\Rightarrow(x+a)(x+b)=0.$

19. 15 hours, 25 hours

Hint : The tank is filled by the two pipes together in $9\frac{3}{8}$ hours

i.e. in $\frac{75}{8}$ hours

$\therefore$ The part of tank filled in one hour $=\frac{8}{75}$.

Let the time taken by the pipe of larger diameter to fill the tank separately be x hours, then

$\frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}\Rightarrow 4x^2-35x-375=0.$

20. If $ax^2+bx+c=0$ and $a'x^2+b'x+c'=0$ have a common root, then it is $\frac{ca'-ac'}{ab'-ba'}$

Here, $a=1$, $b=1$, $c=-k$ and $a'=1$, $b'=-10$, $c'=(2k-3)$

So, $\frac{(-k)(1)-1(2k-3)}{1(-10)-1(1)}=3$

$\Rightarrow\ \frac{-k-2k+3}{-10-1}=3\Rightarrow\frac{-3k+3}{-11}=3$

$\Rightarrow\ -3k+3=-33\Rightarrow\ k=12$

21. $(x+2)(6x+1)(2x+1)(3x+5)+2=0$

$\Rightarrow(6x^2+13x+2)(6x^2+13x+5)+2=0$

$\Rightarrow a(a+3)+2=0\qquad[a=6x^2+13x+2]$

$\Rightarrow a^2+3a+2=0$

$\Rightarrow(a+2)(a+1)=0$

$\Rightarrow a=-2$ or -1

So, $6x^2+13x+2=-2; 6x^2+13x+2=-1$

$\Rightarrow 6x^2+13x+4=0$ and $6x^2+13x+3=0$

$$x=\frac{-13\pm\sqrt{169-96}}{12} \text{ and } x=\frac{-13\pm\sqrt{169-72}}{12}$$

22. $-3,-15,3,15$

23. **Hint:**

(i) Let the two digit number be $10x+y$

(ii) Given $xy=18$ and $10x+y+27=10y+x$ $\Rightarrow y-x=3$

(iii) Substitute $y=x+3$ in first equation and solve for x and y.

LONG ANSWER QUESTIONS :

1. Let the total number of girls who planned the picnic be $=x$

Total budget for eatables = ₹ 2400

$\therefore$ Original share of money for every girl = ₹ $\frac{2400}{x}$

The actual number of girls who attend the picnic $=(x-10)$

$\therefore$ New share of money for the girls attending the picnic

$=$ ₹ $\frac{2400}{x-10}$

The difference between two shares of money = ₹ 8

$$\therefore \frac{2400}{x-10}-\frac{2400}{x}=8 \Rightarrow \frac{2400x-2400(x-10)}{x(x-10)}=8$$

$\Rightarrow 24000=8x(x-10) \Rightarrow 3000=x^2-10x$

$\Rightarrow x^2-10x-3000=0 \Rightarrow x^2-60x-50x-3000=0$

$\Rightarrow x(x-60)+50(x-60)=0$

$\Rightarrow (x-60)(x+50)=0 \quad \therefore x=60,-50$

$\therefore$ Number of girls = 60

($\because$ the number of girls cannot be negative)

2. Let the usual speed of the plane by x km/hour

Increased speed $=(x+400)$ km/h

$$\text{Speed}=\frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time}=\frac{\text{Distance}}{\text{speed}}$$

Time taken by the plane to cover 1600 km. with usual speed $=\frac{1600}{x}$ hours.

Again time taken by the plane to cover 1600 km. with increased speed $=\frac{1600}{x+400}$ hours.

According to given information, we get,

$$\frac{1600}{x}-\frac{1600}{x+400}=\frac{40}{60} \Rightarrow 1600\left[\frac{x+400-x}{x(x+400)}\right]=\frac{2}{3}$$

$\Rightarrow x^2+400x-960000=0$

$\Rightarrow (x+1200)(x-800)=0$

$\therefore x=800$ ($x=-1200$ not permissible)

Hence, usual speed of plane = 800 km/hr

3. Let the number of students $=x$ and cost of food per student $=$ ₹ y

Given that the total budget = ₹ 240

$\therefore xy=240 \Rightarrow y=\frac{240}{x}$

When 4 students did not go, the cost of food per member increased by ₹ 10

$$\therefore (x-4)(y+10)=240 \Rightarrow (x-4)\left(\frac{240}{x}+10\right)=240$$

$$\Rightarrow 240+10x-\frac{960}{x}-40=240$$

$\Rightarrow 10x^2-40x-960=0 \Rightarrow x^2-4x-96=0$

$\Rightarrow (x-12)(x+8)=0 \Rightarrow x=12$ or $x=-8$

Since, x cannot be negative, $x=12$

$\therefore$ number of students = 12

Therefore, the number of students who went for the picnic $=12-4=8$

4. Let the time taken by B to do the work $=x$ days.

Given that A takes 12 days less than B to do the work.

$\therefore$ time taken by A to do the work $=(x-12)$ days

Given also that the time taken by A and B = 8 days.

$\therefore$ work done by A in 1 day $=\frac{1}{x-12}$

work done by B in 1 day $=1/x$

and work done by A and B in 1 day $=1/8$

$$\therefore \frac{1}{x}+\frac{1}{x-12}=\frac{1}{8} \Rightarrow \frac{x-12+x}{x(x-12)}=\frac{1}{8} \Rightarrow x^2-28x+96=0$$

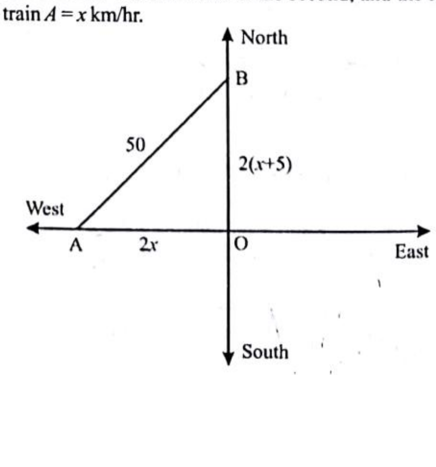
$\Rightarrow (x-24)(x-4)=0 \Rightarrow x=24$ or $x=4$

Since time taken by $A=(x-12)$ days, $x>12$

$\therefore x=24$

$\therefore$ time taken by B = 24 days.

5. Let A be the first train and B the second, and the speed of train $A=x$ km/hr.



$\therefore$ speed of train $B = (x+5)$ km/hr.
Distance covered by A in 2 hours $= OA = (2x)$ km
Distance covered by B in 2 hours $= OB = 2(x+5)$ km
Given that distance $AB = 50$ km.
$\therefore$ By pythagoras's theorm, $[2(x+5)]^2 + (2x)^2 = 50^2$
$\Rightarrow 4(x^2+25+10x)+4x^2 = 2500$
$\Rightarrow 8x^2+40x-2400 = 0 \Rightarrow x^2+5x-300 = 0$
$\Rightarrow (x+20)(x-15) = 0 \Rightarrow x = -20$ or $x = 15$
Since, x cannot be negative, $x = 15$
$\therefore$ Speed of the 1st train A = 15 km/hr
and speed of the 2nd train $= (15+5) = 20$ km/hr

6. Here, $\alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

If roots are $\left(\alpha+\frac{1}{\beta}, \beta+\frac{1}{\alpha}\right)$, then sum of roots are

$$=\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right) = (\alpha+\beta)+\frac{\alpha+\beta}{\alpha\beta} = -\frac{b}{ac}(a+c)$$

and product $= \left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$$= \alpha\beta+1+1+\frac{1}{\alpha\beta} = 2+\frac{c}{a}+\frac{a}{c}$$

$$= \frac{2ac+c^2+a^2}{ac} = \frac{(a+c)^2}{ac}$$

Hence, required equation is given by

$$x^2+\frac{b}{ac}(a+c)x+\frac{(a+c)^2}{ac} = 0$$

$$\Rightarrow acx^2+(a+c)bx+(a+c)^2 = 0$$

7. We have $\frac{p+q-x}{r}+\frac{q+r-x}{p}+\frac{r+p-x}{q} = \frac{-4x}{p+q+r}$

$$\frac{p+q+r-x}{r}+\frac{p+q+r-x}{p}+\frac{p+q+r-x}{q} = 4-\frac{4x}{p+q+r}$$

$$\Rightarrow (p+q+r-x)\left[\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right] = 4\left(\frac{p+q+r-x}{p+q+r}\right)$$

$$\Rightarrow (p+q+r-x)\left[\frac{1}{p}+\frac{1}{q}+\frac{1}{r}-\frac{4}{p+q+r}\right] = 0$$

$$\Rightarrow x = p+q+r$$

8. Since, α and β are roots of the equation

$$A(x^2+m^2)+Amx+cm^2x^2 = 0$$

or $(A+cm^2)x^2+Amx+Am^2 = 0$ ....(1)

$$\therefore \alpha+\beta = -\frac{Am}{A+cm^2} \text{ and } \alpha\beta = \frac{Am^2}{A+cm^2}$$

Now, $A(\alpha^2+\beta^2)+A\alpha\beta+c\alpha^2\beta^2$

$$= A[(\alpha+\beta)^2-2\alpha\beta]+A\alpha\beta+c\alpha^2\beta^2$$

$$= A\left[\frac{A^2m^2}{(A+cm^2)^2}-\frac{2Am^2}{A+cm^2}\right]+\frac{A^2m^2}{A+cm^2}+\frac{cA^2m^4}{(A+cm^2)^2}$$

$$= \frac{A^3m^2-2A^2m^2(A+cm^2)+A^2m^2(A+cm^2)+cA^2m^2}{(A+cm^2)^2}$$

$$= \frac{0}{(A+cm^2)^2} = 0$$

9. The equation is defined for $x-2 \geq 0$, $4-x \geq 0$ and $6-x \geq 0$
$\Rightarrow x \geq 2, x \leq 4$ and $x \leq 6$
$\therefore 2 \leq x \leq 4$
Now, the given equation is $\sqrt{x-2}+\sqrt{4-x} = \sqrt{6-x}$
Squaring both sides, we obtain

$$x-2+4-x+2\sqrt{(x-2)(4-x)} = 6-x$$

$$\Rightarrow 2\sqrt{(x-2)(4-x)} = (4-x)$$

Again, squaring both sides, we get,

$$4(-x^{2?}+8x-12) = 16+x^2-8x$$

$$\Rightarrow 5x^2-40x+64 = 0 \Rightarrow x = 4+\frac{4}{\sqrt{5}}, 4-\frac{4}{\sqrt{5}}$$

But $2 \leq x \leq 4$

$\therefore$ Solution of the original equation is $x = 4-\frac{4}{\sqrt{5}}$

10. (i) Use, $(x-a)(x-b) > 0 \Rightarrow x < a$ or $x > b$ if $(a < b)$ and
(ii) Find the solutions of given inequations individually and then take their common solution.

11. $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

$$\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta} = \frac{-q}{p} \text{ and } \frac{\alpha}{1-\alpha}\times\frac{\beta}{1-\beta} = \frac{r}{p}$$

$$\Rightarrow \frac{\alpha(1-\beta)+\beta(1-\alpha)}{(1-\alpha)(1-\beta)} = \frac{-q}{p} \text{ and } \frac{\alpha\beta}{1-\alpha-\beta+\alpha\beta} = \frac{r}{p}$$

$$\Rightarrow \frac{\alpha+\beta-2\alpha\beta}{1-\alpha-\beta+\alpha\beta} = \frac{-q}{p} \text{ and } \frac{\alpha\beta}{1-(\alpha+\beta)+\alpha\beta} = \frac{r}{p}$$

$$\Rightarrow \frac{\frac{-b}{a}-\frac{2c}{a}}{1-\left(\frac{-b}{a}\right)+\frac{c}{a}} = \frac{-q}{p} \text{ and } \frac{\frac{c}{a}}{1-\left(\frac{-b}{a}\right)+\frac{c}{a}} = \frac{r}{p}$$

$\Rightarrow \quad \frac{b+2c}{a+b+c}=\frac{q}{p}$ and $\frac{c}{a+b+c}=\frac{r}{p}$

$\Rightarrow \quad \frac{p(b+2c)}{q}=a+b+c=\frac{pc}{r}$

$\Rightarrow \quad \frac{p(b+2c)}{q}=\frac{pc}{r} \Rightarrow r(b+2c)=qc$

12. Let the side of the square = a units

New length = $\left(a+\frac{ax}{100}\right)$, new breadth = $\left(a-\frac{ax}{100}\right)$

Area of rectangle = $\left(a+\frac{ax}{100}\right)\left(a-\frac{ax}{100}\right)=a^2-\frac{x^2a^2}{10000}$

Area of square = a^2. Given condition is

$a^2-\left(a^2-\frac{x^2a^2}{10000}\right)=a^2\times\frac{4}{100}$

$\Rightarrow \quad a^2-\left(\frac{10000a^2-x^2a^2}{10000}\right)=\frac{4a^2}{100}$

$\Rightarrow \quad \frac{10000a^2-10000a^2+x^2a^2}{10000}=\frac{4a^2}{100}$

$\Rightarrow \quad \frac{x^2a^2}{10000}=\frac{4a^2}{100}\Rightarrow\frac{x^2}{100}=\frac{4}{10}$

$\Rightarrow \quad x^2=400\Rightarrow x=20$

13. Let x km per hour be the rate at which he rides when there is no wind.

When there is wind, rate at which he rides on the outward journey = $(x+2)$ km per hour

Rate at which he rides in the return journey = $(x-2)$ km/hr

Time taken for the outward journey = $\frac{24}{x+2}$ hours

Time taken for the return journey = $\frac{24}{x-2}$ hours

$\therefore \frac{24}{x-2}-\frac{24}{x+2}=1$

Multiplying both sides by $(x+2)(x-2)$

$24(x+2)-24(x-2)=(x+2)(x-2)$

i.e., $96=x^2-4\Rightarrow x^2=100$

$\therefore x=10$

$\therefore$ the boy rides at the rate of 10 km per hour when there is no wind.

14. 24 days

Hint : Let A complete the work in x days, then will B complete it in $(x-16)$ days. According to given condition,

$\frac{1}{x}+\frac{1}{x-16}=\frac{1}{15}$

15. $6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0$

$\Rightarrow 6\left[\left(x-\frac{1}{x}\right)^2+2\right]-25\left(x-\frac{1}{x}\right)+12=0$

Substituting $\left(x-\frac{1}{x}\right)=y$, we get,

$6(y^2+2)-25(y)+12=0\Rightarrow 6y^2-25y+24=0$

$\Rightarrow 6y^2-16y-9y+24=0$

$\Rightarrow (3y-8)(2y-3)=0\Rightarrow y=\frac{8}{3},\frac{3}{2}\Rightarrow\left(x-\frac{1}{x}\right)=\frac{8}{3}$

$\Rightarrow 3x^2-8x-3=0\Rightarrow(x-3)(3x+1)=0$

$\Rightarrow x=3,-\frac{1}{3}$ or $x-\frac{1}{x}=\frac{3}{2}$, $\Rightarrow\frac{x^2-1}{x}=\frac{3}{2}$

$\Rightarrow 2x^2-2=3x\Rightarrow 2x^2-3x+2=0\Rightarrow(x-2)(2x+1)=0$

$\Rightarrow x=2,-\frac{1}{2}$. So roots of the equation are, $2,-\frac{1}{2},3,-\frac{1}{3}$.

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (d) Let α,β be the roots of the equation. Then $\alpha+\beta=5$ and $\alpha\beta=-6$. So, the equation is $x^2-5x-6=0$. The roots of the equation are 6 and -1.

2. (b) $\therefore\ \alpha+2\alpha=-\frac{b}{a}$ and $\alpha\times2\alpha=\frac{c}{a}$

$\Rightarrow\ 3\alpha=-\frac{b}{a}\ \Rightarrow\ \alpha=-\frac{b}{3a}$

and $2\alpha^2=\frac{c}{a}\ \Rightarrow\ 2\left(\frac{-b}{3a}\right)^2=\frac{c}{a}$

$\Rightarrow\ \frac{2b^2}{9a^2}=\frac{c}{a}\ \Rightarrow\ 2b^2=9ac$

Hence, the required condition is $2b^2=9ac$

3. (d) $\alpha+\beta=-\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$ and $\alpha^2+\beta^2=\frac{(b^2-2ac)}{a^2}$

Now $\frac{\alpha}{\alpha\beta+b}+\frac{\beta}{a\alpha+b}=\frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$=\frac{\alpha(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+b(\alpha+\beta)+b^2}=\frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$

$=\frac{b^2-ac-b^2}{a^2c-ab^2+ab^2}=\frac{-2ac}{a^2c}=-\frac{2}{a}$

4. (c) Use options or apply the formula $a^2 - b^2 = (a-b)(a+b), x=4$

5. (a) Given equation $2x^2 - 3x^2 + 6x + 1 = 0$,

$$\alpha+\beta+\gamma = \frac{3}{2},\ \alpha\beta\gamma = \frac{-1}{2},\ \Sigma\alpha\beta = 3$$

$$(\alpha^2+\beta^2+\gamma^2) = (\alpha+\beta+\gamma)^2 - 2(\Sigma\alpha\beta)$$

$$= \left(\frac{3}{2}\right)^2 - 2.3 = \frac{9}{4} - 6 = \frac{-15}{4}$$

6. (d) Since the given equation have two roots in common so from the condition

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{1} = \frac{1}{1/2} = \frac{k}{-1} \qquad \therefore k = -2$$

7. (c) $x^2 + y^2 = 25,\ xy = 12$

$$\Rightarrow x^2 + \left(\frac{12}{x}\right)^2 = 25 \Rightarrow x^4 + 144 - 25x^2 = 0$$

$\Rightarrow (x^2-16)(x^2-9) \Rightarrow x^2 = 16$ and $x^2 = 9$

$\Rightarrow x = \pm 4$ and $x = \pm 3$

8 (a) We have, $x = \sqrt{7+4\sqrt{3}}$

$$\therefore \frac{1}{x} = \frac{1}{\sqrt{7+4\sqrt{3}}} = \frac{\sqrt{7-4\sqrt{3}}}{\sqrt{7+4\sqrt{3}}.\sqrt{7-4\sqrt{3}}} = \sqrt{7-4\sqrt{3}}$$

$$\therefore x + \frac{1}{x} = \sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}}$$

$$= (\sqrt{3}+2) + (2-\sqrt{3}) = 4$$

9. (b) Let roots are α and $-\alpha$, then sum of the roots

$$\alpha + (-\alpha) = \frac{3(\lambda-2)}{2} \Rightarrow 0 = \frac{3}{2}(\lambda-2) \Rightarrow \lambda = 2$$

10. (b) Equation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ have real roots then from first

$4q^2 - 4pr \geq 0 \Rightarrow q^2 \geq pr$ . ... (1)

and from second $4(pr) - 4q^2 \geq 0$ (for real root)

$\Rightarrow pr \geq q^2$ .... (2)

From (1) and (2), we get result $q^2 = pr$

11. (b) Roots will be equal in magntidue but opposite in sign if coefficient of $x = 0$

But the equation is $x^2 + 2mx + m^2 - ab = 0$

Hence the result.

12. (d) The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$.

$a = 2, b = 2(p+1)$ and $c = p$

$[2(p+1)]^2 - 4(2p) \Rightarrow 4(p+1)^2 - 8p$

$\Rightarrow 4[(p+1)^2 - 2p] \Rightarrow 4[(p^2+2p+1) - 2p]$

$\Rightarrow 4(p^2+1)$

For any real value of p, $4(p^2+1)$ will always be positive as p^2 cannot be negative for real p.

Hence, the discriminant $b^2 - 4ac$ will always be positive.

When the discriminant is greater than '0' or is positive, then the roots of a quadratic equation will be real.

13. (d) Let 1, 2 be the roots of equation (1) and 2, 4 be the roots of equation (2).

$\therefore$ equations are $x^2 - 3x + 2 = 0$ and $x^2 - 6x + 8 = 0$.

Comparing with $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$,

we get $b = -3, c = 2, q = -6$ and $r = 8$.

Putting these values in the options, we find that option (D) is satisfied.

14. (a) We have, $D = (c+a-b)^2 - 4(b+c-a)(a+b-c)$

$= (a+b+c-2b)^2 - 4(a+b+c-2a)(a+b+c-2c)$

$= (-2b)^2 - 4(-2a)(-2c) = 4(b^2 - 4ac)$

$= 4[(-a-c)^2 - 4ac] = 4(a-c)^2$

$= \{2(a-c)\}^2 =$ perfect square

15. (b) Let the roots be α and β

Then $\alpha+\beta = \frac{-n}{\ell},\ \alpha\beta = \frac{n}{\ell}$ and $\frac{\alpha}{\beta} = \frac{p}{q}$

Now, $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{\ell}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$

$$= \frac{\alpha+\beta+\alpha\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{n}{\ell} + \frac{n}{\ell}}{\sqrt{\frac{n}{\ell}}} = 0$$

16. (b) We have $\alpha+\beta = 2p,\ \alpha\beta = q, \gamma+\delta = 2r$ and $\gamma\delta = s$

$\because \alpha, \beta, \gamma, \delta$ are in A.P.

$\therefore \beta - \alpha = \delta - \gamma \Rightarrow (\beta-\alpha)^2 = (\delta-\gamma)^2$

$\Rightarrow (\beta+\alpha)^2 - 4\beta\alpha = (\delta+\gamma)^2 - 4\delta\gamma$

$\Rightarrow 4p^2 - 4q = 4r^2 - 4s$ or $s - q = r^2 - p^2$

17. (d) The given equation is $x^{2/3} + x^{1/3} - 2 = 0$

Put $x^{1/3} = y$, then $y^2 + y - 2 = 0$

$\Rightarrow (y-1)(y+2) = 0$

$\Rightarrow y = 1$ or $y = -2$

$\Rightarrow x^{1/3} = 1$ or $x^{1/3} = -2$

$\therefore x = (1)^3$ or $x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, –8.

18. (d) Consider $\frac{8x^2+16x-51}{(2x-3)(x+4)} - 3 > 0$

$$\Rightarrow \frac{2x^2+x-15}{2x^2+5x-12} > 0 \Rightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} > 0$$

Hence both Nr and Dr are positive if $x < -4$ or $x > 5/2$ and both negative if $-3 < x < 3/2$

Hence all the statements are true.

19. (b) **Hint :** $x(x+1)+1=(x-2)(x-5)$
$\Rightarrow x^2+x+1=x^2-7x+10$
$\Rightarrow 8x-9=0$, which is not a quadratic equation.

20. (d) **Hint :** If one root is α, then the other is $\frac{1}{\alpha}$
$\therefore \alpha.\frac{1}{\alpha}=$ product of roots $=\frac{c}{a} \Rightarrow 1=\frac{c}{a} \Rightarrow a=c$

21. (b) **Hint :** Check that 3 and $\frac{1}{3}$ both satisfy the given equation.

22. (a) **Hint :** '$b^2>4ac$' $\Rightarrow (-6)^2>4.2.p \Rightarrow 36>8p$
$\Rightarrow 8p<36 \Rightarrow p<\frac{9}{2}$.

23. (a) **Hint :** 'b^2-4ac' $=\left(-2\sqrt{3}\right)^2-(4).(1).(5)=12-20=-8<0$.

24. (b) **Hint :** '$b^2=4ac$'
$\Rightarrow 4(mp+nq)^2=4(m^2+n^2)(p^2+q^2)$
$\Rightarrow m^2q^2+n^2p^2-2mnpq=0$
$\Rightarrow (mq-np)^2=0 \Rightarrow mq-np=0$.

25. (b) **Hint :** $[(-1)^2+a(-1)+b=0; (-1)^2+c(-1)-d=0$
$\Rightarrow -a+b=-c-d$ etc.

26. (d) $\alpha.\ \alpha^n=q \Rightarrow \alpha^{n+1}=q \Rightarrow \alpha=\frac{1}{q^{n+1}}$
$\alpha+\alpha^n=p \Rightarrow \frac{1}{q^{n+1}}+\frac{n}{q^{n+1}}=p$

27. (a) **Hint :** $\alpha+\beta=b, \alpha\beta=c$
$\Rightarrow (\alpha+\beta-4)=b-4$;
$(\alpha-2)(\beta-2)=\alpha\beta-2(\alpha+\beta)+4$
$=c-2b+4$
Now, $2=b-4$; $1=c-2b+4$ etc.]

28. (d)

MORE THAN ONE CORRECT :

1. (c, d)
Let $\alpha+3=x$
$\therefore \alpha=x-3$ (replace x by $x-3$)
So the required equation
$(x-3)^2-5(x-3)+6=0$
$\Rightarrow x^2-6x+9-5x+15+6=0$
$\Rightarrow x^2-11x+30=0$
$(x^2-11x+30)\times 2=0$
$2x^2-22x+60=0$

2. (b, d) 3. (a, b)

4. (b, d)
Let the roots be α and β.
$\alpha+\beta=8, |\alpha-\beta|=10$
$(\alpha-\beta)^2=100$
$(\alpha+\beta)^2-4\alpha\beta=100$
$\alpha\beta=-9$
$\therefore x^2-8x-9=0, \Rightarrow (x^2-8x-9)=0$
or $-(-x^2+8x+9)=0$

5. (b,d) $x^2-3x+2=0$
$x^2-2x-x+2=0$
$x(x-2)-1(x-2)=0$
$(x-1)(x-2)=0$
$x=1, x=2$

6. (a, b)
7. (a, d) Given equality is satisfied if $b>9$ or $b<0$.
8. (a, b)
9. (c, d) $\alpha.\beta=\left(\frac{\text{Constant term}}{\text{Coefficient of } x^2}\right)$
10. (c, d)
11. (a,b) Let α be the common root
Then $3\alpha^2-2m\alpha-4=0$ and $\alpha^2-4m\alpha+2=0$
By cross-multiplication, we get
$$\frac{\alpha^2}{-4m-16m}=\frac{\alpha}{-4-6}=\frac{1}{-12m+2m}$$
$$\Rightarrow \frac{\alpha^2}{-20m}=\frac{\alpha}{-10}=\frac{1}{-10m} \Rightarrow \frac{\alpha^2}{2m}=\frac{\alpha}{1}=\frac{1}{m}$$
or $2m^2=1$ $\therefore m=\pm\frac{1}{\sqrt{2}}$

PASSAGE BASED QUESTIONS :

Passage-I

1. (c) $\alpha+\beta=-3a$
$\alpha\beta=2a^2$
$\alpha^2+\beta^2=5$
$(\alpha+\beta)^2-2\alpha\beta=5$
$9a^2-2(2a^2)=5$
$5a^2=5$
$a=\pm 1$
2. (a) $(3a)^2-4(2a^2)=9a^2-8a^2=a^2=1>0$
3. (a) $\alpha\beta=2a^2=2(1)=2$

Passage-II

The given equation is
$x^2+\lambda x+\lambda+1.25=0$
$a=1, b=\lambda, c=\lambda+1.25$
$b^2-4ac=\lambda^2-4\times 1.(\lambda+1.25)$
$=\lambda^2-4\lambda-5=(\lambda-5)(\lambda+1)$

1. (c) The equation has two distinct roots if
$b^2-4ac>0$
$\therefore (\lambda-5)(\lambda+1)>0$
$\Rightarrow$ Either $\lambda-5>0$ and $\lambda+1>0$
$\Rightarrow \lambda>5$ and $\lambda>-1$
$\Rightarrow \lambda>5$
$\Rightarrow \lambda-5<0$ and $\lambda+1<0$
$\lambda<5$ and $\lambda<-1$
$\Rightarrow \lambda<-1$
Hence the given equation has two distinct roots for $\lambda>5$ or $\lambda<-1$

2. (a) The equation has two coincident roots if

$b^2-4ac=0,$

$(\lambda-5)(\lambda+1)=0$

$\Rightarrow$ Either $\lambda-5=0,\ \lambda=5$

$\Rightarrow\ \lambda+1=0\Rightarrow\lambda=-1$

$\therefore\ \lambda=5$ or -1

Hence the given equation has coincident roots for $\lambda=5$ or -1.

ASSERTION & REASON :

1. (d) **Assertion :** Let $f(x)=(x-p)(x-r)+\lambda(x-q)(x-s)$

$f(p)=\lambda(p-q)(p-s),\ f(q)=(q-p)(q-r),f(s)$
$=(s-p)(s-r),f(r)=\lambda(r-q)(r-s)$

If $\lambda>0$ then $f(p)>0,f(q)<0,f(r)<0$ and $f(s)>0$

$\Rightarrow f(x)=0$ has one real root between p and q and other real root between r and s.

$\Rightarrow$ Statement-2 is obviously true.

2. (b) **Assertion :** Given equation $x^2-bx+c=0$

Let α,β be two roots such that $|\alpha-\beta|=1$

$\Rightarrow(\alpha+\beta)^2-4\alpha\beta=1$

$\Rightarrow b^2-4c=1$

Reason : Given equation :

$4abc\,x^2+(b^2-4ac)x-b=0$

$D=(b^2-4ac)^2+16ab^2c$

$D=(b^2+4ac)^2>0$

Hence roots are real and unequal.

3. (c) If $1\le a\le2\ \Rightarrow\ 0\le a-1\le1$

$\Rightarrow\ \sqrt{a+2\sqrt{a-1}}+\sqrt{a-2\sqrt{a-1}}$

$=\sqrt{1}+\sqrt{a-1}+\sqrt{1}+\sqrt{a-1}=2$

Statement 1 is true. Statement 2 is false.

4. (a) $x=\sqrt3-\sqrt2\,;\quad x^2=5-2\sqrt6\,;\quad (x^2-5)^2=24$

$x^4-10x^2+25=24\Rightarrow x^4-10x^2+1=0$

For polynomial equation with rational coefficients, irrational roots occurs in pairs.

5. (a) Assertion and Reason, both are correct. Reason is the correct explanation for Assertion.

6. (b)

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ r, u; (B) $\to$ p, s; (C) $\to$ q ; (D) $\to$ t
2. (A) $\to$ q, s ; (B) $\to$ p, s ; (C) $\to$ q, r ; (D) $\to$ p, t

HOTS SUBJECTIVE QUESTIONS :

1. $12x^4-56x^3+89x^2-56x+12=0$

Dividing both sides by x^2, we get $12x^2-56x+89-\dfrac{56}{x}+\dfrac{12}{x^2}=0$

$$\Rightarrow 12\left(x^2+\frac{1}{x^2}\right)-56\left(x+\frac{1}{x}\right)+89=0$$

$$\Rightarrow 12\left[\left(x+\frac{1}{x}\right)^2-2\right]-56\left(x+\frac{1}{x}\right)+89=0$$

$$\Rightarrow 12\left(x+\frac{1}{x}\right)^2-56\left(x+\frac{1}{x}\right)+65=0\Rightarrow 12y^2-56y+65=0,$$

where $y=x+\dfrac{1}{x}$

$\Rightarrow 12y^2-26y-30y+65=0\Rightarrow(6y-13)(2y-5)=0$

$\Rightarrow y=\dfrac{13}{6}$ or $y=\dfrac{5}{2}$

If $y=\dfrac{13}{6}$, then $x+\dfrac{1}{x}=\dfrac{13}{6}$

$\Rightarrow 6x^2-13x+6=0\Rightarrow(3x-2)(2x-3)=0\Rightarrow x=\dfrac{2}{3},\dfrac{3}{2}$

If $y=\dfrac{5}{2}$, then $x+\dfrac{1}{x}=\dfrac{5}{2}\Rightarrow 2x^2-5x+2=0$

$\Rightarrow(x-2)(2x-1)=0\Rightarrow x=2,\dfrac{1}{2}$

Hence, the roots of the given equation are $2,\dfrac{1}{2},\dfrac{2}{3},\dfrac{3}{2}$.

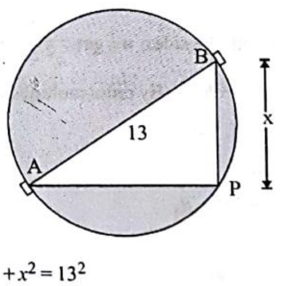
2. We first draw the diagram, as below.

Let P be the required location of the pole. Let the distance of the pole from the gate B be x m, i.e., BP = x m. Now the difference of the distances of the pole from the two gates = AP – BP (or, BP – AP) = 7 m. Therefore, AP = $(x+7)$ m.

Now, AB = 13m, and since AB is a diameter,

$\angle APB=90°$

Therefore, $AP^2+PB^2=AB^2$ (By Pythagoras theorem)



i.e., $(x+7)^2+x^2=13^2$

i.e., $x^2+14x+49+x^2=169$

i.e., $2x^2+14x-120=0$

So, the distance 'x' of the pole from gate B satisfies the equation

$x^2+7x-60=0$

Discriminant, $D = 7^2 + 4 \times 60 = 289 > 0$

So, the given quadratic equation has two real roots, and it is possible to erect the pole on the boundary of the park.

Solving the quadratic equation $x^2 + 7x - 60 = 0$, by the quadratic formula, we get $x = \frac{-7 \pm \sqrt{289}}{2} = \frac{-7 \pm 17}{2}$

Therefore, $x = 5$ or -12.

Since x is the distance between the pole and the gate B, it must be positive.

Therefore, $x = -12$ is ignored. So, $x = 5$. So, BP = 5 and AP = 12.

Taking $BP - AP = 7$ or $AP = x - 7$

we get $x = 12, -5$ and take $x = 12 \Rightarrow AP = 12$ i.e. $AP = 12$ & $BP = 5$

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12m from the gate A or 12 m from gate B and 5 m from gate A.

3. Consider the equation $x^2 - 2ax + b = 0$ .....(i)

Roots of this equation are $a + \sqrt{a^2 - b}$ and $a - \sqrt{a^2 - b}$

Similarly, roots of equation $x^2 - 2cx + d = 0$

are, $c + \sqrt{c^2 - d}$ and $c - \sqrt{c^2 - d}$ ....(ii)

Now, ratio of roots of first equation = ratio of roots of second equation.

$$\Rightarrow \frac{a + \sqrt{a^2 - b}}{a - \sqrt{a^2 - b}} = \frac{c + \sqrt{c^2 - d}}{c - \sqrt{c^2 - d}}$$

Using componendo and dividendo, we get

$$\frac{a + \sqrt{a^2 - b} + a - \sqrt{a^2 - b}}{a + \sqrt{a^2 - b} - a + \sqrt{a^2 - b}} = \frac{c + \sqrt{c^2 - d} + c - \sqrt{c^2 - d}}{c + \sqrt{c^2 - d} - c + \sqrt{c^2 - d}}$$

$$\Rightarrow \frac{2a}{2\sqrt{a^2 - b}} = \frac{2c}{2\sqrt{c^2 - d}} \Rightarrow \frac{a}{\sqrt{a^2 - b}} = \frac{c}{\sqrt{c^2 - d}}$$

Squaring both the sides, we get $\frac{a^2}{a^2 - b} = \frac{c^2}{c^2 - d}$

$\Rightarrow \quad a^2 d = c^2 b$. (By cross multiplication)

$$\Rightarrow \frac{a^2}{c^2} = \frac{b}{d}.$$

4. Cost of scenery $= R_1$

$$R_2 = R_1\left(1 - \frac{x}{100}\right)$$

A further discount of x% on R_2 reduces it by ₹ 415.

$$\Rightarrow \frac{x}{100} \cdot R_2 = 415 \Rightarrow \frac{x}{100} \cdot R_1\left(1 - \frac{x}{100}\right) = 415 \ldots\text{(i)}$$

Further $R_3 = R_1\left(1 - \frac{x}{100}\right)\left(1 - \frac{x}{100}\right) = R_1\left(1 - \frac{x}{100}\right)^2$

Again $R_4 = 3362.8 = R_1\left(1 - \frac{x}{100}\right)^2\left(1 - \frac{x}{100}\right)$

$$\Rightarrow 3362.8 = R_1\left(1 - \frac{x}{100}\right)^3 \quad \ldots\text{(ii)}$$

Dividing (ii) by (i),

$$\frac{3362.8}{415} = \frac{R_1\left(1 - \frac{x}{100}\right)^3}{\frac{xR_1}{100}\left(1 - \frac{x}{100}\right)} \Rightarrow 8.1 = \frac{\left(1 - \frac{x}{100}\right)^2}{\frac{x}{100}}$$

$$\Rightarrow 8.1 = \frac{y^2}{1 - y} \qquad \left[\text{using, } y = 1 - \frac{x}{100}\right]$$

$\Rightarrow y^2 + 8.1y - 8.1 = 0$

$$\Rightarrow y = \frac{-8.1 \pm \sqrt{8.1^2 - 4 \times 1 \times (-8.1)}}{2} = \frac{-8.1 \pm 9.9}{2} = 0.9$$

or -9

$$\Rightarrow 1 - \frac{x}{100} = 0.9 \text{ or } 1 - \frac{x}{100} = -9$$

$\Rightarrow x = 0.1 \times 100 = 10$ or $x = 10 \times 100 = 1000$

But $x = 1000$ is not possible.

using, $x = 10$ in (i), we get $\frac{10}{100} R_1\left(1 - \frac{10}{100}\right) = 415$

$$\Rightarrow R_1 = \frac{415 \times 10 \times 10}{9} = ₹\ 4611.1 \simeq ₹\,4611$$

5. Let the original price of each item = ₹ x.

∴ The number of items bought by the businessman

$$= \frac{600}{x}.$$

He sells $\left(\frac{600}{x} - 10\right)$ items at the rate of ₹ $(x + 5)$ per item.

∴ The total amount received by him in this deal

$$= \left(\frac{600}{x} - 10\right)(x + 5)$$

Now, the amount required to buy 15 more item, i.e.,

$\left(\frac{600}{x} + 15\right)$ items at the rate of ₹ x per item

$$= x\left(\frac{600}{x} + 15\right)$$

Then according to the question

$$x\left(\frac{600}{x}+15\right)=\left(\frac{600}{x}-10\right)(x+5)$$

$\Rightarrow \quad 600+15x=600-10x+\frac{3000}{x}-50$

$\Rightarrow \quad 600+15x-600+10x-\frac{3000}{x}+50=0$

$\Rightarrow \quad 25x-\frac{3000}{x}+50=0$

$\Rightarrow \quad 25x^2+50x-3000=0$

$\Rightarrow \quad x^2+2x-120=0$

$\Rightarrow \quad (x+12)(x-10)=0$

$\Rightarrow \quad x+12=0$ or $x-10=0$

$\therefore \quad x=-12$ or $x=10$.

But the price of an item cannot be –ve.

Hence the original price of each item is ₹ 10.

6. Let the time taken by the second pipe alone to fill the pool be x hrs.

As the second pipe fills the pool 5 hours faster than the first pipe so time taken by the first pipe to fill the pool $=(x+5)$hours.

Again as the second pipe fills the pool four hours slower than the third pipe, so time taken by the third pipe to fill the pool $x=(x-4)$ hours.

$\therefore$ In 1 hour first pipe can fill $\frac{1}{x+5}$ of the pool

In 1 hour second pipe can fill $\frac{1}{x}$ of the pool

In 1 hour third pipe can fill $\frac{1}{x-4}$ of the pool

Since the time taken by first two pipes together to fill the pool is the same as that taken by the third pipe alone

$\therefore \quad \frac{1}{x+5}+\frac{1}{x}=\frac{1}{x-4}$

$\Rightarrow \quad \frac{x+x+5}{x(x+5)}=\frac{1}{x-4}$

$\Rightarrow \quad (x+4)(2x+5)=x(x+5)$

$\Rightarrow \quad 2x^2+5x-8x-20=x^2+5$

$\Rightarrow \quad x^2-8x-20=0$

$\Rightarrow \quad x^2-10x+2x-20=0$

$\Rightarrow \quad x(x-10)+2(x-10)=0$

$\Rightarrow \quad (x-10)(x+2)=0$

$\Rightarrow \quad x=10,-2$

But $x \neq -2$ (time can not be – ve)

$\therefore \quad x=10$

Hence, time taken by first pipe $=x+5=15$ hrs.

Time taken by second pipe $=x=10$ hrs.

Time taken by third pipe $=x-4=6$ hrs.

7. Let the total number of birds be x.

$\therefore$ Number of birds moving about in lotus plants $=\frac{x}{4}$

Number of birds moving on a hill

$$=\frac{x}{9}+\frac{x}{4}+7\sqrt{x}$$

Number of birds in Vakula tree $=56$.

Using the given informations, we have

$$\frac{x}{4}+\left(\frac{x}{9}+\frac{x}{4}+7\sqrt{x}\right)+56=x$$

$\Rightarrow \quad x-\frac{x}{4}-\frac{x}{9}-\frac{x}{4}-7\sqrt{x}-56=0$

$\Rightarrow \quad \frac{36x-9x-4x-9x}{36}-7\sqrt{x}-56=0$

$\Rightarrow \quad \frac{14x}{36}-7\sqrt{x}-56=0$

$\Rightarrow \quad \frac{7x}{18}-7\sqrt{x}-56=0$

$\Rightarrow \quad \frac{x}{18}-\sqrt{x}-8=0$

$\Rightarrow \quad x-18\sqrt{x}-144=0$ .... (i)

Putting $\sqrt{x}=y$, we have

$y^2-18y-144=0$

$\Rightarrow \quad y^2-24y+6y-144=0$

$\Rightarrow \quad y(y-24)+6(y-24)=0$

$\Rightarrow \quad (y-24)(y+6)=0$

$\Rightarrow \quad y=24,-6$

But $y \neq -6$, since $\sqrt{x}=y$ is positive.

$\therefore \quad y=24 \Rightarrow \sqrt{x}=24 \therefore x=576$

Hence the total number of birds is 576.

8. $\frac{x^2+k^2}{k(6+x)} \geq 1$

$\Rightarrow \quad \frac{x^2-kx+k^2-6k}{k(6+x)} \geq 0$ ....(1)

Now the discriminant of the numerator is $24k-3k^2=3k(8-k)$ is negative for all $k<0$ and for all $k>8$. For these values of k, the numerator is positive.

(i) For $k<0$, inequality (1) is true only if $x<-6$.

But $x \in (-1, 1)$ ...(2)

Hence for $k < 0$, the inequality is not valid.

(ii) For $k > 8$, inequality (1) is true only if $x > -6$ ...(3) and $x \in (-1, 1)$ and hence the inequality is valid for all $k > 8$.

For k = 0, the inequality is indeterminate.

9. The numbers are 7 and 3

10. Given $\alpha^2 + \beta^2 - \alpha\beta = 3\frac{1}{4} = \frac{13}{4}$ ......(i)

If α and β are roots of the equation $x^2 + px + 1 = 0$

Sum of roots $\alpha + \beta = \frac{-b}{a} = -p$

Product of roots, $\alpha\beta = \frac{c}{a} = 1$

Substituting $\alpha\beta = 1$ in (1), we get $\alpha^2 + \beta^2 - 1 = \frac{13}{4}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{13}{4} + 1 = \frac{17}{4}$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = \frac{17}{4}$ i.e., $(\alpha^2+\beta^2) - 2 = \frac{17}{4}$

$[\because \alpha\beta = 1]$

or, $(\alpha+\beta) = \frac{17}{4} + 2 = \frac{25}{4}$

$\therefore \alpha + \beta = \frac{5}{2} = -p$

$\Rightarrow p = -\frac{5}{2}$

11. Dividing the equation by x^2. We get

$3x^2 - 20x - 94 - \frac{20}{x} + \frac{3}{x^2} = 0$

Grouping equidistant terms we have,

$3\left(x^2 + \frac{1}{x^2}\right) - 20\left(x + \frac{1}{x}\right) - 94 = 0$

Let $x + \frac{1}{x} = y$

Then $x^2 + \frac{1}{x^2} = y^2 - 2$

The equation becomes

$3(y^2 - 2) - 20y - 94 = 0$

$\Rightarrow 3y^2 - 20y - 100 = 0$

$\Rightarrow (3y + 10)(y - 10) = 0$

$\Rightarrow y = \frac{-10}{3}$ or 10

When $y = \frac{-10}{3}$, we have $x + \frac{1}{x} = \frac{-10}{3}$

$\Rightarrow 3x^2 + 10x + 3 = 0$

$\Rightarrow x = \frac{-10 \pm \sqrt{100-36}}{6} = \frac{-10 \pm 8}{6}$

$\Rightarrow x = -3$ or $-\frac{1}{3}$

When $y = 10$, we have $x + \frac{1}{x} = 10$

$\Rightarrow x^2 - 10x + 1 = 0$

$\Rightarrow x = \frac{10 \pm \sqrt{100-4}}{2} = \frac{10 \pm 2\sqrt{24}}{2}$

$\Rightarrow x = 5 \pm \sqrt{24}$

Hence the roots are -3, $\frac{-3}{3}$, $5+\sqrt{24}$, $5-\sqrt{24}$,

chapter 5

SEQUENCE AND SERIES (A.P, G.P.)

Introduction

In practical life you must have observed many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple on a pipe cone etc.

In our day-to-day life, we see patterns of geometric figures on clothes, pictures, posters etc. They make the learners motivated to form such new patterns.

Number patterns are faced by learners in their study. Number patterns play an important role in the field of mathematics. Let us study the following number patterns :

(i) $2, 4, 6, 8, 10, \ldots$ (ii) $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, \ldots\ldots$ (iii) $10, 7, 4, 1, -2, \ldots$ (iv) $2, 4, 8, 16, 32, \ldots$

(v) $4, \frac{1}{2}, \frac{1}{16}, \frac{1}{128}, \ldots\ldots$ (vi) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots$ (vii) $1, 11, 111, 1111, 11111, \ldots$

It is an interesting study to find whether some specific names have been given to some of the above number patterns and the methods of finding some next terms of the given patterns.

Observing various patterns various sequences were defined to solve various summation problems.

Among various sequences A.P.(Arithmetic progression),G.P.(Geometric progression) and H.P(Harmonic progression) are most common.

Idea on A.P. was given by mathematician Carl Friedrich Gauss, who, as a young boy, stunned his teacher by adding up

$1 + 2 + 3 + \ldots + 99 + 100$ within a few minutes. Here's how he did it:

He realised that adding the first and last numbers, 1 and 100, gives, 101; and adding the second and second last numbers, 2 and 99, gives 101, as well as $3 + 98 = 101$ and so on.

Thus he concluded that there are 50 sets of 101. So the sum of the series is:

$50 (1 + 100) = 5050.$

In this chapter, you will study only Arithmetic Progression (A.P.) and Geometric Progression (G.P.)

SEQUENCE :

The number patterns or arrangement of numbers according to definite rule or a set of rules is called a SEQUENCE.
The various numbers occuring in a sequence are called its terms. The n^{th} term of the sequence is denoted by x_n. The n^{th} term is also called the GENERAL TERM of the sequence. For example,

(i) The numbers $\langle 1, 4, 9, 16,....\rangle$ represent a sequence written according to the rule $x_n = n^2,\ n \in \mathbf{N}$.

(ii) The numbers $\left\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5},.........\right\rangle$ represent a sequence written according to the rule $x_n = \frac{n}{n+1},\ n \in \mathbf{N}$.

(iii) The numbers $\langle 1,3,5,7,.........\rangle$ represent a sequence written according to the rule $x_n = 2n-1,\ n \in \mathbf{N}$.

(iv) The numbers $\langle 1,3,7,13,21.........\rangle$ represent a sequence written according to the rule $x_n = n^2 - n + 1,\ n \in \mathbf{N}$.

(v) The numbers $\langle 1,1,2,3,5,8,.........\rangle$ represent a sequence written according to the following set of rules

$x_1 = x_2 = 1,\ x_n = x_{n-1} + x_{n-2},\ n > 2,\ n \in \mathbf{N}$.

This sequence of numbers is called the Fibonacci sequence.

(vi) The numbers $\langle 2,3,5,7,11,13,.........\rangle$ represent a sequence of prime numbers.

In every sequence it is not always possible to write a specific formula.

METHODS OF DESCRIBING A SEQUENCE :

(i) A sequence may be described by writing first few terms till the rule for writing down the other terms is evident.

(ii) A sequence may be described by giving a formula for its general term (the n^{th} term)

(iii) a sequence may be described by specifying first few terms and a formula (or a set of formulae) giving a relation between successive terms. Such a formula is called RECURSIVE FORMULA (or RECURRENCE RELATION).

(iv) Some sequences may not be described by any rule

SERIES :

If $\langle x_1, x_2, x_3,.........\rangle$ is a sequence, then the expression $x_1 + x_2 + x_3 + ...$ is called the series associated with the given sequence.

PROGRESSION :

A sequence is said to be a PROGRESSION if its terms numerically increase or numerically decrease continuously.
In this chapter, you will study two types of progressions (i) Arithmetic Progression (A.P.) and (ii) Geometric Progression (G.P.)

ARITHMETIC PROGRESSION (A.P.) :

The sequence $\langle x_1, x_2, x_3, ..., = x_n,.......\rangle$ is called an arithmetic progression (A.P.), if $x_2 - x_1 = x_3 - x_2 = ... = x_n - x_{n-1} = ...$

In general $x_{n+1} - x_n =$ Constant (say,d) $n \in \mathbf{N}$

The constant difference d is called the *common difference* of the A.P. First term x_1 of the A.P. is taken as 'a'. Then the standard form of A.P. is $\langle a, a+d, a+2d,.........\rangle$

Where x_1 = First term = a, Common difference = d, and n^{th} term = a_n (symbol)

Formula for General Term of an A.P. :

The n^{th} term of the A.P., written in standard form is given by

$a_n = a + (n-1)d,\ n \in N$

Formula for Sum of First n Terms of an A.P.:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + \ell)$$

Where ℓ = Last term up to which the sum of the A.P. is to find.

Important Characteristic of A.P. :

(i) $a_n = S_n - S_{n-1}$

(ii) If three terms to be selected in A.P., choose them $a-d,\ a,\ a+d$

(iii) If four terms to be selected in A.P., choose them $a-3d,\ a-d, a+d,\ a+3d$

(iv) Three numbers a, b, c are in A.P. if and only if $b-a=c-b$, i.e., if and only if $a+c=2b$

Arithmetic Mean (A.M.) of Two Terms *a* & *b* of an A.P.:

If a, A, b are in A.P. then A is called ARITHMETIC MEAN of numbers a and b, we get $A=\frac{a+b}{2}$

Inserting *n* Arithmetic Means Between Two Terms *a* and *b* :

Let $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ be n arithmetic means between two terms a and b then a, $A_1, A_2, \ldots\ldots A_n$, b will be in A.P.

Clearly $b = x_{n+2} = a+[(n+2)-1]d \Rightarrow d=\frac{b-a}{n+1}$

Thus the n arithmetic means between a and b are as follow :

$$A_1 = a+d = a+\frac{b-a}{n+1};\quad A_2 = a+2d = a+\frac{2(b-a)}{n+1};\ldots\ldots\quad A_n = a+nd = a+\frac{n(b-a)}{n+1}$$

GEOMETRIC PROGRESSION (G. P.) :

The sequence $\langle x_1, x_2, x_3, \ldots\ldots\ldots x_n \ldots\ldots \rangle$ is called a geometric progression (G.P.) if $\frac{x_2}{x_1}=\frac{x_3}{x_2}=\ldots\ldots=\frac{x_n}{x_{n-1}}=\ldots\ldots$, where none of $x_1, x_2, \ldots\ldots, x_n, \ldots\ldots$ is zero.

In general $\frac{x_{n+1}}{x_n}$ = constant (say, r), $n \in \mathbf{N}$.

The constant ratio r is called the *common ratio* of the G.P. If the first term x_1 of the G.P. be taken as a, then the standard form of G.P. is $\langle a, ar, ar^2, \ldots\ldots\rangle$

Formula for General Term of a G.P. :

The n^{th} term of the G.P. written in standard form is given by $a_n = ar^{n-1},\ n \in \mathbf{N}$

Formula for Sum of First n Terms of a G.P. :

The sum of first n terms of the geometric series,

$$S_n = \frac{a(r^n-1)}{r-1}, \text{ if } |r|>1 \text{ and } S_n = \frac{a(1-r^n)}{1-r}, \text{ if } |r|<1$$

NOTE: When r = 1, then

$S_n = a+a+a+\ldots$ upto n terms $= na$.

Formula for the Sum of Infinite Terms of a G.P. :

If $|r|<1$, the sum of infinite terms (S) of the G.P.,

$$S = \frac{a}{1-r}$$

Important characteristic of G.P. :

(i) If three terms to be selected in G.P., choose them $\frac{a}{r}, a, ar$.

(ii) If four terms to be selected in G.P. choose them $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

(iii) Three numbers a, b, c are in G.P. if and only if $\frac{b}{a} = \frac{c}{b}$ or if and only if $b^2 = a.c$

Geometric Mean (G. M.) of Two Terms a and b :

If a, G, b are in G.P. (a and b are positive), then G is the GEOMETRIC MEAN of numbers a and b.

We get $G = \sqrt{ab}$

Inserting n Geometric means between Two Terms a and b :

Let a and b be positive numbers. Let $G_1, G_2, ..., G_n$ be such that $a, G_1, G_2, ..., G_n, b$ is a G.P.

Then $b = x_{n+2} = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Thus the n geometric means between a and b are :

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}; G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}; ...., G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

RELATIONSHIP BETWEEN A.M. AND G.M. :

Let A and G be A.M. and G.M. of two given positive real numbers a and b respectively, then

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

Thus, we have

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0$$

$$\Rightarrow A - G \geq 0$$

Hence, $A \geq G$.

MISCELLANEOUS SOLVED EXAMPLES

1. If the first term and common difference in an A.P. are 8 and –1 respectively, then find:
(i) General term
(ii) The progression
(iii) The 10th term and
(iv) The expression for sum to n terms and hence sum to 10 terms.

Sol. Given the first term $\Rightarrow$ $a = 8$ and

The common difference $\Rightarrow$ $d = -1$

(i) The nth term of an A.P. $\Rightarrow$ $t_n = a + (n-1)d$

Substituting a and d in t_n

$\Rightarrow$ $t_n = 8 + (n-1)(-1)$

$\Rightarrow$ $t_n = 8 - n + 1 = 9 - n$

(ii) By substituting $n = 1, 2, 3, ...$ in the general term $t_n = 9 - n$ we can generate the arithmatic progression

$n = 1 \Rightarrow t_1 = 9 - 1 = 8$ $\quad$ $n = 2 \Rightarrow t_2 = 9 - 2 = 7$

$n = 3 \Rightarrow t_3 = 9 - 3 = 6$ $\quad$ $n = 4 \Rightarrow t_4 = 9 - 4 = 5$

Hence the progression is 8, 7, 6, 5,......

(iii) The 10th term of the A.P. can be calculated by substituting $n = 10$ in the nth term $\Rightarrow t_{10} = 9 - 10 = -1$

(iv) The sum to 'n' terms of an A.P. = $S_n = \frac{n}{2}[2a+(n-1)d]$

Substituting 'a' and 'd' in S_n we get

$$S_n = \frac{n}{2}[2\times8+(n-1)(-1)] = \frac{n}{2}[16-(n-1)] = \frac{n}{2}[16+n+1] = \frac{n}{2}[16-(n-1)] = \frac{n}{2}[17-n]$$

We get the sum to 10 terms by substituting n = 10 is the above expression

$$\Rightarrow \quad S_{10} = \frac{10}{2}[17-10] = 35.$$

2. If the sum of 'n' terms of an A.P. is $2n+3n^2$, generate the progression and find the nth term.

Sol. Given $S_n = 2n+3n^2$

Substitute $n=1 \Rightarrow S_1 = 2(1)+3(1)^2 = 5$

$n=2 \Rightarrow S_2 = 2(2)+3(2)^2 = 16$

$n=3 \Rightarrow S_3 = 2(3)+3(3)^2 = 33$

$n=4 \Rightarrow S_4 = 2(4)+3(4)^2 = 56$

S_1 = Sum to 1st term is nothing but the first term itself (t_1)

S_2 = Sum to first two terms t_1 and t_2

Similarly, S_3 = Sum of first three terms t_1, t_2 and t_3

S_4 = Sum of first four terms t_1, t_2, t_3 and t_4

$\Rightarrow \quad S_1 = t_1 = 5$

$S_2 = t_1 + t_2 = 16$

$S_3 = t_1 + t_2 + t_3 = 33$

$S_4 = t_1 + t_2 + t_3 + t_4 = 56$

$\Rightarrow \quad S_2 - S_1 = [t_1 + t_2] - t_1 = 16 - 5 = 11 = t_2$

$S_3 - S_2 = [t_1 + t_2 + t_3] - [t_1 + t_2] = 33 - 16 = 17 = t_3$

Hence the sequence is 5, 11, 17, ...

where $a = 5$ and $d = 6$

The general term $= t_n = a + (n-1)d \Rightarrow t_n = 5 + (n-1)6 = 6n - 1$

3. How many odd integers beginning will 15 must be taken for their sum to be equal to 975 ?

Sol. The odd integers beginning with 15 are as follows 15, 17, 19....

This forms an A.P. with first term, $a = 15$ and the common difference, $d = 17 - 15 = 2$

Let 'n' terms of the A.P. be taken to make the sum 975

$\Rightarrow 15 + 17 + 19 + ...\, n \text{ terms} = 975$

$$\Rightarrow S_n = \frac{n}{2}\left[2a+(n-1)d\right] = 975$$

Substituting the value of 'a' and 'd' in S_n

$$\Rightarrow \quad \frac{n}{2}\left[2\times15+(n-1)2\right] = 975 \Rightarrow 15n+(n-1)n = 975 \Rightarrow 15n+n^2-n = 975 \Rightarrow n^2+14n-975 = 0$$

$$\Rightarrow \quad n^2+39n-25n-975 = 0 \Rightarrow n(n+39)-25(n+39) = 0 \Rightarrow (n-25)(n+39) = 0 \Rightarrow n = 25 \text{ or } n = -39$$

But $n = -39$ is rejected since number of terms cannot be negative

$\therefore$ Number of odd integers beginning with 15 to make the sum equal to 975 = 25.

4. Find the value of 'k' if $2k+7, 6k-2, 8k-4$ are in A.P. Also find the sequence.

Sol. Given that $2k+7$, $6k-2$ and $8k-4$ are in A.P. The difference between successive terms in an A.P. is same.

$\Rightarrow \quad t_2 - t_1 = t_3 - t_2$

$\Rightarrow \quad [6k-2]-[2k+7]=[8k-4]-[6k-2] \Rightarrow 4k-9=2k-2 \Rightarrow 2k=7 \Rightarrow k=\dfrac{7}{2}$

Substituting the value of 'k' in $2k+7, 6k-2, 8k-4$ we get,

$2\times\frac{7}{2}+7,\ 6\times\frac{7}{2}-2,\ 8\times\frac{7}{2}-4$ i.e., 14, 19, 24

$\therefore$ The sequence is 14, 19, 24, ...

5. The number of terms in A.P is even. The sum of odd and even numbers are 24 and 30, respectively. If the last term exceeds the first term by 10.5, then find number of terms in the A.P.

Sol. Let the number of terms in the A.P be 'n' the two conditions given in the problem are :

The sum of the odd numbered terms in an A.P is $S_0 = 24$ and the sum of even numbered terms is $S_e = 30$.

The last terms exceeds first term by 10.5.

We know that the general representation of an A.P is

$a, a+d, a+2d, a+3d$ ......

$\therefore$ Sum of odd numbers

$S_0 = t_1 + t_3 + t_5 + t_7 + ... = 24$

$\Rightarrow \quad a + a + 2d + a + 4d .... = 24 \qquad$ ....(i)

This is in A.P with a common difference '$2d$' and there will be $\frac{n}{2}$ terms in the above A.P since we have even number of terms in the A.P.

Similarly the sum of even numbered terms,

$S_e = t_2 + t_4 + t_6 + ... = 30$

$\Rightarrow \quad a + d + a + 3d + a + 5d + ... = 30 \qquad$ ....(ii)

This will be in A.P with a common difference $2d$ and there will be $\frac{n}{2}$ terms in the above A.P since we have even number of terms in the A.P.

Subtracting (i) from (ii),

i.e., $[a+d+a+3d+a+5d+.......]-[a+a+2d+a+4d+...]=30-24$

$\Rightarrow \quad [d+d+d+........\frac{n}{2} \text{ terms}]=6$

$\Rightarrow \quad \frac{n}{2}\times d = 6 \Rightarrow nd = 12 \qquad$ ....(iii)

The other condition given in the problem is that the last term exceeds the first term by 10.5. Let nth term be the last term.

$\Rightarrow \quad t_n - t_1 = 10.5$

$\Rightarrow \quad a+(n-1)d-a=10.5$

$\Rightarrow \quad (n-1)d = 10.5 \qquad$ ....(iv)

With the two conditions in the problem, we got two equations (iii) and (iv). On solving these equations we can find the variables 'd' and 'n'. i.e., subtracting (iv) and (iii), we get

$nd-(n-1)d = 12-10.5 \Rightarrow nd - nd + d = 1.5 \Rightarrow d = 1.5$

$\therefore \quad n = \frac{12}{1.5} = 8$ [using (iii)]

Number of terms in A.P is 8.

6. Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots$, if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

Sol. We know that in an A.P. the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of the first and last term.

i.e., $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \ldots$ so, if an A.P. consists of 24 terms, then

$a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$. Now, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow \quad (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow \quad 3(a_1 + a_{24}) = 225 \Rightarrow a_1 + a_{24} = \frac{225}{3} = 75 \quad \ldots(i)$

$\therefore \quad S_{24} = \frac{24}{2}(a_1 + a_{24}) \quad \left[\text{Using } S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= 12(75) = 900 \quad$ [Using (i)]

7. $x_1, x_2, x_3, \ldots$ are in A.P. If $x_1 + x_7 + x_{10} = -6$ and $x_3 + x_8 + x_{12} = -11$, find $x_3 + x_8 + x_{22}$.

Sol. Let the common difference $= d$.

$x_1 + x_7 + x_{10} = -6$

$x_1 + x_1 + 6d + x_1 + 9d = -6 \quad \ldots\ldots\ldots(i)$

and $x_1 + 2d + x_1 + 7d + x_1 + 11d = -11 \quad \ldots\ldots\ldots(ii)$

(i) becomes $3x_1 + 15d = -6$, (ii) becomes $3x_1 + 20d = -11$

(i) – (ii) gives $-5d = 5 \Rightarrow d = -1$

From (i), $3x_1 + 15(-1) = -6 \Rightarrow x_1 = 3$

Now, $x_3 + x_8 + x_{22} = x_1 + 2d + x_1 + 7d + x_1 + 21d$

$\Rightarrow 3x_1 + 30d = 3(3) + 30(-1) = -21.$

8. Between 3 and 65, a certain number of A.M.s are inserted. The sum of the resulting sequence is 1088. Find the number of A.M.s inserted.

Sol. Let the number of A.M.s be n.

After insertion, there will be $(n+2)$ terms in A.P.

$\frac{n+2}{2}\{2(3) + (n+2-1)d\} = 1088 \quad [a = 3]$

Again, $T_{n+2} = 3 + (n+2-1)d = 65$

So, we get $\frac{n+2}{2}\{6 + (n+1)d\} = 1088 \quad \ldots\ldots(i)$

And, $3 + (n+1)d = 65 \quad \ldots\ldots(ii)$

From (ii), we get $(n+1)d = 62$

From (i), we get $\frac{n+2}{2}\{6 + 62\} = 1088$

$\Rightarrow \frac{n+2}{2} \times 68 = 1088 \Rightarrow n + 2 = 32 \Rightarrow n = 30$

No. of A.M.'s = 30.

9. If x, y are the A.M. and G.M. of two numbers respectively, find the numbers in terms of x and y.

Sol. $x + \sqrt{x^2 - y^2}$, $x - \sqrt{x^2 - y^2}$

Let the numbers be a, b.

$\frac{a+b}{2} = x; \sqrt{ab} = y$

$\Rightarrow \quad a+b=2x;\ ab=y^2$

The roots of $z^2-(a+b)z+ab=0$ are a, b.

$\Rightarrow z^2-2xz+y^2=0$

$$\Rightarrow z=\frac{2x\pm\sqrt{4x^2-4y^2}}{2}=\frac{\left(x\pm\sqrt{x^2-y^2}\right)2}{2}=x\pm\sqrt{x^2-y^2}$$

The numbers are $x+\sqrt{x^2-y^2}$, $x-\sqrt{x^2-y^2}$

10. If α,β are the roots of $x^2-x+k=0$ and γ,δ are roots of $x^2-9x+l=0$, find the values of k, l taking $\alpha,\beta,\gamma,\delta$ in G.P.

Sol. Let $\alpha,\beta,\gamma,\delta=a, ar, ar^2, ar^3$ [Since they are in G.P.]

$\Rightarrow \quad a, ar$ are the roots of $x^2-x+k=0$

$\Rightarrow \quad a+ar=1$ and $a\times ar=k$ ...(i)

Again, ar^2, ar^3 are the roots of $x^2-9x+l=0$

$\Rightarrow \quad ar^2+ar^3=9,\ ar^2.\ ar^3=l$ ...(ii)

From (i) and (ii)

$a+ar=1$ and $ar^2+ar^3=9 \Rightarrow r^2(a+ar)=9$

$\Rightarrow r^2(1)=9 \Rightarrow r\pm 3$

Case 1 : so $a+3a=1$ [Taking $r=3$]

$$\Rightarrow a=\frac{1}{4}$$

$$k=a^2r=\frac{1}{16}\times 3=\frac{3}{16}$$

$$l=a^2r^5=\left(\frac{1}{4}\right)^2\times 3^5=\frac{3^5}{16}=\frac{243}{16}$$

Case 2 : Let $r=-3$

$$a+(-3a)=1=a=\frac{-1}{2}$$

$$k=a^2r=\frac{1}{4}(-3)=\frac{-3}{4}$$

$$l=a^2r^5=\frac{1}{4}(-3)^5=\frac{-243}{4}$$

11. If a, b, c are in A.P. and x, y, z are in G.P., show that $x^b y^c z^a=x^c y^a z^b$.

Sol. $2b=a+c$ ...(i) [a, b, c are in A.P.]

$y^2=xz$ ...(ii) [x, y, z are in G.P.]

$$\Rightarrow \quad \frac{y}{x}=\frac{z}{y}$$

$$\text{Now, } \frac{x^b y^c z^a}{x^c y^a z^b}=\left(\frac{x}{z}\right)^b\left(\frac{y}{x}\right)^c\left(\frac{z}{y}\right)^a=\left(\frac{x}{z}\right)^b\left(\frac{y}{x}\right)^c\left(\frac{y}{x}\right)^a\left[\frac{z}{y}=\frac{y}{x}\right]=\left(\frac{x}{z}\right)^b\left(\frac{y}{x}\right)^{c+a}$$

$$\Rightarrow \frac{x^b}{z^b}\cdot\left(\frac{y}{x}\right)^{2b} \qquad [c+a=2b]$$

$$= \frac{x^b}{z^b} \cdot \frac{(y^2)^b}{x^{2b}} = \frac{1}{z^b} \cdot \frac{(zx)^b}{x^b} \qquad [y^2 = zx]$$

$$= \frac{1}{z^b} \cdot \frac{z^b x^b}{x^b} = 1$$

So, $x^b y^c z^a = x^c y^a z^b$.

12. Given A = 1, 8, 15, ... 1975, B = 2, 13, 24, ... 1982. Find the number of terms common to both the arithmetic progressions.

Sol. In A let there be 'n' terms.

$\Rightarrow \quad a = 1, d = 7$, so, $1975 = 1 + (n-1)7$

$\Rightarrow \quad 1975 = 7n - 6$

$\Rightarrow \quad 1981 = 7n \Rightarrow n = 283$

In B let there be k terms

$\Rightarrow \quad a = 2, d = 11, 1982 = 2 + (k-1)11$

$\Rightarrow \quad 1982 = 11k - 9$

$\Rightarrow \quad 1991 = 11k \Rightarrow k = 181$

To find common terms, $T_n = T_k$ (say)

$7n - 6 = 11k - 9$

$\Rightarrow \quad 7n + 3 = 11k \qquad \Rightarrow \quad 7n + 3 + 11 = 11k + 11$

$\Rightarrow \quad 7n + 14 = 11k + 11 \qquad \Rightarrow \quad 7(n+2) = 11(k+1)$

$\Rightarrow \quad \frac{n+2}{11} = \frac{k+1}{7} = m$ (say)

$\Rightarrow \quad n + 2 = 11m ; k + 1 = 7m \Rightarrow \quad n = 11m - 2 ; k = 7m - 1$

Now, $n \le 283$ and $k \le 181$

$\Rightarrow \quad 11m - 2 \le 283 ; 7m - 1 \le 181$

$\Rightarrow \quad 11m < 285 ; 7m \le 182$

$\Rightarrow \quad m \le 25\frac{10}{11} ; m \le 26$

The number satisfying the above two conditions is $m = 25$

So, the number of common terms = 25

13. Find the value of : $1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 ... + 1999 + 2000 - 2001$.

Sol. Given sequence can be grouped as

$(1 + 2 - 3) + (4 + 5 - 6) + (7 + 8 - 9) + 10 + 11 - 12) + ... (1999 + 2000 - 2001)$

$\Rightarrow \quad 0 + 3 + 6 + 9 + ... 1998 \qquad \left[\frac{1998}{3} = 666; 667 \text{ terms in A.P.}\right]$

$\Rightarrow \quad \frac{667}{2}(0 + 1998)\left[S_n = \left(\frac{a+l}{2}\right)n\right] \Rightarrow 667 \times 999 \Rightarrow 666333$

14. Given a sequence of numbers such that $T_1 = 1, T_n = (n-1) + T_{n-1}$ for all positive numbers n. Find T_{2000}.

Sol. $T_n = (n-1) + T_{n-1}$ [given]

$\Rightarrow T_n - T_{n-1} = n - 1$

Put, $n = 2 ; T_2 - T_1 = 1$

$n = 3\,;\ T_3 - T_2 = 2$
$n = 4\,;\ T_4 - T_3 = 3$
....
$n = 2000\,;\ T_{2000} - T_{1999} = 1999$
Adding $T_{2000} - T_1 = 1 + 2 + \ldots 1999$
$T_{2000} = T_1 + (1 + 2 + \ldots 1999)$

$$= 1 + \left(\frac{1999 \times 2000}{2}\right) = 1999001$$

15. Find the sum of : $1, (1+2), (1+2+2^2), (1+2+2^2+2^3) \ldots (1+2+\ldots 2^{2000})$

Sol. $T_1 = 2^1 - 1$
$T_2 = 2^2 - 1$
$T_3 = 2^3 - 1$
....
$T_{2001} = 2^{2001} - 1$ [Since there are 2001 terms]
Adding, $T_1 + T_2 + \ldots T_{2001}$
$= 2^1 + 2^2 + 2^{2001} - 2001$

$$= 2\left(\frac{2^{2001} - 1}{2 - 1}\right) - 2001 = 2^{2002} - 2 - 2001 \Rightarrow 2^{2002} - 2003$$

16. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.

Sol. Suppose the work is completed in n days when the workers started dropping. Since 4 workers are dropped on every day except the first day. Therefore, the total number of workers who worked all the n days is the sum of n terms of an A.P. with first term 150 and common difference –4 i.e.

$$\frac{n}{2}\left[2 \times 150 + (n-1) \times -4\right] = n(152 - 2n)$$

Had the workers not dropped then the work would have finished in $(n - 8)$ days with 150 workers working on each day. Therefore, the total number of workers who would have worked all the n days in $150(n - 8)$.

$\therefore\ n(152 - 2n) = 150(n - 8)$
$\Rightarrow\ n^2 - n - 600 = 0$
$\Rightarrow\ (n - 25)(n + 24) = 0$
$\Rightarrow\ n = 25.$

Thus, the work is completed in 25 days.

17. The $(m + n)$th and $(m - n)$th terms of a G.P. are p and q respectively. Show that the mth and nth terms are $\sqrt{pq}$ and $p\left(\frac{q}{p}\right)^{m/2n}$ respectively.

Sol. Let a be the first term and r be the common ratio. Then,
$a_{m+n} = p$ and $a_{m-n} = q$
$\Rightarrow\ ar^{m+n-1} = p$ and $ar^{m-n-1} = q$

$$\Rightarrow\ \frac{ar^{m+n-1}}{ar^{m-n-1}} = \frac{p}{q}$$

$$\Rightarrow r^{2n} = \frac{p}{q} \Rightarrow r = \left(\frac{p}{q}\right)^{1/2n} \Rightarrow \frac{1}{r} = \left(\frac{q}{p}\right)^{1/2n}$$

Now, $a_m = ar^{m-1}$

$$\Rightarrow a_m = ar^{(m+n-1)}\left(\frac{1}{r}\right)^n$$

$$\Rightarrow a_m = a_{m+n}\left(\frac{1}{r}\right)^n \qquad [\because a_{m+n} = ar^{m+n-1}]$$

$$\Rightarrow a_m = p\left(\frac{q}{p}\right)^{n/2n} \qquad \left[\because a_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p}\right)^{1/2n}\right]$$

$$\Rightarrow a_m = p\left(\frac{q}{p}\right)^{1/2} \Rightarrow a_m = \sqrt{pq} \quad \text{and} \quad a_n = ar^{n-1}$$

$$\Rightarrow a_n = ar^{(m+n-1)}\left(\frac{1}{r}\right)^m = a_{m+n}\left(\frac{1}{r}\right)^m \qquad [\because a_{m+n} = ar^{m+n-1}]$$

$$\Rightarrow a_n = p\left(\frac{q}{p}\right)^{m/2n} \qquad \left[\because a_{m+n} = p \text{ and } \frac{1}{r} = \left(\frac{q}{p}\right)^{1/2n}\right]$$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. In the following table, given that a is the first term, d the common difference and a_n the nth term of the AP.

	a	d	n	a_n
(i)	7	3	8	.....
(ii)	–18	.....	10	0
(iii)	.....	– 3	18	–5
(iv)	–18.9	2.5	.....	3.6
(v)	3.5	0	105	.....

2. 4, 10, 16, 22, ..., ...
3. 1, –1, –3, – 5, ..., ...
4. 11^{th} term from last term of AP 10, 7, 4........., –62, is ....
5. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. Number of rows in the flower bed is ....
6. Sum of 1 + 3 + 5 + ... + 1999 is ....
7. If the nth term of an A.P. = $7n + 4$, then S_n = ...
8. The sum of 8 A.Ms between 3 and 15 is = ...
9. The first and last terms of a G.P. are 5 and 5120. If the common ratio is 2, then S_n = ...
10. The sum of n terms of an A.P. is $4n^2 - n$. The common difference = ...
11. The difference of corresponding terms of two A.P's will be.................... .
12. Sum of all the integers between 100 and 1000 which are divisible by 7 is _____.

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. In an AP with first term a and common difference d, the nth term (or the general term) is given by $a_n = a + (n-1)d$.
2. If ℓ is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by :

$$S = \frac{n}{2}(a+\ell)$$

3. The balance money (in ₹) after paying 5% of the total loan of ₹ 1000 every month is 950, 900, 850, 800, . . . 50. respresented A.P.
4. The total savings (in ₹) after every month for 10 months when ₹ 50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500. represent GP.
5. 2, 4, 8, 16, ... is not an AP.
6. 10^{th} term of AP : 2, 7, 12, ... is 45.
7. 301 is a term of AP 5, 11, 17, 23, ...
8. A.M. of any n positive numbers a_1, a_2, a_3, ..., a_n is

$$A = \frac{a_1 + a_2 + ... + a_n}{n}$$

9. Given series :
15, 30, 60, 120, 240, is in GP.
10. 184 is a term of the sequence 3, 7, 11, ...
11. In an A.P., sum of terms equidistant from the beginning and end is constant which is the same as the sum of the first and last term.
12. The A.M. and G.M. of the numbers $a+\sqrt{a^2-b^2}$ and $a-\sqrt{a^2-b^2}$ are a, b respectively.
13. The third term of a G.P. is 6. Then the product of the first five terms is 6667.
14. We can find a set of three numbers forming an A.P. and a G.P. at the same time.
15. Zero can be the common ratio of a G.P.
16. The G.M. between 1.8 and 7.2 is 3.6.
17. The A.M. between $(a-b)^2$ and $(a+b)^2$ is a^2+b^2.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II give common difference for A.P. given in column I, match them correctly.

Column I	Column II
(A) $1, \frac{3}{2}, 2, \frac{5}{2}, ...$	(p) -4
(B) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$	(q) 0.2
(C) 1.8, 2.0, 2.2, 2.4	(r) 4/3
(D) 0, −4, −8, −12	(s) 1/2

2. Column II give n^{th} term for AP given in column I, match them correctly.

Column I	Column II
(A) 119, 136, 153, 170	(p) $13-3n$
(B) 7, 11, 15, 19, ...	(q) $9-5n$
(C) 4, −1, −6, −11, ...	(r) $3+4n$
(D) 10, 7, 4, 3	(s) $17n+102$

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find 10^{th} term of given A.P. 10, 20, 30, 40............
2. Find the sum of all multiples of 9 between 300 and 700.
3. Show that the sequence log a, $\log(ab)$, $\log(ab^2)$, $\log(ab^3)$, ... is an A.P. Find its n^{th} term.
4. Find the sum of the series $x + (x+y) + (x+2y) +$ .... to n terms.
5. Find the 1st four terms of the sequence whose first term is 1 and whose $(n+1)^{th}$ term is obtained by subtracting n from its n^{th} term. $t_{n+1} = t_n - n$.
6. Check whether $t_n = 2n^2 + 1$ is an A.P. or not.
7. Which term of the sequence 72, 70, 68, 66, ... is 40?
8. If m times the m^{th} term of an A.P. is equal to n times its n^{th} term. Show that the $(m+n)^{th}$ term of the A.P. is zero.
9. Find the common difference of the A.P. for which 11th term is 5 and 13th term is 79.
10. Find the number of terms of an A.P. if the last term is 43, first term is 7 and common difference is 6.
11. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the A.M. of a and b, then n?
12. If A_1, A_2 be two AM's and G_1, G_2 be two GM's between two numbers a and b, then find the value of $\frac{A_1+A_2}{G_1 G_2}$
13. Find the sum of n terms of the series 8 + 88 + 888 + ...
14. 2nd term of a G.P. is 30 and 4th term is 750. Find the 3rd term.
15. If a, b, c are in A.P., then the straight line $ax + by + c = 0$ will always pass through which point?
16. A man gets ₹30 for his first month's work and is given a rise of ₹ 2 each succeeding month. How much money does he earn over a period of ten years ?
17. The n^{th} term of a GP is 128 and the sum of its n terms is 255. If its common ratio is 2 then find its first term.
18. If the nth term of an A.P. is $3n+5$, find the sum of the first 12 terms.
19. Find the two numbers whose product is 135 and whose arithmetic mean is 12.

20. Find the arithmetic mean between $10\frac{1}{2}$ and $25\frac{1}{2}$.
21. Find the number of terms in the arithmetic progression 6, 9, 12, ..., 78.
22. Find the sum from the sixth term to the twelfth term of the arithmetic progression 6, 10, 14, ...
23. Which term of the sequence 4, 9, 14, 19, ... is 124 ?
24. The nth term of a sequence is given by $a_n = 2n + 7$. Show that it is an A.P. Also, find its 7th term.
25. Find the 5th term of the progression

$$1, \frac{(\sqrt{2}-1)}{2\sqrt{3}}, \left(\frac{3-2\sqrt{2}}{12}\right), \left(\frac{5\sqrt{2}-7}{24\sqrt{3}}\right), ...$$

26. Find the sixth term from end of the A.P. 17, 14, 11, ..., – 40.
27. The geometric progression 6, – 12, 24, ..., 6144 consists of n terms. Find the value of n.
28. 4th term of an A.P. is 8. Find the sum of its first 7 terms
29. If the sum of the n terms of the series 54, 51, 48, ... is 513, then find the value of n .
30. 1st and 4th term of a G.P. are 9 and 243 respectively. Find the sum of its first 5 terms .

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$.
2. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.
3. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5°, then find the number of sides.
4. A ladder has rungs 25 cm apart. The rungs decrease uniformly in the length from 45 cm. at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs?
5. If x, y, z are in G.P. and $a^x = b^y = c^z$ then, find the relation between a, b and c.
6. If p^{th}, q^{th} and r^{th} terms of an A.P. are equal to corresponding terms of a G.P. and these are respectively x, y, z, then find the value of $x^{y-z}, y^{z-x}, z^{x-y}$.
7. If the p^{th} term of an A.P. is a q and q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.
8. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.
9. A clock strikes the number of times of the hour. How many strikes does it make in one day?
10. There are four arithmetic means between 2 and –18. Find the means.
11. If the first, second and the last terms of an A.P. are a, b, c respectively, then find its sum.
12. If $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in A.P. then prove $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.
13. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then find its first term.
14. If a is the A.M. of b and c and the two geometric means are G_1 and G_2, then find the value of $G_1^3 + G_2^3$.
15. If $y = 3^{x-1} + 3^{-x-1}$ (x real), then find the least value of y.
16. There are 25 trees at equal distances of 5 meters in a line with a well, the distance of the well from the nearest tree being 10 metres. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.
17. Three numbers are in A.P. Their sum is 15 and their product is 45. Find the numbers.
18. The sum of the six numbers in A.P. is 345 and the difference between the first and sixth term is 55. Find the numbers.
19. Which term of the sequence 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$,... is first negative term ?
20. If pth, qth and rth terms of an A.P. are a, b, c respectively, then show that : $a(q - r) + b(r - p) + c(p - q) = 0$.
21. Determine the number of terms in the A.P. 3,7, 11, ... 407. Also, find its 20th term from the end.
22. Divide 32 into four parts which are in A.P. such that the product of extremes is to the product of means is 7 : 15.
23. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7th term is two more than thrice of its 3rd term.
24. If the first term of an A.P. is 2 and the sum of first five terms is equal to one-fourth of the sum of the next five terms, find the sum of first 30 terms.
25. Sum of all natural numbers between 1 and 98 which are multiples of 6 is ....
26. Find a G.P. for which the sum of first two terms is – 4 and the fifth term is 4 times the third term.
27. Determine the A.P. whose third term is 16 and the difference of 5^{th} term from 7^{th} term is 12.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

2. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., find

$$\frac{S_1}{n_1}(n_2-n_3)+\frac{S_2}{n_2}(n_3-n_1)+\frac{S_3}{n_3}(n_1-n_2)$$

3. Sums of the first p, q, r terms of an A.P. are a, b, c respectively. Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$.

4. The sum of n, $2n$ and $3n$ terms of an A.P. are x, y, z. Prove that $z = 3(y-x)$.

5. If in an A.P. the sum of m terms in n and sum of n terms is m, prove that the sum of $(m+n)$ terms is $-(m+n)$.

6. A manufacture of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :
   (i) The production in the 1st year
   (ii) The production in the 10th year
   (iii) The total production in first 7 years

7. If $(b-c)^2$, $(c-a)^2$, $(a-b)^2$ are in A.P., prove that $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in A.P.

8. Let S_n denote the sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then find the ratio S_{3n}/S_n.

9. If S be the sum, P be the product and R be the sum of the reciprocals of n terms of a GP, then find the value of P^2.

10. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7th and $(m-1)^{th}$ means is 5 : 9, then find the value of m.

11. If $a_1, a_2, a_3, a_4, \ldots\ldots, a_{n-2}, a_{n-1}, a_n$ are in A.P., then show that

$$\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\frac{1}{a_3a_{n-2}}+\cdots+\frac{1}{a_na_1}$$

$$=\frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right]$$

12. Find the sum of the series $85^2-83^2+84^2-82^2+83^2-81^2+82^2-80^2$ .... to 30 terms.

13. The 8th term of an A.P. is 17, and 19th term is 39. Find the A.P. and the 25th term?

14. The pth term of an A.P. is a and qth term is b. Prove that the sum of its $(p+q)$ terms is $\frac{p+q}{2}\left\{a+b+\frac{a-b}{p-q}\right\}$

15. If $S_1, S_2, S_3, \ldots S_m$ are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3, ..., m and common differences are 1, 3, 5, ..., $(2m-1)$ respectively. Show that

$$S_1+S_2+\cdots+S_m=\frac{mn}{2}(mn+1)$$

16. If a^2, b^2, c^2 are in A.P., then prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are also in A.P.

17. If the pth, qth and rth terms of a G.P. are a, b, c respectively, prove that: $a^{(q-r)} \cdot b^{(r-p)} \cdot c^{(p-q)} = 1$.

18. The ratio of sum of m and n terms of an A.P. is $m^2 : n^2$, then the ratio of m^{th} and n^{th} term.

19. If xth term of an A.P. be $1/y$ and yth term be $1/x$, then show that its (xy)th term is 1.

20. If $a_1, a_2, \ldots\ldots\ldots a_{n+1}$ are in A.P, then find the value of $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\cdots+\frac{1}{a_na_{n+1}}$ –

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The sum to 200 terms of the series 1 + 4 + 6 + 5 + 11 + 6 + ... is
   (a) 30,200 (b) 29,800
   (c) 30,200 (d) None of these

2. If the sum of the series 2 + 5 + 8 + 11 .... is 60100, then the number of terms are –
   (a) 100 (b) 200
   (c) 150 (d) 250

3. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. The third term of the progression if the sum of the first and the fifth term is equal to 10, is
   (a) 15 (b) 5
   (c) 8 (d) 10

4. What is the common difference of four terms in AP such that the ratio of the product of the first fourth term to that of the second and third term is 2 : 3 and the sum of all four terms is 20 –
   (a) 3 (b) 1
   (c) 4 (d) 2

5. If the sum of the series 54 + 51 + 48 + ... is 513, then the number of terms are –
   (a) 18 (b) 20
   (c) 17 (d) None of these

6. There are 60 terms in an A.P. of which the first term is 8 and the last term is 185. The 31st term is
(a) 56 (b) 94
(c) 85 (d) 98

7. There are four arithmetic means between 2 and −18. The means are
(a) $-4,-7,-10,-13$ (b) $1,-4,-7,-10$
(c) $-2,-5,-9,-13$ (d) $-2,-6,-10,-14$

8. If the first, second and the last terms of an A.P. are a, b, c respectively, then the sum is –
(a) $\dfrac{(a+b)(a+c-2b)}{2(b-a)}$ (b) $\dfrac{(b+c)(a+b-2c)}{2(b-a)}$
(c) $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$ (d) None of these

9. The sum of 11 terms of an A.P. whose middle term is 30, is–
(a) 320 (b) 330
(c) 340 (d) 350

10. The first and last term of an A.P. are a and ℓ respectively. If S is the sum of all the terms of the A.P. and the common difference is $\dfrac{\ell^2-a^2}{k-(\ell+a)}$, then k is equal to –
(a) S (b) $2S$
(c) $3S$ (d) None of these

11. The sum of two numbers is $2\frac{1}{6}$. If an even number of arithmetic means are inserted between them and their sum exceeds their number by 1, then number of means inserted is –
(a) 12 (b) 8
(c) 6 (d) None of these

12. If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are–
(a) 5, 10, 15, 20 (b) 4, 10, 16, 22
(c) 3, 7, 11, 15 (d) None of these

13. If m arithmetic means are inserted between 1 and 31 so that the ratio of the 7th and (m – 1)th means is 5 : 9, then the value of m is
(a) 9 (b) 11
(c) 13 (d) 14

14. Let T_r be the r^{th} term of an A.P. for $r = 1, 2, 3, ...$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals–
(a) $\frac{1}{mn}$ (b) $\frac{1}{m}+\frac{1}{n}$
(c) 1 (d) 0

15. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61......., then n is equal to –
(a) 10 (b) 12
(c) 11 (d) 13

16. If G be the geometric mean of x and y, then $\dfrac{1}{G^2-x^2}+\dfrac{1}{G^2-y^2}=$
(a) G^2 (b) $\dfrac{1}{G^2}$
(c) $\dfrac{2}{G^2}$ (d) $3G^2$

17. If a, b, c are in G.P., then
(a) a^2, b^2, c^2 are in G.P.
(b) $a^2(b+c), c^2(a+b), b^2(a+c)$ are in G.P.
(c) $\dfrac{a}{b+c}, \dfrac{b}{c+a}, \dfrac{c}{a+b}$ are in G.P.
(d) None of the above

18. If fifth term of a GP is 2, then the product of its 9 terms is:
(a) 256 (b) 512
(c) 1024 (d) none of these

19. If 4 GM's be inserted between 160 and 5, then third GM will be -
(a) 8 (b) 118
(c) 20 (d) 40

20. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then its common difference is -
(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

21. The number of terms of the series 5, 7, 9, ... that must be taken in order to have sum of 1020 is
(a) 20 (b) 30
(c) 40 (d) 50

22. If the nth term of an A.P. is $4n + 1$, then the common difference is :
(a) 3 (b) 4
(c) 5 (d) 6

23. If a, b, c, d, e, f are in A.P., then $e - c$ is equal to:
(a) $2(c-a)$ (b) $2(d-c)$
(c) $2(f-d)$ (d) $(d-c)$

24. The number of common terms to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is –
(a) 19 (b) 20
(c) 21 (d) 91

25. The number of two digit numbers which are divisible by 3 is
(a) 33 (b) 31
(c) 30 (d) 29

26. If the nth term of an A.P. is given by $a_n = 5n - 3$, then the sum of first 10 terms is
(a) 225 (b) 245
(c) 255 (d) 270

27. If there exists a geometric progression containing 27, 8 and 12 as three of its terms (not necessarily consecutive) then no. of progressions possible are
(a) 1 (b) 2
(c) infinite (d) none of these.

28. The fourth, seventh and tenth terms of a G.P. are p, q, r respectively, then :
(a) $p^2 = q^2 + r^2$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $pqr + pq + 1 = 0$

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following represents an A.P.
(a) $0.2, 0.4, 0.6, ...$ (b) $29, 58, 87, 116....$
(c) $15, 45, 135, 405...$ (d) $3, 3.5, 4.5, 8.5 ....$

2. If $t_n = 6n + 5$, then $t_{n+1} =$
(a) $6(n+1) + 17$ (b) $6(n-1) + 17$
(c) $6n + 11$ (d) $6n - 11$

3. 15th term of the series 243, 81, 27, ... is
(a) $\left(\frac{1}{3}\right)^9$ (b) $\left(\frac{1}{3}\right)^{10}$
(c) $\left(\frac{1}{3}\right)^{10}\left(\frac{1}{3}\right)^{-1}$ (d) $\left(\frac{1}{3}\right)^{10}\left(\frac{1}{3}\right)$

4. Summation of n terms of an A.P. is
(a) $\frac{n}{2}(a+l)$ (b) $\frac{n}{2}[2a+(n-1)d]$
(c) $\frac{a(r^n-1)}{(r-1)}$ (d) $\frac{a(1-r^n)}{(1-r)}$

5. $S_n = 54 + 51 + 48 + ...$ n terms $= 513$. Value of n is
(a) 18 (b) 19
(c) 15 (d) None of these above

6. Which of the following is not a G.P.?
(a) $2, 4, 6, 8......$ (b) $5, 25, 125, 625 ....$
(c) $1.5, 3.0, 6.0, 12.0.....$ (d) $8, 16, 24, 32, ...$

7. If the nth term of an A.P. be $(2n - 1)$, then the sum of its first n terms will be
(a) $n^2 - 1$ (b) $(n-1)^2 + (2n-1)$
(c) $(n-1)^2 - (2n-1)$ (d) n^2

8. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., then which of the following is in A.P.?
(a) a, b, c (b) a^2, b^2, c^2
(c) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ (d) bc, ac, ab

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Following two given series are in A.P.
$2, 4, 6, 8$ ....
$3, 6, 9, 12$ ....

First series contains 30 terms, while the second series contains 20 terms. Both of the above given series contains some terms, which are common to both of them.

1. The last term of both the above given A.P. are
(a) 57 (b) 60
(c) 50 (d) 54

2. The sum of both the above given A.P. are
(a) (930, 630) (b) (630, 930)
(c) (870, 580) (d) (580, 870)

3. No. of terms identical to both the above given A.P. is
(a) 5 (b) 1
(c) 0 (d) 10

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** $1, 2, 4, 8, ...$ is a G.P., $4, 8, 16, 32$ is a G.P. and $1+4, 2+8, 4+16, 8+32, ...$ is also a G.P.
Reason : Let general term of a G.P. with common ratio r be T_{k+1} and general term of another G.P. with common ratio r be T_{k+1} then the series whose general term $T''_{k+1} = T'_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.

2. **Assertion :** 11 11 ...... 1 (up to 91 terms) is a prime number.
Reason : If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

3. **Assertion :** let the positive numbers a, b, c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P.
Reason : If each term of an A.P. is divided by abc, then the resulting sequence is also in A.P.

4. **Assertion :** Let three distinct positive real numbers a,b,c are in G.P., then a^2, b^2, c^2 are in G.P.

Reason : If we square each term of a G.P., then the resulting sequence is also in G.P.

5. **Assertion :** The sum of the series with the nth term, $t_n = (9-5n)$ is (465), when no. of terms $n = 15$.

Reason : Given series is in A.P. and sum of n terms of an A.P. is $S_n = \frac{n}{2}[2a+(n-1)d]$

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

	Column I	Column II
(A)	$S_n = 54+51+48+\dots$ n terms $= 513$, Value of $n = ?$	(p) 20
(B)	$21, 42, 63, 84 \dots\dots$, $t_n = 420$, Value of $n = ?$	(q) 18
		(r) 32
(C)	$3, 3.6, 4.2, 4.8 \dots\dots\dots$, $t_{26} = ?$	(s) 19
(D)	$\frac{1}{2}, 1, 2, 4 \dots\dots$, $t_7 = ?$	(t) 64

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Find the sum of the integers lying between 1 and 100 (both inclusive) and divisible by 3, 5 or 7.
2. If a, b, c are in A.P., then prove that $a^2(b+c)+b^2(c+a)+c^2(a+b)$ is equal to $\frac{2}{9}(a+b+c)^3$.
3. The digits of a positive integer having three digits are in A.P. and their sum is 15. If the number obtained by reversing the digits is 594 less than the original number, then find the number.
4. If the ratio of the sum of n terms of two A.P.s is $(3n-13):(5n+21)$, then find the ratio of 24th terms of the two progression.
5. If the pth term of an A.P. is $\frac{1}{q}$ and qth term $\frac{1}{p}$. Prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$.
6. The sum of n terms of two arithmetic series are in the ratio $2n+3 : 6n+5$, then find the ratio of their 13th terms.
7. The sum of the third and the seventh terms of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.
8. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of m such that the sum of numbers of the houses preceding the house marked m is equal to the sum of numbers of the houses following it. Find this value of m.
9. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Find the sum of n terms of the G.P.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. (i) $a_n = 28$ (ii) $d = 2$ (iii) $a = 46$ (iv) $n = 10$ (v) $a_n = 3.5$
2. 28, 34 3. –7, –9 4. –32
5. $n = 10$
6. $\frac{1000}{2}[2(1) + (1000-1)2]$
7. $\frac{n}{2}\{15+7n\}$ [$T_1 = 7(1)+4 = 11, d = T_2 - T_1 = 18 - 11 = 7 \Rightarrow S_n = \frac{n}{2}\{22+\overline{n-1}7\}$ etc.]
8. $72\left[8\left(\frac{3+15}{2}\right)\text{etc.}\right]$
9. $10235\left[S_n = \frac{lr-a}{r-1}\text{ etc.}\right]$
10. 11 [$S_2 = 4(2)^2 - 2 \Rightarrow 14$
$S_1 = 4(1)^2 - 1 \Rightarrow 3$ etc.]
11. another A.P.
12. 70336 [**Hint :** $S = 105 + 112 + ... 994$ and $105 + (n-1)7 = 994 \Rightarrow 105 + 7n - 7 = 994 \Rightarrow n = 128$ etc.]

TRUE / FALSE

1.	True	2.	True	3.	True
4.	False	5.	True	6.	False
7.	False	8.	True	9.	True
10.	False	11.	True	12.	True
13.	False	14.	True	15.	False
16.	True	17.	True		

MATCH THE FOLLOWING :

1. (A) Common difference $= d = \frac{3}{2} - 1 = \frac{1}{2}$

(B) $d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(C) $d = 2 - 1.8 = 0.2$

(D) $d = -4 - 0 = -4.$

$\therefore$ (A) → (s); (B) → (r); (C) → (q); (D) → (p)

2. $13 - 3n = 13 - 3(1) = 10$

$9 - 5n = 9 - 5(1) = 4$

$3 + 4n = 3 + 4(1) = 7$

$17n + 102 = 17(1) + 102 = 119$

$\therefore$ (A) → (s); (B) → (r); (C) → (q); (D) → (p)

VERY SHORT ANSWER QUESTIONS :

1. $a_{10} = 10$
2. The A.P. formed is 306, 315, 324, ..., 693

Here first term $= a = 306$, common difference $= d = 9$ and last term $= l = 693$

$693 = 306 + (n-1)9$

$77 = 34 + n - 1$

$n = 44$

Sum $= n/2(a + l) = 22(306 + 693) = 21978$

3. If $\log a$, $\log(ab)$, $\log(ab^2)$, $\log(ab^3)$,........... is an A.P.

then $\log(ab) - \log a = \log\left(\frac{ab}{a}\right) = \log b$

$\log(ab^2) - \log(ab) = \log\left(\frac{ab^2}{ab}\right) = \log b$

Since, common difference is constant. i.e. log b so it is an A.P.

Now, $t_n = A + (n-1)d = \log a + (n-1)\log b$
$= \log a + \log b^{n-1} = \log(ab^{n-1})$

4. $\frac{n}{2}[2x + (n-1)y]$
5. $t_1 = 1, \quad \therefore t_2 = t_1 - t \Rightarrow t_2 = 1 - 1 \Rightarrow t_2 = 0$

next $t_3 = t_2 - 2 = 0 - 2 = 2$

$t_4 = t_3 - 3 = -2 - 3 = -5$

6. $t_n = 2n^2 + 1$

then $t_{n+1} = 2(n+1)^2 + 1$

$\therefore t_{n+1} - t_n = 2n^2 + 4n + 2 + 1 - 2n^2 - 1$
$= 4n + 2,$ which is not constant

$\therefore$ The above sequence is not an A.P.

7. 17^{th} term is 40.
8. $t_{m+n} = 0$
9. $t_n = a + (n-1)d$

11th term = 5 and 13th term = 79

$\therefore 5 = a + (11-1)d \Rightarrow a + 10d = 5$ ....(1)

and $79 = a + (13-1)d \Rightarrow a + 12d = 79$ ....(2)

Solving eq. (1) and (2), we get, d = 37

10. $n = 7$
11. $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a+b}{2}$

$\Rightarrow a^{n+1} - ab^n + b^{n+1} - ba^n = 0 \Rightarrow (a-b)(a^n - b^n) = 0$

If $a^n - b^n = 0$. The $\left(\frac{a}{b}\right)^n = 1 \Rightarrow \left(\frac{a}{b}\right)^0$. Hence $n = 0$

12. By the property of AP and GP, we have

$A_1 + A_2 = a + b$

$G_1 . G_2 = ab$

$$\therefore \frac{A_1 + A_2}{G_1 G_2} = \frac{a+b}{ab}$$

13. $\text{Sum} = \frac{8}{9}[9 + 99 + 999 + \ldots n \text{ terms}] = \frac{8}{9}[(10-1) + (100-1) + (1000-1) + \ldots n \text{ terms}]$

$$= \frac{8}{9}[(10 + 10^2 + 10^3 + \ldots + 10^n) - n] = \frac{8}{9}\left[\frac{10(10^n - 1)}{10-1} - n\right]$$

$$= \frac{8}{81}[10^{n+1} - 9n + 10]$$

14. Let 'a' be the 1st term and 'r' the common ratio.

Now, $ar = 30$ ....(i)

$ar^3 = 750$ ....(ii)

Dividing (ii) by (i),

$r^2 = 25, r = 5$

$$\therefore a = \left(\frac{30}{5}\right) = 6$$

$\therefore$ 3rd term $= ar = 6(5)^2 = 150$

15. a, b, c are in A.P.

So $2b = a + c$, then straight line $ax + by + c = 0$ will pass through $(1, -2)$ because if the line satisfies the condition $a - 2b + c = 0$ or $2b = a + c$.

16. $S_{120} =$ ₹ 17880.

17. Let a be the first term. Then as given

$T_n = 128$ and $S_n = 255$

But $S_n = \frac{rT_n - a}{r-1} \Rightarrow 255 = \frac{2(128) - a}{2-1} \Rightarrow a = 1$

18. 294. **19.** 15 and 9 **20.** 18

21. 25 **22.** 266

23. 25^{th} term $= 124$. **24.** 21

25. Clearly, the given progression is a G.P. with first term $a = 1$ and common ratio $\left(\frac{\sqrt{2}-1}{2\sqrt{3}}\right)$. So, its 5th term is given by

$$a_5 = ar^{(5-1)} = 1 \times \left(\frac{\sqrt{2}-1}{2\sqrt{3}}\right)^4 = \frac{(\sqrt{2}-1)^4}{144}$$

26. -25. **27.** 11

28. $S_7 = \frac{7}{2}(2)8 = 56$ **29.** $n = 18$ or 19

30. $ar^3 = 243$

$$r^3 = \frac{243}{9} = 27$$

$r = 3$

$$S = \frac{a(r^n - 1)}{r-1} = \frac{9(3^5 - 1)}{3-1} = \frac{9(242)}{2} = 1089$$

SHORT ANSWER QUESTIONS :

1. As $a_n = 3 + 2n$,

so, $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

....

List of numbers becomes 5, 7, 9, 11, . . .

Here, $7 - 5 = 9 - 7 = 11 - 9 = 2$ and so on.

So, it forms an AP with common difference $d = 2$.

To find S_{24}, we have $n = 24$, $a = 5$, $d = 2$.

Therefore, $S_{24} = \frac{24}{2}[2 \times 5 + (24-1) \times 2] = 12[10 + 46] = 672$

So, sum of first 24 terms of the list of numbers is 672.

2. The sum of integers from 1 to 100 that are divisible by 2 or 5

= sum of series divisible by 2 + sum of series divisible by 5 − sum of series divisible by both 2 and 5

$= (2 + 4 + 6 + \ldots + 100) + (5 + 10 + 15 + \ldots + 100) - (10 + 20 + 30 + \ldots + 100)$

$$= \frac{50}{2}\{2 \times 2 + (50-1)2\} + \frac{20}{2}\{2 \times 5 + (20-1)5\} - \frac{10}{2}\{10 \times 2 + (10-1)10\}$$

$= 2550 + 1050 - 550 = 3050$

3. Let the number of sides of the polygon be n.

Then the sum of interior angles of the polygon

$= (2n - 4)\frac{\pi}{2} = (n-2)\pi$

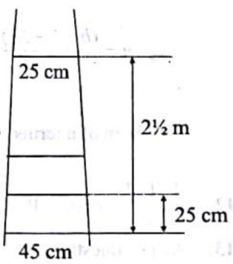
Since the angles are in A.P. and $a = 120°, d = 5$,

therefore, $\frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$

$\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow (n-9)(n-16) = 0 \Rightarrow n = 9, 16$

But $n = 16$ gives $T_{16} = a + 15d = 120° + 75.5° = 195.5°$ which is impossible as interior angle cannot be greater than 180°. Hence $n = 9$.

4. The length of the wood required for the rungs is $45 + \ldots + 25$



There are $\frac{250}{25} = 10$ rungs in all.

Using the formula, $S = \frac{n}{2}[a + \ell]$

a = first term, ℓ = last term,

$S = 350$ cm.

5. $\log_a b = \log_b c \Rightarrow \log_b a = \log_c b$

6. Let first term of an A.P. be a and c.d. be d and first term of a G.P. be A and c.r. be R, then

$a + (p-1)d = AR^{p-1} = x \Rightarrow p - 1 = (x - a)/d$ ..... (1)

$a + (q-1)d = AR^{q-1} = y \Rightarrow q - 1 = (y - a)/d$ ..... (2)

$a + (r-1)d = AR^{r-1} = z \Rightarrow r - 1 = (z - a)/d$ ..... (3)

$\therefore$ Given expression

$= (AR^{p-1})^{y-z}, (AR^{q-1})^{z-x}, (AR^{r-1})^{x-y}$

$= A^{o}, R^{[(p-1)(y-z)+(q-1)(z-x)+(r-1)(x-y)]}$

$= A^{o}\, R^{[(x-a)(y-z)+(y-a)(z-x)+(z-a)(x-y)]/d}$

[By (1), (2) and (3)]

$= A^{o}R^{o} = 1$

7. $\therefore t_n = A + (n-1)D = (p+q-1) + (n-1)(-1) = p+q-n$

8. The numbers between 250 and 1000 which are divisible by 3 will be : 252, 255, 258, ..., 999.

This is an A.P. whose first term $= a = 252$, $d = 255 - 252 = 3$ and last term $= 999$.

Now last term $= \ell = a + (n-1)d$

$\Rightarrow 999 = 252 + (n-1)3$

$\Rightarrow \dfrac{999-252}{3} + 1 = n \quad \therefore n = 250$

$\therefore$ Required sum

$= \dfrac{n}{2}[a+\ell] = \dfrac{250}{2}(252+999) = 125 \times 1251 = 156375$

9. For the first 12 hours of the day, the clock will strike

$1+2+3+\cdots+12 = \dfrac{12}{2}(1+12) = 78$ times

For the next 12 hours, there will be another 78 times, so in one day, the clock will strike 156 times

10. Let the means be X_1, X_2, X_3, X_4 and the common difference be b; then $2, X_1, X_2, X_3, X_4, -18$ are in A.P.;

$\Rightarrow -18 = 2 + 5b \Rightarrow 5b = -20 \Rightarrow b = -4$

Hence, $X_1 = 2 + b = 2 + (-4) = -2$; $X_2 = 2 + 2b = 2 - 8 = -6$

$X_3 = 2 + 3b = 2 - 12 = -10$; $X_4 = 2 + 4b = 2 - 16 = -14$

The required means are $-2, -6, -10, -14$.

11. We have, first term $= a$, $\quad \therefore T_1 = a$

Second term $= b$, $\quad \therefore T_2 = b$

The common difference, $d = T_2 - T_1 = b - a$

Also, last term $= c$

$\Rightarrow c = a + (n-1) \Rightarrow n = \dfrac{c-a+d}{d}$

$\Rightarrow n = \dfrac{(b+c-2a)}{(b-a)} \qquad (\because d = b-a)$

$\therefore$ Sum of n terms $S_n = \dfrac{n}{2}(a+\ell) = \dfrac{(b+c-2a)(a+c)}{2(b-a)}$

12. $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

13. As per question,

$a + ar = 12 \quad \ldots(1)$

$ar^2 + ar^3 = 48 \quad \ldots(2)$

$\Rightarrow \dfrac{ar^2(1+r)}{a(1+r)} = \dfrac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are = + ve and –ve alternately)

$\Rightarrow a = -12$.

14. Put $b = 1$ and $c = 8$ so that $a = 4.5$ and $G_1 = 2, G_2 = 4$. Now $G_1^3 + G_2^3 = 72$.

15. Let $y = 3^{x-1} + 3^{-x-1}$

$\Rightarrow y = \dfrac{3^x}{3} + \dfrac{1}{3.3^x} = \dfrac{1}{3}(3^x + 3^{-x})$

Using A.M. $\geq$ G.M.

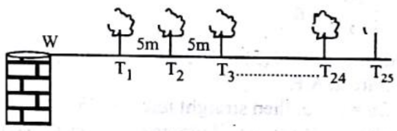
i.e., $\dfrac{a+b}{2} \geq \sqrt{ab}$

We get $\dfrac{3^x + 3^{-x}}{2} \geq \sqrt{3^x . 3^{-x}}$

$\Rightarrow 3^x + 3^{-x} \geq 2 \Rightarrow 3y \geq 2 \Rightarrow y \geq \dfrac{2}{3}$

Therefore, least value of y is $\dfrac{2}{3}$

16. Obviously the well (W) must be on one side of the trees $T_1, T_2, \ldots\ldots, T_{25}$,



The total distance covered by the gardener

$= WT_1 + (2WT_1 + T_1T_2) + (2WT_2 + T_2T_3) + \ldots\ldots + (2WT_{24} + T_{24}T_{25})$

$= 10 + (2\times10+5) + (2\times15+5) + \ldots\ldots\ldots\ldots +$ to 25 terms

$= 10 + (25+35+45+\ldots\ldots\ldots$ *to* 24 terms)

$= 10 + \dfrac{24}{2}[2\times25 + (24-1)\times10] = 10 + 12[50+230] = 3370$ m

17. 9, 5, 1.

In both the cases we get the same set of numbers.

18. 30, 41, 52, 63, 74 and 85

19. The given sequence is an A.P. in which first term a = 20 and common difference $d = -3/4$. Let the nth term of the given A.P. be the first negative term. Then,

$a_n < 0$

$\Rightarrow a + (n-1)d < 0$

$\Rightarrow 20 + (n-1)\times(-3/4) < 0$

$\Rightarrow \dfrac{83}{4} - \dfrac{3n}{4} < 0 \Rightarrow 83 - 3n < 0 \Rightarrow 3n > 83 \Rightarrow n > 27\dfrac{2}{3}$.

Since, 28 is the natural number just greater than $27\dfrac{2}{3}$.

So, $n = 28$.

Thus, 28th term of the given sequence is the first negative term.

20. **Hint :** Put the value of a, b and c in the L.H.S. expression and calculate the value of L.H.S.

21. Clearly, the given sequence is an A.P. with first term 3 and the common difference 4. Let there be n terms in the given A.P. Then,

$407 = n\text{th term} \Rightarrow 407 = 3 + (n-1) \times 4$

$\Rightarrow 4n = 408 \Rightarrow n = 102$

Now, 20th term from the end

$= [102 - 20 + 1]$th term from the beginning

$= 83$rd term from the beginning $= 3 + (83-1) \times 4 = 331$

After, to find 20th term from the end, we consider the given sequence as an A.P. with first term = 407 and common difference -4.

$\therefore$ 20th term from the end $= 407 + (20-1) \times (-4) = 331$.

22. Let the four parts be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$. Then,

Sum = 32

and, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

$\Rightarrow \dfrac{a^2-9d^2}{a^2-d^2} = \dfrac{7}{15} \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

Thus, the four parts are 2, 6, 10, 14.

23. $S_{20} = 740$

24. Let $a_1, a_2, a_3, \ldots$ be given A.P. with common difference d. It is given that $a_1 = 2$

and $a_1 + a_2 + a_3 + a_4 + a_5 = \dfrac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$

$\Rightarrow 4(a_1 + a_2 + a_3 + a_4 + a_5) = (a_6 + a_7 + a_8 + a_9 + a_{10})$

$\Rightarrow 5(a_1 + a_2 + a_3 + a_4 + a_5) = (a_1 + a_2 + \ldots + a_{10})$

$\Rightarrow 5S_5 = S_{10}$

$\Rightarrow 5\left[\dfrac{5}{2}\{2\times2+(5-1)d\}\right] = \dfrac{10}{2}[2\times2+(10-1)d]$

$\Rightarrow 50(1+d) = 20 + 45d \Rightarrow d = -6$

$\therefore$ Required sum $= S_{30} = \dfrac{30}{2}[2\times2+(30-1)\times -6]$

$= -2550$

25. Total no. of such numbers = 16

$S = \dfrac{16}{2}[2(6)+(16-1)6] = 8(102) = 816$

26. $-\dfrac{4}{3}, -\dfrac{8}{3}, -\dfrac{16}{3}, \ldots$ or $4, -8, 16, -32, \ldots$

27. $\therefore$ A.P. is 4, 10, 16, 22, 28, ...

LONG ANSWER QUESTIONS :

1. Let n = number of sides of the polygon sum of all the interior angles of a polygon of n sides = $(2n-4) \times 90$

Here the interior angles form an A.P. with a = 120° and d = 5°

Now $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\therefore \dfrac{n}{2}[2(120°) + (n-1)5] = 2(n-2) \times 90$

$\therefore \dfrac{n}{2}[240 + (n-1)5] = 180(n-2)$

$\therefore n = 9$ or $n = 16$

But note that if n = 16 then greatest angle $= a + (n-1)d = 120 + (16-1)5 = 195$

Greatest angle is 195° and common difference is 5°

$\therefore$ One of the angle would be 180° which is not possible in a polygon.

Hence this is to be omitted.

$\therefore$ The only possible value of n is $n = 9$.

2. We have, $\dfrac{2S_1}{n_1} = 2a + (n_1 - 1)d$

$\dfrac{2S_2}{n_2} = 2a + (n_2 - 1)d$

$\dfrac{2S_3}{n_3} = 2a + (n_3 - 1)d$

$\therefore \dfrac{2S_1}{n_1}(n_2 - n_3) + \dfrac{2S_2}{n_2}(n_3 - n_1) + \dfrac{2S_3}{n_3}(n_1 - n_2)$

$= [2a + (n_1 - 1)d](n_2 - n_3) + [2a + (n_2 - 1)d](n_3 - n_1) + [2a + (n_3 - 1)d](n_1 - n_2) = 0$

3. $S_p = a \therefore \dfrac{p}{2}[2A + (p-1)d] = a$

$\Rightarrow \left[A + \dfrac{(p-1)d}{2}\right](q-r) = \dfrac{a}{p}(q-r)$ .... (1)

$S_q = b \therefore \dfrac{q}{2}[2A + (q-1)d] = b$

$\Rightarrow \left[A + \dfrac{(q-1)d}{2}\right](r-p) = \dfrac{b}{q}(r-p)$ .... (2)

$S_r = c \therefore \dfrac{r}{2}[2A + (r-1)d] = c$

$\Rightarrow \left[A + \dfrac{(r-1)d}{2}\right](p-q) = \dfrac{c}{r}(p-q)$ .... (3)

Adding eqs (1), (2) and (3), we get

$\dfrac{a}{p}(q-r) + \dfrac{b}{q}(r-p) + \dfrac{c}{r}(p-q)$

$= A(q - r + r - p + p - q) + \dfrac{d}{2}[(q-r)(p-1) + (r-p)(q-1) + (p-q)(r-1)]$

$= A \times 0 + \dfrac{d}{2} \times 0 = 0$

4. **Hint :** Put the value of x, y and z in the given equation and calculate the value of R.H.S.

5. $S_{m+n}=\frac{m+n}{2}[2a+(m+n-1)d]=\left(\frac{m+n}{2}\right)(-2)=-(m+n)$

6. (i) Let the number of TV sets manufactured in the n^{th} year $=a_n$.

$\therefore a_3=600$ and $a_7=700$

$a+2d=600$ ....(1)

$a+6d=700$ ....(2)

Now subtracting (2) from (1), we get

$d=25$

Putting the value of d in (1), we get, $a=550$

$\therefore$ Production of TV sets in first year = 550

(ii) Production of TV sets in 10^{th} year = 775

(iii) $S_7=\frac{7}{2}[2\times550+(7-1)\times25]=4375$

$\therefore$ Total production of TV sets in first 7 years = 4375

7. Show that $\frac{1}{b-c},\frac{1}{c-a},\frac{1}{a-b},\frac{1}{a-b}$ has a common difference.

8. Given, $S_{2n}=3S_n$

$\therefore$ From given equation, we have

$$\frac{2n}{2}[2a+(2n-1)d]=\frac{3n}{2}[2a+(n-1)d]$$

$\Rightarrow$ $2a=(n+1)d$

Now, consider

$$\frac{S_{3n}}{S_n}=\frac{\frac{1}{2}(3n)[2a+(3n-1)d]}{\frac{1}{2}(n)[2a+(n-1)d]}=\frac{3[2a+3nd-d]}{[2a+nd-d]}$$

Put value of $2a=(n+1)$ d, we get, $\frac{S_{3n}}{S_n}=6$

9. We assume that $a, ar, ar^2, \ldots, ar^{n-1}$ be n terms of a GP.

Given : S denotes the sum, P represents the product and R be the sum of the reciprocals.

$\therefore$ $S=a+ar+ar^2+\ldots+ar^{n-1}=\frac{a(1-r^n)}{1-r}$ $\quad(\because r<1)$

$P=a\,.\,ar\,.\,ar^2\ldots.ar^{n-1}=a^n r^{1+2+\ldots+(n-1)}=a^n r$

and $R=\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\ldots+\frac{1}{ar^{n-1}}=\frac{\frac{1}{a}(1-r^n)}{r^{n-1}(1-r)}$

$\left(\because \frac{1}{r}>1\right)$

Now, $\left(\frac{S}{R}\right)^n=\{a^n r^{\frac{n(n-1)}{2}}\}^2=P^2$.

10. $m=\frac{1022}{73}=14$

11. $\frac{1}{a_1}+\frac{1}{a_n}=\frac{a_1+a_n}{a_1a_n}$

$$\frac{1}{a_2}+\frac{1}{a_{n-1}}=\frac{a_2+a_{n-1}}{a_2a_{n-1}}=\frac{(a_1+d)+(a_n-d)}{a_2a_{n-1}}=\frac{a_1+a_n}{a_2a_{n-1}}$$

$$\frac{1}{a_3}+\frac{1}{a_{n-2}}=\frac{a_3+a_{n-2}}{a_3a_{n-2}}=\frac{(a_1+2d)+(a_n-2d)}{a_3a_{n-2}}=\frac{a_1+a_n}{a_3a_{n-2}}$$

....

$$\frac{1}{a_{n-1}}+\frac{1}{a_2}=\frac{a_2+a_{n-1}}{a_{n-1}a_2}=\frac{(a_1+d)+(a_n-d)}{a_{n-1}a_2}=\frac{a_1+a_n}{a_{n-1}a_2}$$

$$\frac{1}{a_n}+\frac{1}{a_1}=\frac{a_1+a_n}{a_na_1}$$

Adding, $2\left[\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right]$

$$=(a_1+a_n)\left[\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\cdots+\frac{1}{a_{n-2}a_3}+\frac{1}{a_{n-1}a_2}+\frac{1}{a_na_1}\right]$$

$$\therefore \quad \frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\cdots+\frac{1}{a_{n-2}a_3}+\frac{1}{a_{n-1}a_2}+\frac{1}{a_na_1}$$

$$=\frac{2}{(a_1+a_n)}\left[\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right]$$

12. The required sum $S=(85+83)(85-83)+(84+82)(84-82)+(83+81)(83-81)+(82+80)(82-80)\ldots$ to 15 terms (as the summation consists of pairs of terms)

$S=(85+83)2+(84+82)2+(83+81)2+(82+80)2\ldots$ to 15 terms

$=2[(85+84+83\ldots$ to 15 terms$)+(83+82+81\ldots$ to 15 terms$)]$

$=2\times\frac{15}{2}[2(85)+(15-1)(-1)]+2\times\frac{15}{2}[2(83)+(15-1)(-1)]$

$=4620.$

13. Given $t_8=17$ and $t_{19}=39$

We known $t_n=a+(n-1)d$

$\Rightarrow t_8=a+7d=17$ ..... (i)

$\Rightarrow t_{19}=a+18d=39$ ..... (ii)

(ii) − (i) $\Rightarrow 11d=22$ ..... (iii)

$\Rightarrow d=2 \Rightarrow a=3$

$\therefore$ The first term of the A.P., $a=3$ and common difference $d=2$

The A.P. is 3, 5, 7 .....

$\therefore$ t_{25} of the A.P. is 51.

14. Let A and D be the first term and common difference respectively of the given A.P. Then,

$a=p$th term $\Rightarrow a=A+(p-1)D$ ....(i)

$b=q$th term $\Rightarrow b=A+(q-1)D$ ....(ii)

Subtracting (ii) from (i), we get $D=\frac{a-b}{p-q}$

Adding (i) and (ii), we get

$$(a+b)+\frac{a-b}{p-q}=2A+(p+q-1)D \quad \text{....(iii)}$$

Now, S_{p+q} = Sum of $(p+q)$ terms

$$\Rightarrow \quad S_{p+q}=\frac{p+q}{2}\{2A+(p+q-1)D\}$$

$$=\frac{p+q}{2}\left\{a+b+\frac{a-b}{p-q}\right\} \quad \text{[Using (iii)]}$$

15. We have

First terms	Common differences	Sums of n terms
1	1	$S_1=\frac{n}{2}[2\times1+(n-1)\times1]$
2	3	$S_2=\frac{n}{2}[2\times2+(n-1)\times3]$
3	5	$S_3=\frac{n}{2}[2\times3+(n-1)\times5]$
⋮	⋮	
m	$2m-1$	$S_m=\frac{n}{2}[2m+(n-1)(2m-1)]$

$$\therefore S_1+S_2+\cdots+S_m=\frac{n}{2}[2\times1+(n-1)\times1]$$

$$+\frac{n}{2}[2\times2+(n-1)\times3]+\cdots+\frac{n}{2}[2m+(n-1)(2m-1)]$$

$$=\frac{n}{2}[2\times(1+2+3+\cdots+m)+(n-1)(1+3+5+\cdots+(2m-1))]$$

$$=\frac{mn}{2}(mn+1)$$

16. $\frac{a}{b+c},\frac{b}{c+a},\frac{c}{a+b}$ will be in A.P.

if $\frac{a}{b+c}+1,\frac{b}{c+a}+1,\frac{c}{a+b}+1$ are in A.P.

[On adding 1 to each term]

i.e. if $\frac{a+b+c}{b+c},\frac{a+b+c}{c+a},\frac{a+b+c}{c+a}$ are in A.P.

i.e. if $\frac{1}{b+c},\frac{1}{c+a},\frac{1}{a+b}$ are in A.P.

[On dividing each term by $a+b+c$]

i.e. if $\frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{a+b}-\frac{1}{c+a}$

i.e. if $\frac{b-a}{(c+a)(b+c)}=\frac{c-b}{(a+b)(c+a)}$

i.e. if $\frac{b-a}{b+c}=\frac{c-b}{a+b}$

i.e. if $b^2-a^2=c^2-b^2$ i.e. if $2b^2=a^2+c^2$ i.e. if a^2, b^2, c^2 are in A.P.

Thus, a^2, b^2, c^2 are in A.P. $\Rightarrow \frac{a}{b+c},\frac{b}{c+a},\frac{c}{a+b}$ are in A.P.

17. **Hint :** Put the value of a, b, c in the L.H.S. expression and calculate it.

18. Given that : $\dfrac{\frac{m}{2}[2a+(m-1)d}{\frac{n}{2}[2a+(n-1)d}=\dfrac{m^2}{n^2}$

$$\Rightarrow \quad an+\frac{1}{2}(m-1)nd=am+\frac{1}{2}(n-1)md$$

So required ratio, $\frac{T_m}{T_n}=\frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1}$

19. According to given conditions, we get

$$t_x=a+(x-1)d=\frac{1}{y} \quad \text{...........(1)}$$

$$t_y=a+(y-1)d=\frac{1}{x} \quad \text{...........(2)}$$

$$\therefore (x-1)d-(y-1)d=\frac{1}{y}-\frac{1}{x} \Rightarrow d=\frac{1}{xy}$$

Substituting the value of d in (1), we get

$$a=\frac{x-(x-1)}{xy}=\frac{1}{xy}$$

Now (xy)th term will be

$$t_{xy}=a+(xy-1)d$$

20. $a_1, a_2, \ldots\ldots a_{n+1}$ are in A.P. and common difference $=d$

Let $S=\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots..+\frac{1}{a_na_{n+1}}$

$$\Rightarrow S=\frac{1}{d}\left\{\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\ldots..+\frac{a_{n+1}-a_n}{a_na_{n+1}}\right\}$$

$$\Rightarrow S=\frac{1}{d}\left\{\frac{1}{a_n}-\frac{1}{a_{n+1}}\right\} \Rightarrow S=\frac{1}{d}\left(\frac{nd}{a_1a_{n+1}}\right)=\frac{n}{a_1a_{n+1}}$$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (c) Above series is a combination of two APs.

The 1st AP is $(1+6+11+\ldots\ldots..)$ and the 2nd AP is $(4+5+6+\ldots\ldots...)$

Since the terms of the two series alternate, $S=(1+6+11+\ldots\ldots..$to 100 terms$)+(4+5+6+\ldots\ldots..$to 100 terms$)$

$$=\frac{100[2\times1+99\times5]}{2}+\frac{100[2\times4+99\times1]}{2}$$

(Using the formula for the sum of an AP)

$$=50[497+107]=50[604]=30200$$

Alternatively, we can treat two consecutive terms as one. So we will have a total of 100 terms of the nature :

$$(1+4)+(6+5)+(11+6)\ldots.. \Rightarrow 5, 11, 17, \ldots\ldots$$

Now, $a = 5, d = 6$ and $n = 100$
Hence the sum of the given series is

$$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6] = 50\,[604] = 30200$$

2. (b) **3.** (b)

4. (d) Take the four terms as $a - 3x, a - x, a + x, a + 3x$
The sum $= 4a = 20 \Rightarrow a = 5$
Also, $3(a^2 - (3x)^2) = 2(a^2 - x^2) \Rightarrow x = 1$
However, the common difference is $2x$ and not x
$\therefore$ When $x = 1, d = 2x = 2$

5. (a)

6. (d) Let d be the common difference; then 60th term $= 8 + 59d = 185$
$\Rightarrow 59d = 177 \Rightarrow d = 3 \Rightarrow$ 31st term $= 8 + 30 \times 3 = 98.$

7. (d) Let the means be X_1, X_2, X_3, X_4 and the common difference be b; then $2, X_1, X_2, X_3, X_4, -18$ are in A.P.;
$\Rightarrow -18 = 2 + 5b \Rightarrow 5b = -20 \Rightarrow b = -4$
Hence, $X_1 = 2 + b = 2 + (-4) = -2; X_2 = 2 + 2b = 2 - 8 = -6$
$X_3 = 2 + 3b = 2 - 12 = -10; X_4 = 2 + 4b = 2 - 16 = -14$
The required means are $-2, -6, -10, -14$.

8. (c) **9.** (b)

10. (b) We have, $S = \frac{n}{2}(a + \ell) \Rightarrow \frac{2S}{a + \ell} = n$ .... (1)

Also, $\ell = a + (n-1)\,d \Rightarrow d = \frac{\ell - a}{n - 1} = \frac{\ell - a}{\frac{2S}{a + \ell} - 1}$

$= \frac{\ell^2 - a^2}{2S - (\ell + a)}$ $\therefore k = 2S$

11. (a) Let a and b be the two numbers so that

$a + b = \frac{13}{6}$ (given) .... (1)

Let $2n$ (even) means be inserted between them so that $a, x_1, x_2, ...x_{2n}, b$ is an A.P. of $(2n + 2)$ terms whose first term is a and last term is b and whose sum is

$\frac{2n+2}{2}[a + b] = (n+1)(a+b)$ .... (2)

$\therefore$ Sum of the means = Sum of the series $- (a + b)$
$\Rightarrow (2n + 1)$ (given) $= (n + 1)(a + b) - (a + b) = n\,(a + b)$

$\Rightarrow 2n + 1 = n \cdot \frac{13}{6}$ by (1)

$\Rightarrow 12n + 6 = 13n,$ $\therefore n = 6$
Hence, the number of means inserted $= 2n = 12$

12. (a)

13. (d) Let the means be $x_1, x_2, ...x_m$ so that $1, x_1, x_2, ...x_m, 31$ is an A.P. of $(m + 2)$ terms.
Now, $31 = T_{m+2} = a + (m + 1)d = 1 + (m + 1)d$

$\therefore d = \frac{30}{m+1}$ Given; $\frac{x_7}{x_{m-1}} = \frac{5}{9}$

$\therefore \frac{T_8}{T_m} = \frac{a + 7d}{a + (m-1)d} = \frac{5}{9}$

$\Rightarrow 9a + 63d = 5a + (5m - 5)d$

$\Rightarrow 4.1 = (5m - 68)\frac{30}{m+1}$

$\Rightarrow 2m + 2 = 75m - 1020 \Rightarrow 73m = 1022$

$\therefore m = \frac{1022}{73} = 14$

14. (c)

15. (c) Given, $\frac{2n}{2}\{2.2 + (2n-1)3\} = \frac{n}{2}\{2.57 + (n-1)2\}$
or $2(6n + 1) = 112 + 2n$ or $10n = 110$ $\therefore n = 11$

16. (b) As given $G = \sqrt{xy}$

$\therefore \frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2} = \frac{1}{xy - x^2} + \frac{1}{xy - y^2}$

$= \frac{1}{x - y}\left\{-\frac{1}{x} + \frac{1}{y}\right\} = \frac{1}{xy} = \frac{1}{G^2}$

17. (a) **18.** (b)

19. (c) $r = \left(\frac{5}{160}\right)^{\frac{1}{4+1}} = \left(\frac{1}{32}\right)^{\frac{1}{5}} = \frac{1}{2}$

$G_3 = ar^3 \Rightarrow 160 \times \frac{1}{2^3} = 20$

20. (c) Let roots be α, β, γ and $a = a - d, b = a, g = a + d$. Then
$\alpha + \beta + \gamma = 3a = -(-12) \Rightarrow a = 4$
$\alpha\beta\gamma = a(a^2 - d^2) = -(-28) \Rightarrow d = \pm 3$

21. (b) **22.** (b)

23. (b) Let x be the common difference of the A.P. a, b, c, d, e, f.
$\therefore e = a + (5 - 1)x$ $[\because a_n = a + (n-1)d]$
$\Rightarrow e = a + 4x$ ...(1)
and $c = a + 2x$ ...(2)
$\therefore$ Using equation (1) and (2), we get
$e - c = a + 4x - a - 2x$
$\Rightarrow e - c = 2x = 2(d - c).$

24. (b) Common terms will be $21, 41, 61,$ ....
$21 + (n - 1)\,20 \le 417 \Rightarrow n \le 20.8 \Rightarrow n = 20$

25. (c) Two digit numbers which are divisible by 3 are 12, 15, 18,..., 99;
So, $99 = 12 + (n - 1) \times 3.$

26. (b) Putting $n = 1, 10$, we get $a = 2, l = 47$.

$\therefore S_{10} = \frac{10}{2}(2 + 47) = 5 \times 49 = 245.$

27. (c) Let there be an increasing G.P., with first term 8, m^{th} term 12 and n^{th} term 27. then

$12 = 8r^{m-1}$ ...(i) and $27 = 8r^{n-1}$ ...(ii)
Dividing, we get

$\Rightarrow \left(\frac{3}{2}\right)^2 = r^{n-m}$ ....(iii)

Also from (i) $\Rightarrow r^{m-1} = \frac{3}{2}$

Substituting in (iii) we get $\left(r^{m-1}\right)^2 = r^{n-m}$

$\Rightarrow \frac{m}{1} = \frac{n+2}{3} = k\ (say)$

$\therefore$ Corresponding to $k = 2, 3, 4,$........ we get sets of distinct positive integral values of m, n, So there exists innumerable G.P's which have 27,8 and 12 as three of their terms.

28. (b) Let a be the first term and r be common ratio.
i.e. given conditions
Fourth term of G.P. : $p = T_4 = ar^3$ ...(i)
Seventh term of G.P. : $q = T_7 = ar^6$ ...(ii)
Tenth term of G.P. : $r = T_{10} = ar^9$ ...(iii)
Equ. (i) × Equ. (iii) :
$pr = ar^3 \times ar^9 \Rightarrow pr = a^2r^{12} \Rightarrow pr = (ar^6)^2 \Rightarrow pr = q^2$

MORE THAN ONE CORRECT :

1. (a,b) Since there is a common difference option (a),
$d = 0.4 - 0.2 = 0.6 - 0.4 = 0.2$
Similarly for option (b), $d = 58 - 29 = 87 - 58 = 29$

2. (b,c) Put $n+1$ in place of n in $Tn = 6n + 5$

3. (a,c) Common ratio $= \frac{81}{243} = \frac{27}{81} = \frac{1}{3}$
Hence, the above series is in G.P.

$$t_{15} = 243\left(\frac{1}{3}\right)^{15-1} = \frac{3^5}{3^{14}} = \frac{1}{3^9} = \frac{3}{3^9(3)} = \left(\frac{1}{3}\right)^{10}\left(\frac{1}{3}\right)^{-1}$$

4. (a,b)

5. (a,b) $S_n = 513$

$\frac{n}{2}[2(54) + (n-1)(-3)] = 513$
$n(108 - 3n + 3) = 1026$
$n^2 - 37n + 342 = 0$
$n^2 - 19n - 18n + 342 = 0$
$n(n-19) - 18(n-19) = 0$
$(n-18)(n-19) = 0$
$n = 18$ or $n = 19$

6. (a,d) Both (a) and (d) are in A.P.

7. (b,d) $t_1 = 2(1) - 1 = 1$
$t_2 = 2(2) - 1 = 3, t_3 = 2(3) - 1 = 5$
and so on.
$\therefore\ t_1 + t_2 + t_3 + \cdots + t_n = 1 + 3 + 5 + \cdots\ [2(n) - 1]$
$= \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}(2 + 2n - 2) = n^2$
$(n-1)^2 + (2n-1) = n^2 - 2n + 1 + 2n - 1 = n^2$

8. (c,d) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.
Adding 2 to each term
$\frac{b+c-a}{a} + 2, \frac{c+a-b}{b}, \frac{a+b-c}{c} + 2$ are in A.P.

$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.
Dividing each term by $(a+b+c)$,
$\frac{a+b+c}{a(a+b+c)}, \frac{a+b+c}{b(a+b+c)}, \frac{a+b+c}{c(a+b+c)}$ are in A.P.
$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.
Multiplying each term by abc
$\frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in A.P.
bc, ac, ab are in A.P.

PASSAGE BASED QUESTIONS :

Passage I

1. (b) 2, 4, 6, 8 ....
Last term, $t_{30} = 2 + (30-1)2 = 2 + 2(29) = 60$
3, 6, 9, 12 ......
Last term, $t_{20} = 3 + (20-1)3 = 3 + 57 = 60$

2. (a) For 2, 4, 6, 8.........
$S_{30} = \frac{30}{2}(2 + 60) = 930$
For 3, 6, 9, 12 ....
$S_{20} = \frac{20}{2}(3 + 60) = 630$

3. (d) Let mth term of the first series is common with the nth term of the second series.
$t_m = t_n$
$2 + (m-1)2 = 3 + (n-1)3$
$2 + 2m - 2 = 3 + 3n - 3$
$2m = 3n$
$\frac{m}{3} = \frac{n}{2} = k$ (let)
$m = 3k, n = 2k$

$1 \le m \le 30$	$1 \le n \le 20$
$1 \le 3k \le 30$	$1 \le 2k \le 20$
$\frac{1}{3} \le k \le 10$	$\frac{1}{2} \le k \le 10$

Hence, $k = 1, 2, 3, \ldots$ 10. For each value of k, we get one identical term.
Thus, no of identical terms = 10

ASSERTION & REASON :

1. (a) Let $T_{k+1} = ar^k$ and $T'_{k+1} = br^k$
Since $T''_{k+1} = ar^k + br^k = (a+b)r^k$,
$\therefore\ T''_{k+1}$ is general term of a G.P.

2. (a) Since 11 11 .... 1 (up to 91 terms)
$= \frac{(10^{91} - 1)}{10 - 1}$ = divisible by 9.
$\Rightarrow$ the given number is not prime. But reason is true.

3. (a) **4.** (a) **5.** (d)

MULTIPLE MATCHING QUESTIONS :

1. (A) → (q), (s); (B) → (p); (C) → (q); (D) → (r)

(A) $S_n = 513$

$\frac{n}{2}[2(54)+(n-1)(-3)] = 513$

$n(108-3n+3) = 1026$

$n^2-37n+342=0$

$n^2-19n-18n+342=0$

$n(n-19)-18(n-19)=0$

$(n-18)(n-19)=0$

$n = 18$ or $n = 19$

(B) $t_n = 420$

$21+(n-1)21 = 420$

$1+n-1 = 20$

$n = 20$

(C) $t_{26} = 3+(26-1)(0.6) = 3+25(0.6) = 3+15 = 18$

(D) $t_7 = \frac{1}{2}(2)^{7-1} = \frac{2^6}{2} = 2^5 = 32$

HOTS SUBJECTIVE QUESTIONS :

1. Integers divisible by 3 from 1 to 100 are 3, 6, 9, ...99, *i.e.* total 33 in number

Integers divisible by 5 are, 5, 10, 15, ... 100 (20 in number).

Integers divisible by 7 are 7, 14, ...98 (14 in number)

Integers, divisible by both 3 and 5 are 15, 30, ...90, (6 in number).

Integers divisible by both 3 and 7 are 21, 42, 63, & 84. (4 in number)

and Integers divisible by both 5 and 7 are 35, 70 (2 in number).

So, sum of numbers divisible by 3, 5 or 7 is

$$= \frac{33}{2}(3+99)+\frac{20}{2}(5+100)+\frac{14}{2}(7+98)$$

$$-\frac{6}{2}(15+90)-\frac{4}{2}(21+84)-\frac{2}{2}(35+70) = 2838$$

2. Let $x = a^2(b+c)+b^2(c+a)+c^2(a+b)$

$\Rightarrow x = a^2b+a^2c+b^2c+b^2a+c^2a+c^2b;$

$\Rightarrow x = a^2b+b^2c+b^2a+c^2a+c^2b+2abc-2abc$ [Adding $2abc$ and subtracting]

$= abc+a^2b+ac^2+a^2c+b^2c+ab^2+bc^2+abc-2abc$

$= ab(c+a)+ac(c+a)+b^2(c+a)+bc(c+a)-2abc$

$\Rightarrow x = (c+a)(ab+ac+b^2+bc)-2abc$

$= (c+a)\{a(b+c)+b(b+c)\}-2abc$

$= (a+b)(b+c)(c+a)-2abc$

$\Rightarrow x = (a+b)(b+c)2b-2abc$ $[\because a+c=2b]$

$= 2ab\{(a+b)(b+c)-ac\}$

$= 2b(ab+ac+b^2+bc-ac)$

$= 2b(ab+b^2+bc) = 2b^2(a+b+c)$

Since, a, b, c are in AP, $a+b+c = b+2b = 3b$

and $b = \frac{a+b+c}{3}$

$\Rightarrow 2b^2(a+b+c) = 2.\left(\frac{a+b+c}{3}\right)^2(a+b+c)$

$\Rightarrow x = \frac{2}{9}(a+b+c)^3$

3. Let the digit in the unit's place be $a-d$

Digit in the ten's place = a and the digit in the hundred's place be $a+d$

Sum of digits $= (a-d)+a+(a+d) = 3a$

Also sum = 15 (Given)

$\therefore$ $3a = 15 \Rightarrow a = 15/3 = 5$

Original number $= (a-d)+10a+100(a+d)$

$= 111a+99d = 111\times5+99d = 555-99d$

Number formed by reversing the digits

$= (a+d)+10a+100(a-d)$

$= 111a+99d = 111\times5-99d = 555-99d$

$\therefore (555+99d)-(555-99d) = 594$

$\Rightarrow 198d = 594 \Rightarrow d = 594 \div 198 = 3$

Thus the digit in the unit's place is 5 – 3 = 2, in the ten's place is 5 and in the hundred's place is 5 + 3 = 8

Hence the number is 852

4. $\therefore$ The required ratio is 1 : 2

5. $S_{pq} = \frac{pq}{2}\left[\frac{2}{pq}+1-\frac{1}{pq}\right] = \frac{pq}{2}\left[\frac{pq+1}{pq}\right] = \frac{1}{2}(pq+1)$

6. Let S_{n_1} be the sum of n terms of I[st] A.P.

and S_{n_2} be the sum of n terms of II[nd] A.P.

Given that the sum of n terms of two arithmetic series is in the ratio $2n+3 : 6n+5$

$\Rightarrow \frac{S_{n_1}}{S_{n_2}} = \frac{2n+3}{6n+5}$ .....(i)

From Eq. (i), we get

$$\frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{2n+3}{6n+5}$$

For $a = 13$, $n = 2a-1 = 2\times13-1 = 25$

$\therefore \frac{2a_1+(25-1)d_1}{2a_2+(25-1)d_2} = \frac{53}{155} \Rightarrow \frac{a_1+12d_1}{a_2+12d_2} = \frac{53}{155}$

Note : 13[th] term of A.P. $= T_{13} = a+(12-1)d$

Therefore, the ratio of their 13[th] terms is 53 : 155

7. 76 or 20;

Hint: $(a+2d)+(a+6d) = 6 \Rightarrow a = 3-4d$,

also $(a+2d)(a+6d) = 8 \Rightarrow a^2+8ad+12d^2 = 8$

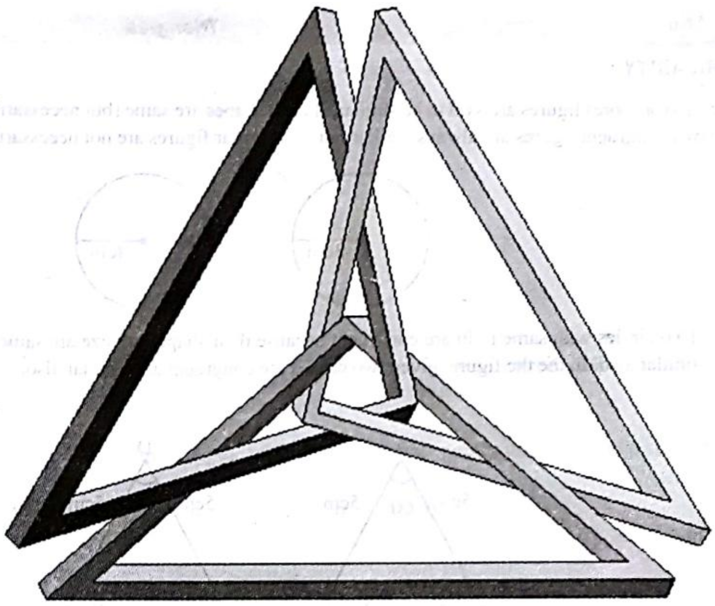
$S_{16} = \frac{16}{2}\left[2\times1+15\times\frac{1}{2}\right]$ or $\frac{16}{2}\left[5\times2+15\times\left(-\frac{1}{2}\right)\right]$

8. 35,

Hint: $S_{m-1} = S_{49}-S_m \Rightarrow S_{m-1}+S_m = S_{49}$

9. $S_n = a\left(\frac{r^n-1}{r-1}\right) \Rightarrow S_n = \frac{16}{7}\left(\frac{2^n-1}{2-1}\right) = \frac{16}{7}(2^n-1)$

chapter 6



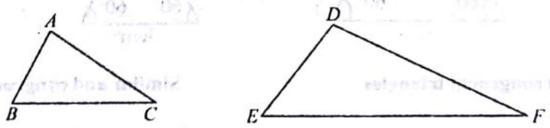
TRIANGLES

Introduction

In class IX, you have studied the congruency of two triangles. Two triangles are said to be congruent if their shape and size are same. In two congruent triangles, each angle and side of one triangle is equal to their corresponding angle and side of other triangle respectively. In this chapter, you will study the similarity of two triangles and some important theorems related to similarity of two triangles.

Two triangles having the same shape (but not necessarily the same size) are called Similar Triangles.

In two similar triangles, each angle of one triangle is equal to the corresponding angle of other triangle but corresponding sides of two triangles are proportional. The sign '~' is used to represent the similarity of two triangles. $\Delta ABC \sim \Delta DEF$ means ΔABC similar to ΔDEF.



Hence by definition of similarity of two triangles,

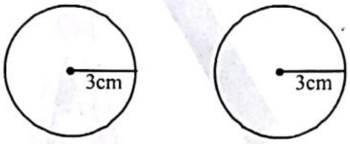
$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F \text{ and } \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Similarity of two triangles is used in proof of some theorems and solving many problems.

SIMILARITY :

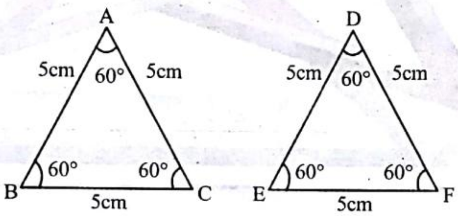
Any two (or more) figures are said to be similar, if their shapes are same (but necessarily the same size) are called similar. Any two congruent figures are always similar but two similar figures are not necessarily congruent. For example

(i)



Two circles with same radii are congruent because their shape and size are same. Since their shapes are same, hence they are similar also. In the the figure, given two circles are congruent and similar also.

(ii)



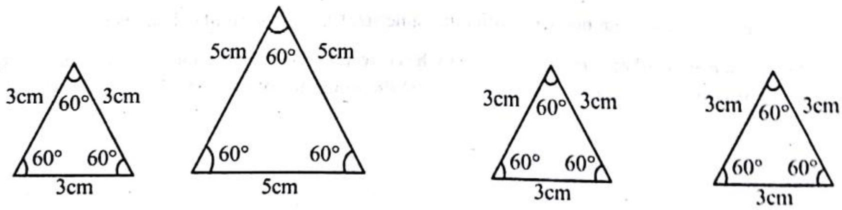
Two equilateral triangles with equal side length are congruent because their shape and size are same. Since their shapes are same, hence they are similar also. In the figure, given two triangle ΔABC and ΔDEF are congruent and similar also.

(iii)

Similar but not congruent circles **Similar and congruent circles**

Two circles are always similar because their shapes are same. But they may or may not be congruent because their size depends on their radius and their radius may or may not be equal. If their radii are equal then they are congruent also, other wise they are not congruent.

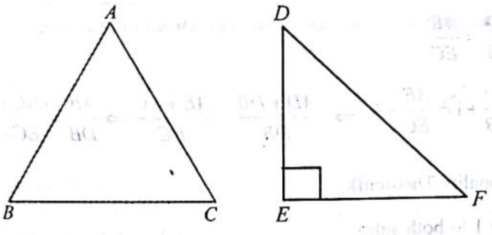
(iv)



Similar but not congruent triangles **Similar and congruent triangles**

Two equilateral triangles are always similar because their shapes are always same but may or may not be congruent because their sizes depend on length of their sides. But length of their sides may or may not be equal. If length of their sides are equal then they are congruent also, otherwise they are not congruent.

Some figures are always neither congruent nor similar. For examples : As acute angled triangle and a right angle triangle are always neither congruent not similar.

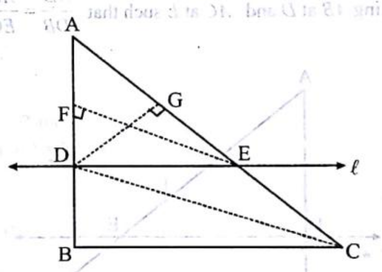


In figure, acute angled ΔABC and right angled ΔDEF are neither similar nor congruent.

BASIC PROPORTIONALITY THEOREM :

Statement : In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.

Given : In ΔABC, ℓ is drawn parallel to BC which intercepts AB and AC at D and E respectively.



To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw $EF \perp AB$ and DG lines AC

Proof : ΔDBE and ΔCDE are on the same base DE and between the same parallel lines DE and BC.

$\therefore$ Area (ΔBDE) = Area (ΔCDE) .... (1)

Now ΔADE and ΔBDE have the bases AD and DB are on the same straight line AB and their opposite vertices is also same i.e. point E, hence the height of both triangles are EF.

$$\therefore \quad \frac{Area\ (\Delta ADE)}{Area\ (\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} \quad \text{.......... (2)}$$

$$\text{Similarly} \quad \frac{Area\ (\Delta ADE)}{Area\ (\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \quad \text{.......... (3)}$$

Hence from (1), (2) and (3), we get $\frac{AD}{DB} = \frac{AE}{EC}$

Corollary : In ΔABC, DE is parallel to BC and intersects AB and AC at D and E respectively, then

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ and (ii) $\frac{AB}{AD} = \frac{AC}{AE}$

Proof :

(i) By proportionality Theorm $\frac{AD}{DB}=\frac{AE}{EC}$

On adding 1 to both sides $\frac{AD}{DB}+1=\frac{AE}{EC}+1 \Rightarrow \frac{AD+DB}{DB}=\frac{AE+EC}{EC} \Rightarrow \frac{AB}{DB}=\frac{AC}{EC}$

(ii) $\frac{AD}{DB}=\frac{AE}{EC}$ (By basic proportionality Theorem)

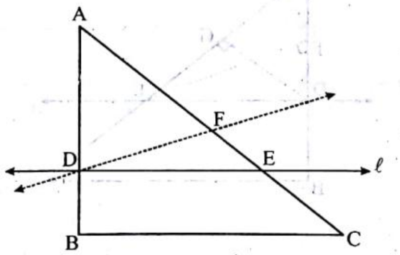
Taking inverse and then adding 1 to both sides

$1+\frac{DB}{AD}=1+\frac{EC}{AE}$ or $\frac{AD+DB}{AD}=\frac{AE+AC}{AE}$ or $\frac{AB}{AD}=\frac{AC}{AE}$

CONVERSE OF BASIC PROPORTIONALITY THEOREM :

Statement : If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third line.

Given : A triangle ABC and line ℓ intersecting AB at D and AC at E such that $\frac{AD}{DB}=\frac{AE}{EC}$



To prove : $DE \parallel BC$

Proof : Let us suppose that DE is not parallel to BC. Then, through D there must be some other line DF (let) parallel to BC. Since $DF \parallel BC$, by basic proportionality theorem, we get

$\frac{AD}{DB}=\frac{AF}{FC}$ ....(1)

But, $\frac{AD}{DB}=\frac{AE}{EC}$ (given) ....(2)

From (1) and (2), $\frac{AF}{FC}=\frac{AE}{EC}$

On adding 1 to both sides

$\frac{AF}{FC}+1=\frac{AE}{EC}+1 \Rightarrow \frac{AF+FC}{FC}=\frac{AE+EC}{EC} \Rightarrow \frac{AC}{FC}=\frac{AC}{EC}$. Hence, $FC=EC$

But this is impossible unless the points F and E coincide. i.e., DF and DE are coincident lines.

Hence, $DE \parallel BC$

SIMILARITY OF TWO TRIANGLES :

Two triangles are said to be similar if their shape and size are same. But the shape and size of two triangles are same only if each angle of one triangle is equal to the corresponding angle of other triangle and ratio of corresponding sides of two triangles are also equal.

Note: Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).

THEOREM : ANGLE-ANGLE-ANGLE SIMILARITY

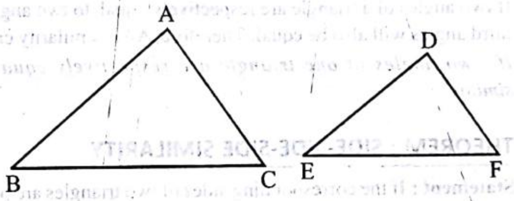
Statement : In two triangles, if the corresponding angles are equal then their corresponding sides are in the same ratio (or proportion) and hence two triangles are similar.

OR, Two equiangular triangles are similar.

Given : ΔABC and ΔDEF are equiangular.

Hence $\angle A = \angle D, \angle B = \angle F$ and $\angle C = \angle F$

To prove : $\Delta ABC \sim \Delta DEF$



Proof : Here, ΔABC and ΔDEF are equiangular,

i.e. $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$ .... (1)

Three cases arises for sides AB of ΔABC and DE of ΔDEF :

(i) $AB = DE$ (ii) $AB > DE$ (iii) $AB < DE$

Case (1) : When $AB = DE$

Proof : In ΔABC and ΔDEF

$\angle A = \angle D$ (Given)

$AB = DE$ (Given)

$\angle B = \angle E$ (Given)

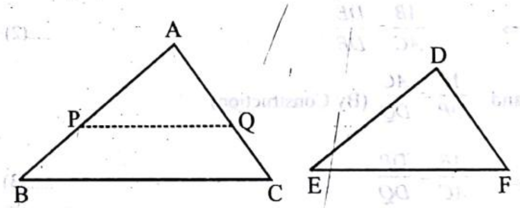
Then by ASA rule of congruence, $\Delta ABC \cong \Delta DEF$

Therefore $BC = EF,\ AC = DF,\ AB = DE$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \Delta ABC \sim \Delta DEF$$

Case (2) : When $AB > DE$

Construction : As in figure, taking the point P and Q on side AB and AC such that $AP = DE$ and $AQ = DF$.



Proof : In ΔAPQ and ΔDEF

$AP = DE$ (By Construction)

$AQ = DF$ (By Construction)

$\angle A = \angle D$ (Given)

Therefore by Side-Angle-Side Rule for congruency

$\Delta APQ \cong \Delta DEF$

So, $\angle APQ = \angle E$ .... (1)

But $\angle B = \angle E$ (Given) .... (2)

$\Rightarrow$ $\angle APQ = \angle B$, which is corresponding angle

Consequently, $PQ \parallel BC$

Hence $\frac{AP}{AB} = \frac{AQ}{AC}$ (By Basic Proportionality Theorem)

$$\Rightarrow \frac{AP}{AQ} = \frac{AB}{AC} \quad .... (3)$$

Also, $\frac{AP}{DE} = \frac{AQ}{DF}$ (By Construction)

$$\Rightarrow \frac{AP}{AQ} = \frac{DE}{DF} \quad ..... (4)$$

From (3) and (4),

$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} = \frac{AB}{DE} = \frac{AC}{DF} \quad .... (5)$$

Similarly, $\frac{AB}{DE} = \frac{BC}{EF}$ .... (6)

From (5) and (6), we get, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Hence $\Delta ABC \sim \Delta DEF$

Case (3) : When $AB < DE$. Proof is the same as for case (2).

Taking points P and Q on the side DE and DF respectively one can prove $\Delta ABC \sim \Delta DEF$

Corollary : (AA similarity) :

If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

THEOREM : SIDE-SIDE-SIDE SIMILARITY

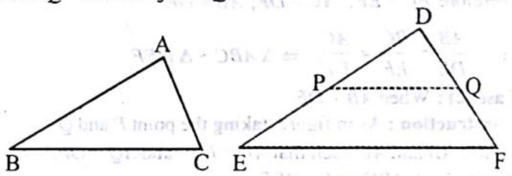
Statement : If the corresponding sides of two triangles are proportional (i.e., in the same ratio), their corresponding angles are equal and hence the two triangles such that are similar.

Given: ΔABC and ΔDEF

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}<1$$

To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on DF such that $DP = AB$ and $DQ = AC$ then join PQ.



Proof : In ΔABC and ΔDEF, $\frac{AB}{DE}=\frac{AC}{DF}=\frac{BC}{EF}$ ...... (1)

$\Rightarrow \quad \frac{AB}{AC}=\frac{DE}{DF}$ .....(2)

and $\frac{AB}{DP}=\frac{AC}{DQ}$ (By Construction)

$\Rightarrow \quad \frac{AB}{AC}=\frac{DP}{DQ}$ ......(3)

From (2) and (3), $\frac{DE}{DF}=\frac{DP}{DQ}=\frac{DP}{DE}=\frac{DQ}{DF}$

Therefore, by basic proportionality theorem, $PQ \parallel EF$

So $\angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$ (corresponding angles)

Hence by AA similarity, $\Delta DPQ \sim \Delta DEF$ .... (4)

Hence the corresponding sides of similar triangles ΔDPQ and ΔDEF are proportional.

i.e., $\frac{DP}{DE}=\frac{PQ}{EF} \Rightarrow \frac{AB}{DE}=\frac{PQ}{EF}$ .... (5)

From (1) and (5), $\frac{PQ}{EF}=\frac{BC}{EF} \Rightarrow PQ = BC$ .... (6)

Now, in ΔABC and ΔDPQ

$AB = DP$ (By Construction)

$AC = DQ$ (By Construction)

$BC = PQ$ [From (6)]

So by SSS congruence rule

$\Delta ABC \cong \Delta DPQ$ .... (7)

From (4) and (7)

$\Delta ABC \sim \Delta DPQ \sim \Delta DEF \Rightarrow \Delta ABC \sim \Delta DEF$

THEOREM : SIDE-ANGLE-SIDE SIMILARITY

Statement : If one angle of one triangle is equal to an angle of other triangle and if the sides including the angles are proportional, then the two triangles are similar.

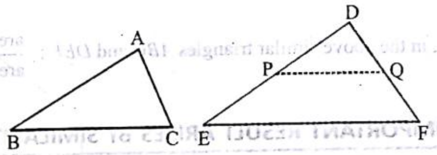
Given: ΔABC and ΔDEF, such that

$\angle A = \angle D$

and $\frac{AB}{DE} = \frac{AC}{DF} < 1$

To prove: $\Delta ABC \sim \Delta DEF$

Construction: Taking points P on DE and Q on sides DE and DF respectively such that $AB = DP$ and $AC = DQ$, join PQ.



Proof: In ΔABC and ΔDPQ

$AB = DP$ (By Construction)

$AC = DQ$ (By Construction)

$\angle A = \angle D$ (Given)

By SAS rule of congruence

$\Delta ABC \cong \Delta DPQ$ ....(1)

$\frac{AB}{DP} = \frac{AC}{DQ}$ (Given) ....(2)

and $\frac{AB}{DE} = \frac{AC}{DF}$ (By Construction) ....(3)

From (2) and (3), $\frac{DP}{DE} = \frac{DQ}{DF}$

By converse of basic Proportionality theorem

$PQ \parallel EF$

So $\angle DPQ = \angle E$ and $\angle DQP = \angle F$ (corresponding angles)

Consequently, by AA similarity, $\Delta DPQ \sim \Delta DEF$ ....(4)

From (1) and (4), we get, $\Delta ABC \sim \Delta DPQ \sim \Delta DEF$

$\Rightarrow$ $\Delta ABC \sim \Delta DEF$

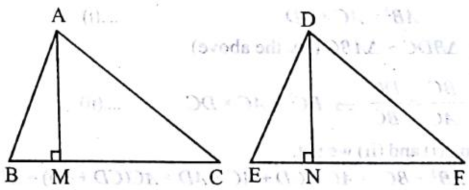
THEOREM : RELATION BETWEEN AREAS OF TWO SIMILAR TRIANGLES

Statement : The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

Given : $\Delta ABC \sim \Delta DEF$

To prove : $\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{CA^2}{FD^2}$

Construction : Draw $AM \perp BC$ and $DN \perp EF$



Proof : In ΔAMB and ΔDNE,

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

$\angle M = \angle N = 90°$ [By construction]

$\Rightarrow$ $\Delta AMB \sim \Delta DNE$ [By AA similarity]

$\therefore$ $\frac{AM}{DN} = \frac{AB}{DE}$

= Ratio of corresponding sides of two similar triangles

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore$ $\frac{AM}{DN} = \frac{BC}{EF}$

$\therefore$ $\frac{\text{area } \Delta ABC}{\text{area } \Delta DEF} = \frac{1/2.BC.AM}{1/2.EF.DN} = \left(\frac{BC}{EF}\right)$

[Area of a $\Delta = \frac{1}{2}$ base $\times$ ht.]

= Ratio of corresponding sides of two similar triangles

$\therefore \quad \dfrac{\text{area } \Delta\, ABC}{\text{area } \Delta\, DEF} = \dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{CA^2}{FD^2}$

$\left[\left[\dfrac{BC}{EF} = \dfrac{AB}{DE} = \dfrac{CA}{FD}\right]\right.$, From eq. (i)]

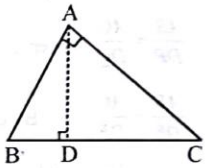
Thus, in the above similar triangles ABC and DEF: $\dfrac{\text{area } \Delta\, ABC}{\text{area } \Delta\, DEF} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$

AN IMPORTANT RESULT ARISES BY SIMILARITY OF TRIANGLES :

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Thus in the above figure, $\Delta\, ABC \sim \Delta\, DBA \sim \Delta\, DAC$

Let us now apply this theorem in proving the Pythagoras Theorem:



PYTHAGORAS THEOREM :

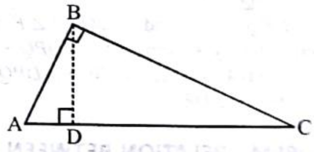
Statement : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : A triangle ABC right angled in at B.

Prove that : $AC^2 = AB^2 + BC^2$

(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

Construction : From B, draw $BD \perp AC$.



Proof: Since $BD \perp AC$.

$\therefore \quad \Delta ADB \sim \Delta ABC$ (By the above)

$\therefore \quad \dfrac{AD}{AB} = \dfrac{AB}{AC}$

$\Rightarrow \quad AB^2 = AC \times AD \qquad$ ....(i)

Also, $\Delta BDC \sim \Delta ABC$ (By the above)

$\therefore \quad \dfrac{BC}{AC} = \dfrac{DC}{BC} \Rightarrow BC^2 = AC \times DC \qquad$ ....(ii)

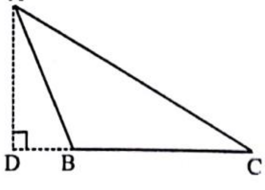
Adding (i) and (ii) we get,

$AB^2 + BC^2 = AC \times CD + AC \times AD = AC(CD + AD) = AC \times AC \,(\because\, CD + AD = AC)$

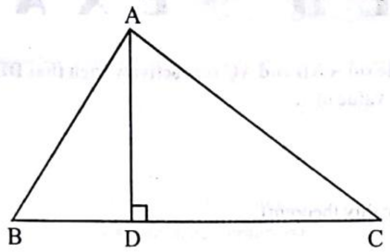
$AB^2 + BC^2 = AC^2$

SOME IMPORTANT RESULTS DEDUCED FROM PYTHAGORAS THEOREM :

(a) In the given $\Delta\, ABC$, obtuse angled at B. If $AD \perp CB$ produced, then $AC^2 = AB^2 + BC^2 + 2BC.BD$



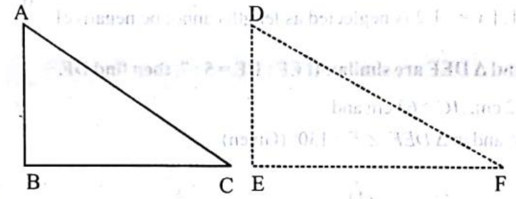
(b) In the given figure, if $\angle B$ of ΔABC is an acute angle and $AD \perp BC$, then
$AC^2 = AB^2 + BC^2 - 2BC.BD$



CONVERSE OF PYTHAGORAS THEOREM :

Statement : In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$



To prove : $\angle B = 90°$

Construction : Construct a triangle DEF such that $DE = AB$, $EF = BC$ and $\angle E = 90°$

Proof : Since Δ DEF is a right-angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have :

$DF^2 = DE^2 + EF^2$

$\Rightarrow$ $DF^2 = AB^2 + BC^2$

[$\because$ $DE = AB$ and $EF = BC$ (By construction)]

$\Rightarrow$ $DF^2 = AC^2$ [$\because$ $AB^2 + BC^2 = AC^2$ (Given)]

$\Rightarrow$ $DF = AC$ .... (1)

Thus, in ΔABC and ΔDEF, we have

$AB = DE,\ BC = EF$ [By construction]

and $AC = DF$ [From eq. (1)]

$\therefore$ $\Delta ABC \cong \Delta DEF$

[By SSS criteria of congruency]

$\Rightarrow$ $\angle B = \angle E = 90°$

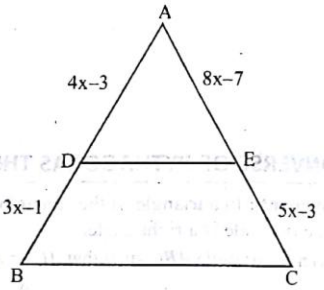
Hence ΔABC is right angled at B.

SOME IMPORTANT RESULTS AND THEOREMS :

1. The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
2. In a triangle ABC, if D is a point on BC such that D divides BC in the ratio $AB:AC$, then AD is the bisector of $\angle A$.
3. The external bisector of an angle of a triangle divides the opposite sides externally in the ratio of the sides containing the angle.
4. The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.
5. The line joining the mid-points of two sides of a triangle is parallel to the third side.
6. Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.
7. If three or more parallel lines are intersected by two transversals, then the intercepts made by them on the transversals are proportional.

MISCELLANEOUS SOLVED EXAMPLES

1. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, find the value of x.



Sol. In ΔABC, we have $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ (By basic proportionality theorem)

$\Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3} \Rightarrow 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$\Rightarrow \quad 20x^2 - 27x + 9 = 24x^2 - 29x + 7 \Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow \quad 2x^2 - x - 1 = 0 \Rightarrow (2x+1)(x-1) = 0 \Rightarrow x = 1 \text{ or } x = -1/2$

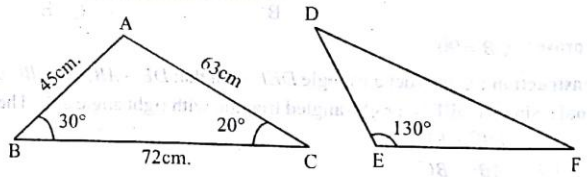
So, the required value of x is 1. [$x = -1/2$ is neglected as length cannot be negative]

2. In figure, prove that ΔABC and Δ DEF are similar. If EF : DE = 5 : 7, then find DF.

Sol. In ΔABC, $AB = 45$ cm, $BC = 72$ cm, $AC = 63$ cm and

$\angle A = 180° - (20° + 30°) = 130°$ and in ΔDEF, $\angle E = 130°$ (Given)

$\frac{AB}{AC} = \frac{45}{63} = \frac{5}{7}$ and $\frac{EF}{DE} = \frac{5}{7}$ .... (1)



Now for ΔABC and ΔDEF,

$\angle A = \angle E = 130°$ and $\frac{AB}{AC} = \frac{EF}{DE}$ [by eq. (1)]

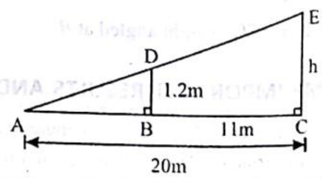
By SAS rule of congruency

$\Delta ABC \sim \Delta EFD \quad \Rightarrow \quad \angle B = \angle F = 30°$ and $\angle D = \angle C$

and $\frac{AB}{EF} = \frac{BC}{DF} \quad \Rightarrow \quad DF = \frac{BC \times EF}{AB} = \frac{72 \times 5}{45} = 8cm$

3. A child 1.2 meters tall is standing 11 meters away from a tall building. A spotlight on the ground is located 20 meters away from the building and shines on the wall. How tall is the child's shadow on the building?

Sol. Let h be the height of the shadow on the building. Then draw a diagram assuming the ground to be flat, as in the diagram. There are two triangles: one formed by the spotlight and the child, and one formed by the spotlight and the height of the shadow, h. These two triangles share a common angle A at the spotlight. If we assume that the child and the wall of the building are perpendicular to the ground, then the angle formed by the child and the ground (angle C) are both right angles. So the triangles have another pair of equal angles. Therefore, the triangles are similar.



Now we must look at the lenghts of the corresponding sides. We know that the child must be 9 meters from the spotlight (i.e. 20 m- 11 m). This length in the smaller triangle corresponds to the distance from the spotlight to the building in the larger triangle (i.e. 20 m). The height of the child in the smaller triangle (1.2 m) corresponds to the height of the shadow in the larger triangle (h). Since the triangles are similar, these lengths are in proportion.

Therefore: $\frac{9}{20} = \frac{1.2}{h}$,

$9h = 20(1.2)$

$h = 24/9 = 8/3 = 2.67$ meters

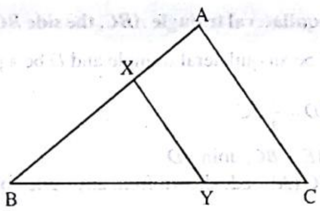
The height of the shadow is 8/3 meters (approx. 2.67 meters).

4. In figure, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$.

Sol. We have, $XY \parallel AC$ (Given)

So, $\angle BXY = \angle A$ and $\angle BYX = \angle C$ (Corresponding angles)

Therefore, $\Delta ABC \sim \Delta XBY$ (AA similarity criterion)



So, $\frac{ar(ABC)}{ar(XBY)} = \left(\frac{AB}{XB}\right)^2$ .... (1)

Also, $ar(ABC) = 2\, ar(XBY)$ (Given)

So, $\frac{ar(ABC)}{ar(XBY)} = \frac{2}{1}$ .... (2)

Therefore, from (1) and (2),

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}, \text{ i.e., } \frac{AB}{XB} = \frac{\sqrt{2}}{1} \quad \text{or} \quad \frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

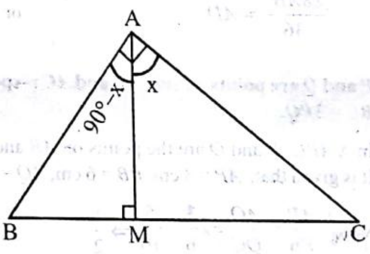
$$\text{or} \quad 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} \text{ i.e., } \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

5. From the diagram, prove that $\Delta ABM \sim \Delta AMC \sim \Delta ABC$.

Sol. Let $\angle B = x$

$\angle BAM = 90 - x\ [\angle x + \angle BAM = 90°]$

$\Rightarrow \angle MAC = x$ $[\angle BAM + \angle MAC = 90°]$



In ΔABM and ΔAMC:

$\angle B = \angle MAC = x$

$\angle M = \angle M = 90°$ [Given]

$\Rightarrow \Delta MBA \sim \Delta MAC$ [A.A.A.]

$\Rightarrow \frac{\Delta ABM}{\Delta AMC} = \frac{AB^2}{AC^2}$

In ΔAMB and ΔABC:

$\angle B = \angle B$ (Common)

$\angle AMB = \angle BAC = 90°$ [Given]

$\Rightarrow \Delta MBA \sim \Delta ABC$ [A.A.A.]

$\Rightarrow \frac{\Delta AMB}{\Delta ABC} = \frac{AM^2}{AC^2}$

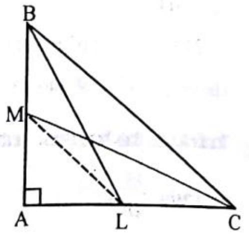
6. BL and CM are medians of ΔABC right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.

Sol. In ΔBAL, $BL^2 = AL^2 + AB^2$ (using Pythagoras theorem) ..... (1)

and, in ΔCAM

$CM^2 = AM^2 + AC^2$ (using Pythagoras theorem) ..... (2)

Adding (1) and (2) and then multiplying by 4, we get



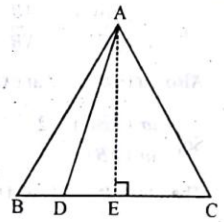
$$4(BL^2 + CM^2) = 4(AL^2 + AB^2 + AM^2 + AC^2)$$
$$= 4\{AL^2 + AM^2 + (AB^2 + AC^2)\} \quad [\because \Delta ABC \text{ is a right triangle}]$$
$$= 4(AL^2 + AM^2 + BC^2)$$
$$= 4(ML^2 + BC^2) \quad [\because \Delta LAM \text{ is a right triangle}]$$
$$= 4ML^2 + 4BC^2$$

(A line joining mid-points of two sides is parallel to the third side and is equal to

$$= BC^2 + 4BC^2 = 5BC^2 \quad (\text{half of it, } ML = BC/2)$$

7. In an equilateral traingle ABC, the side BC is trisected at D. Prove that $9AD^2 = 7AB^2$.

Sol. ΔABC be an quilateral triangle and D be a point on BC such that



$$BD = \frac{1}{3}BC \qquad \text{(Given)}$$

Draw $AE \perp BC$, Join AD

$BE = EC$ (Altitude drown from any vertex of an equilateral triangle bisects the opposite side)

So, $BE = EC = \dfrac{BC}{2}$

In ΔABC, $AB^2 = AE^2 + EB^2$ ....(1)

$AD^2 = AE^2 + ED^2$ ....(2)

From (1) and (2)

$$AB^2 = AD^2 - ED^2 + EB^2$$

$$AB^2 = AD^2 - \frac{BC^2}{36} + \frac{BC^2}{4} \qquad \left(\because BD + DE = \frac{BC}{2} \Rightarrow \frac{BC}{3} + DE = \frac{BC}{2} \Rightarrow DE = \frac{BC}{6}\right)$$

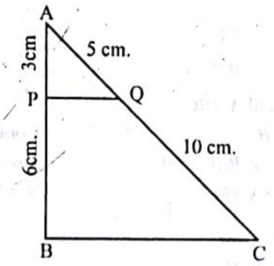
$$AB^2 + \frac{BC^2}{36} - \frac{BC^2}{4} = AD^2 \qquad \text{or} \qquad \frac{36AB^2 + AB^2 - 9AB^2}{36} = AD^2$$

$$\frac{28AB^2}{36} = AD^2 \qquad \text{or} \qquad 7AB^2 = 9AD^2$$

8. P and Q are points on side AB and AC respectively of ΔABC. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm. and $QC = 10$cm, show that $BC = 3PQ$.

Sol. In ΔABC, P and Q are the points on AB and AC.

It is given that, $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm.



Now, $\dfrac{AP}{PB} = \dfrac{AQ}{QC} \Rightarrow \dfrac{3}{6} = \dfrac{5}{10} \Rightarrow \dfrac{1}{2}$

Hence $PQ \parallel BC$

In ΔAPQ and ΔABC

$\angle P = \angle B$ [Corresponding angles]

$\angle Q = \angle C$ [Corresponding angles]

$\angle A = \angle A$ [Common angle]

$\Rightarrow \Delta APQ \sim \Delta ABC$ [AAA Similarity]

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

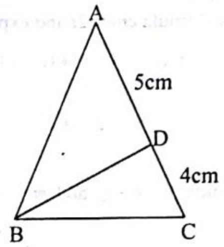
$$\Rightarrow \frac{3}{9} = \frac{5}{15} = \frac{PQ}{BC} \qquad [\because AB = 3 + 6 = 9 \text{ cm.}, AC = 5 + 10 = 15 \text{ cm}]$$

$$\Rightarrow \frac{1}{3} = \frac{PQ}{BC} \Rightarrow BC = 3PQ$$

9. In $\Delta ABC, AB = AC$ and $BC = 6$ cm. D is a point on side AC such that $AD = 5$ cm and $CD = 4$ cm. Show that $\Delta BCD \sim \Delta ACB$ and hence find BD.

Sol. Consider ΔABC and ΔBCD.

It is given that $AB = AC$, $BC = 6$ cm, $AD = 5$ cm and $CD = 4$ cm.



Then, $\frac{BC}{AC} = \frac{6}{5+4} = \frac{6}{9} = \frac{2}{3}$ and $\frac{CD}{AB} = \frac{4}{6} = \frac{2}{3}$

$\therefore \quad \frac{BC}{AC} = \frac{CD}{CB}$

Also, $\angle BCD = \angle ACB$ (common)

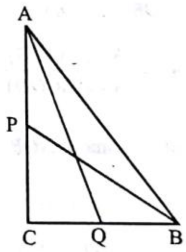
$\therefore \quad \Delta BCD \sim \Delta ACB$ (SAS similarly)

$\therefore \quad \frac{BD}{AB} = \frac{CD}{CB} = \frac{2}{3}$ $\quad \therefore \quad \frac{BD}{AC} = \frac{2}{3}$ $(\because AB = AC)$

$\therefore \quad BD = \frac{2}{3}AC = \frac{2}{3}(5+4) = \frac{2}{3}\times 9 = 6cm.$

10. P and Q are the midpoints of the sides CA and CB respectively of a ΔABC in which C is a right angle. Prove that (i) $4AQ^2 = 4AC^2 + BC^2$ and (ii) $4(AQ^2 + BP^2) = 5AB^2$

Sol. Given $\angle C = 90°$, P is the midpoint of AC, Q is the midpoint of BC.



Proof : $AQ^2 = AC^2 + CQ^2$ (Pythagoras's throerem)

$$= AC^2 + \left(\frac{1}{2}BC\right)^2 = AC^2 + \frac{1}{4}BC^2$$

$\therefore \quad 4AQ^2 = 4AC^2 + BC^2$ .... (1)

Similarly, $4BP^2 = 4BC^2 + AC^2$ .... (2)

Adding (1) and (2),

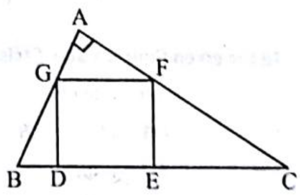
$$4AQ^2 + 4BP^2 = (4AC^2 + BC^2) + (4BC^2 + AC^2) = 5AC^2 + 5BC^2$$
$$= 5(AC^2 + BC^2)$$

$\Rightarrow \quad 4(AQ^2 + BP^2) = 5AB^2$ (Phythagoras's theorem)

11. ΔABC is right-angled at A. $DEFG$ is a square as shown in the figure. Prove that $DE^2 = BD \times EC$.

Sol. Given ΔABC is right-angled at . $DEFG$ is a square

To prove $DE^2 = BD \times EC$.



Proof : In ΔAGF and ΔDBG

$\angle GAF = \angle BDG = 90°$

$\angle AGF = \angle DBG$ (corrsp. angles)

$\therefore \quad \Delta AGF \sim \Delta DBG$ .... (i) (AA similarlity)

In ΔAGF and ΔEFC,

$\angle GAF = \angle CEF = 90°$

$\angle AFG = \angle FCE$ (corrsp. angles)

$\therefore \quad \Delta AFG \sim \Delta EFC$ .... (ii) (AA similarity)

From (i) and (ii)

$\Delta DBG \sim \Delta EFC$

$\therefore \quad \frac{DB}{EF} = \frac{DG}{EC}$, But $EF = DG = DE$ (sides of a square)

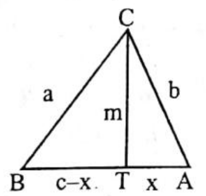
$\therefore \quad \frac{DB}{DE} = \frac{DE}{EC}$

$\therefore \quad DE^2 = DB \times EC$

12. Let a, b, c be side of Δ while t is its area, show that $a^2+b^2+c^2 \geq 4t\sqrt{3}$, when does equality holds ?

Sol. Suppose that the largest angle of the ΔABC is at C. The foot of the altitude m at C is T which is an inner point of the interval AB. Let x denote AT.

Now apply formula cm $= 2t$ and express a^2 and b^2 using the Pythagoras theorem.



$$a^2+b^2+c^2-4t\sqrt{3} = [m^2+(c-x)^2]+(m^2+x^2)+c^2-2\sqrt{3}\,cm$$

$$= 2c^2+2m^2+2x^2-2cx-2\sqrt{3} = \frac{1}{2}[(c-2x)^2+(c\sqrt{3}-2m)^2] \geq 0$$

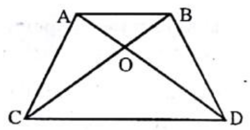
Equality holds if $x=\frac{c}{2}$ and $m=\frac{c\sqrt{3}}{2}$ i.e., Δ is equilateral.

13. $ABCD$ is a trapezium in which $AB \parallel CD$. The diagonal AC and BD intersect at O. Prove that
(i) $\Delta AOB \sim \Delta COD$ **(ii) If $OA = 6$ cm, $OC = 8$ cm.**

Find (a) $\frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)}$ (b) $\frac{\text{Area}(\Delta AOD)}{\text{Area}(\Delta COD)}$

Sol. (i) Let AC, BD meet at the point O.

In ΔAOB and ΔCOD.



$\angle AOB = \angle COD$ [Vertically opposite $\angle$s]

$\angle OAB = \angle OCD$ [Alternate $\angle$s]

$\Delta AOB \sim \Delta COD$ .... (i) [By $A.A.$ rule of similarity]

(ii) (a) $\therefore \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{AO^2}{CO^2} = \frac{6^2}{8^2} = \frac{36}{64}$

(b) Since $\Delta AOB \sim \Delta COD$ $\therefore \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD}$

In ΔAOD and ΔCOD,

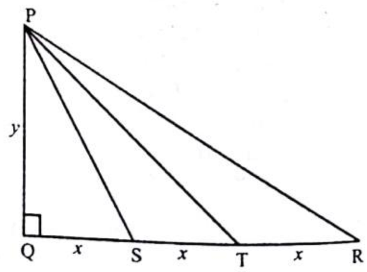
$\therefore \frac{AO}{CO} = \frac{OB}{OD}$ and $\angle AOD = \angle BOC$ (vertically opposite angles)

$\therefore \Delta AOD \sim \Delta COD$

$\therefore \frac{\text{Area}(\Delta AOD)}{\text{Area}(\Delta COD)} = \frac{AO^2}{CO^2} = \frac{36}{64}$

14. In the given figure, S and T trisect the side QR of a right triangle PQR. Prove that $8PT^2 = 3PR^2 + 5PS^2$

Sol. S and T trisect the side QR.



Let $QS = ST = TR = x$ units

Let $PQ = y$ units

In right ΔPQS, $PS^2 = PQ^2 + QS^2$ (By Pythagoras Theorem)

$= y^2 + x^2$ ...(1)

In right ΔPQR, $PT^2 = PQ^2 + QT^2$ (By Pythagoras Theorem)

$= y^2 + (2x)^2 = y^2 + 4x^2$ ...(2)

In right ΔPQR, $PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem)

$= y^2 + (3x)^2 = y^2 + 9x^2$ ...(3)

R.H.S. $= 3PR^2 + 5PS^2$

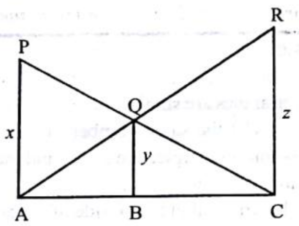
$= 3(y^2 + 9x^2) + 5(y^2 + x^2)$ [From (1) and (3)]

$= 3y^2 + 27x^2 + 5y^2 + 5x^2 = 8y^2 + 32x^2$

$= 8(y^2 + 4x^2) = 8PT^2 = $ L.H.S. [From (2)]

Thus $8PT^2 = 3PR^2 + 5PS^2$

15. In the given figure PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$



Sol. In ΔPAC and ΔQBC

$\angle PAC = \angle QBC$ [Each = 90°]

$\angle PCA = \angle QCB$ [Common]

$\therefore \Delta PAC \sim \Delta QBC$

$\therefore \frac{x}{y} = \frac{AC}{BC}$ i.e. $\frac{y}{x} = \frac{BC}{AC}$ ...(1)

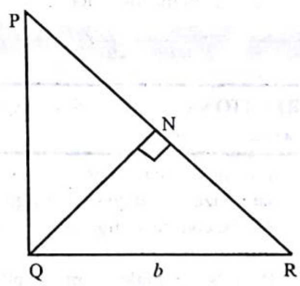
Similarly $\frac{z}{y} = \frac{AC}{AB}$ i.e. $\frac{y}{z} = \frac{AB}{AC}$ ...(2)

Adding (1) and (2), we get

$$\frac{BC + AB}{AC} = \frac{y}{x} + \frac{y}{z} = y\left(\frac{1}{x} + \frac{1}{z}\right)$$

$$\frac{AC}{AC} = y\left(\frac{1}{x} + \frac{1}{z}\right) \Rightarrow 1 = y\left(\frac{1}{x} + \frac{1}{z}\right) \Rightarrow \frac{1}{y} = \frac{1}{x} + \frac{1}{z}$$

16. If A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.



Sol. Let PQR be a right triangle right-angled at Q such that $QR = b$ and A = Area of ΔPQR

Draw QN perpendicular to PR.

We have, A = area of ΔPQR

$\Rightarrow PQ = \frac{2A}{b}$ ...(1)

Now, in Δ's PNQ and PQR, we have

$\angle PNQ = \angle PQR$ [Each equal to 90°]

and $\angle QPN = \angle QPR$ [Common]

So, by AA-criterion of similarity, we have $\Delta PNQ \sim \Delta PQR \Rightarrow \frac{PQ}{PR} = \frac{NQ}{QR}$

By Pythagoras theorem in ΔPQR, we have $PQ^2 + QR^2 = PR^2 \Rightarrow \frac{4A^2}{b^2} + b^2 = PR^2$

$$\Rightarrow PR = \sqrt{\frac{4A^2 + b^4}{b^2}} = \frac{\sqrt{4A^2 + b^4}}{b}$$

From (1) and (2) we have $\frac{2A}{b \times PR} = \frac{NQ}{b} \Rightarrow NQ = \frac{2A}{PR}$

$$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}} \qquad \left[\because PR = \frac{\sqrt{4A^2 + b^4}}{b}\right]$$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. All circles are ....
2. All squares are ....
3. All .... triangles are similar.
4. Two polygons of the same number of sides are similar, if their corresponding angles are .... and their corresponding sides are in the same ....
5. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the .... ratio.
6. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the .... side.
7. All congruent figures are similar but the similar figures need .... be congruent.
8. Two polygons of the same number of sides are similar, if all the corresponding angles are ....
9. The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$. $ABCD$ is a ....
10. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the .... side.
11. Line joining the mid-points of any two sides of a triangle is .... to the third side.

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.
4. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar
5. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar.
6. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar.
7. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
8. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
9. If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.
10. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O, $\frac{OA}{OC}=\frac{OB}{OD}$.
11. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. ΔABE is similar to ΔCFB.

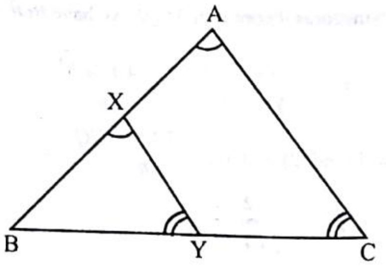
Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. If in a Δ ABC, DE $\parallel$ BC and intersects AB in D and AC in E, then match the column.

Column I	Column II
(A) $\frac{AD}{DB}$	(p) $\frac{AC}{AE}$
(B) $\frac{AB}{AD}$	(q) $\frac{AE}{EC}$
(C) $\frac{DB}{AB}$	(r) $\frac{AE}{AC}$
(D) $\frac{AD}{AB}$	(s) $\frac{EC}{AC}$

2. In figure, the line segment XY is parallel to the side AC of ΔABC and it divides the triangle into two parts of eual areas, then match the column.

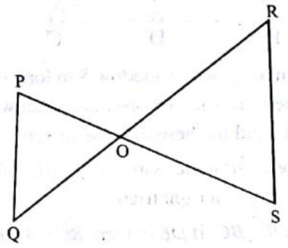


Column I	Column II
(A) $AB : XB$	(p) $\sqrt{2}:1$
(B) ar (ΔABC) : ar (ΔXBY)	(q) $2:1$
(C) $AX : AB$	(r) $(\sqrt{2}-1)^2 : \sqrt{2}$
(D) $\angle X : \angle A$	(s) $1:1$

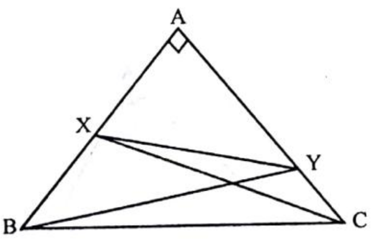
VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

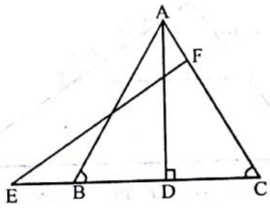
1. In Fig., if $PQ \parallel RS$, prove that $\Delta POQ \sim \Delta SOR$



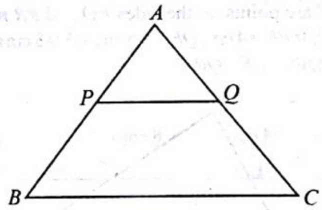
2. A clever outdoorsman whose eye-level is 2 meters above the ground, wishes to find the height of a tree. He places a mirror horizontally on the ground 20 meters from the tree, and finds that if he stands at a point C which is 4 meters from the mirror B, he can see the reflection of the top of the tree. How high is the tree?
3. A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.
4. From the adjoining figure, prove that $BC^2 + YX^2 = BY^2 + C^2$.



5. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Prove that $AB^2 = 2AC^2$, if ΔABC is an isosceles triangle right angled at C.
6. Two line segments AB and CD intersect at the point E such that $\Delta ACE \sim \Delta DBE$. If $AE = 4$ cm., $BE = 3$ cm, $CE = 2$cm and $DE = x$, find x.
7. The areas of two similar triangles ABC and PQR are in the ratio of $9:16$, If $BC = 4.5$ cm., find the length of QR.
8. In the given fig., E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$. Prove that $\Delta ABD \sim \Delta ECF$.



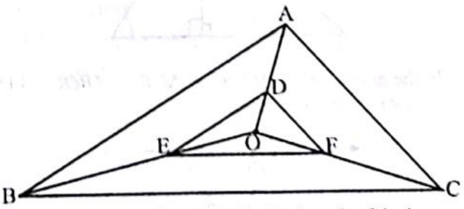
9. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm. find the corresponding median of the other.
10. The areas of two similar triangles are 81 cm^2 and 49 cm^2. If the altitude of the bigger is 4.5 cm. find the corresponding altitude of smaller triangle.
11. Given $\Delta ABC \sim \Delta DEF$. If $AB = 2DE$ and area of ΔABC is 56 cm^2. find the area of ΔDEF.
12. In a triangle ABC, $AD \perp BC$, If $AD^2 = BD.DC$, prove that ΔABC is rt. angle Δ.
13. In the given figure $\Delta ACB \sim \Delta APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, $AP = 2.8$ cm, then find AC.



SAQ Short Answer Questions :

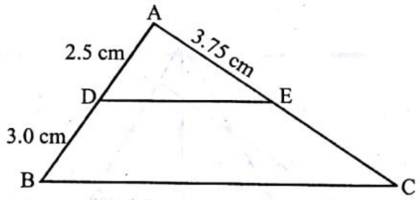
DIRECTIONS : *Give answer in 2-3 sentences.*

1. Any point O, inside ΔABC, is joined to its vertices. From a point D on AO, DE and DF are drawn so that $DE \parallel AB$ and $EF \parallel BC$ as shown in figure. Prove that $DF \parallel AC$.

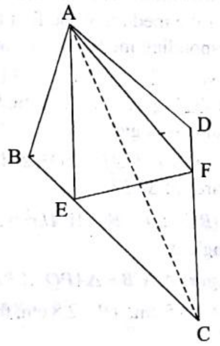


2. If in an isosceles triangle 'a' is the length of the base and 'b' is the length of one of the equal side, then prove that its area is $\frac{a}{4}\sqrt{4b^2 - a^2}$.

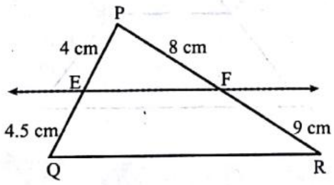
3. In a $\Delta\, ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $AD = 2.5$ cm, $DB = 3.0$ cm and $AE = 3.75$ cm, find the length of AC.



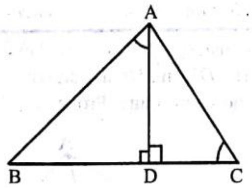
4. In figure, $AB = AD$. AE and AF are angle bisectors of $\angle BAC$ and $\angle DAC$. Prove that $BD \parallel EF$.



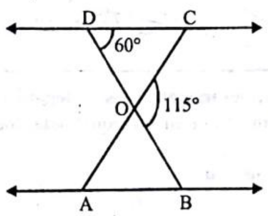
5. E and F are points on the sides PQ and PR respectively of a $\Delta\, PQR$. If $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm, state whether $EF \parallel QR$.



6. In triangles ABD and ADC, prove that $AD^2 = BD \,.\, DC$.

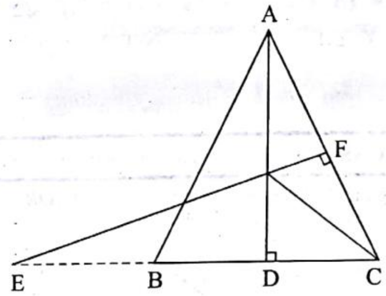


7. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 60°$, find $\angle OAB$.



8. In figure, E is a point on side CB produced of a isosceles triangle ABC with $AB = AC$.

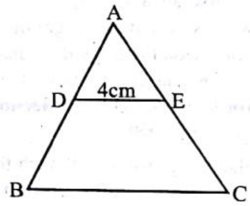
If $AD \perp BC$ and $EF \perp AC$ prove that $\Delta\, ABD \sim \Delta\, ECF$



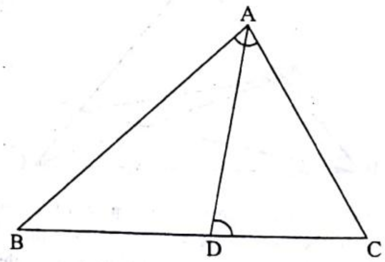
9. A vertical stick 12m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40m long on the ground. Find the height of the tower.

10. $\Delta\, ABC$ is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

11. In the given figure, $DE \parallel BC$. If $DE = 4$ cm, $BC = 8$ cm and area of $\Delta ADE = 25$ sq. cm, find the area of ΔABC.



12. In the given figure, D is a point on the side BC of $\Delta\, ABC$ such that $\angle\, ADC = \angle\, BAC$. Prove that $\dfrac{CA}{CD} = \dfrac{CB}{CA}$.



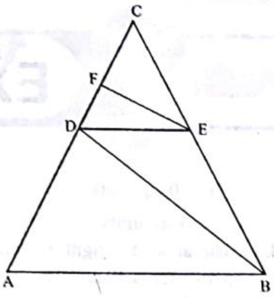
13. In the given figure, $ABCD$ is a trapezium in which $AB \parallel DC$. The diagonals AC and BD intersect at O. Prove that $\dfrac{AO}{OC} = \dfrac{BO}{DO}$.

14. In figure,

$AB \parallel DE$ and $BD \parallel EF$.

Prove that

$DC^2 = CF \times AC$.

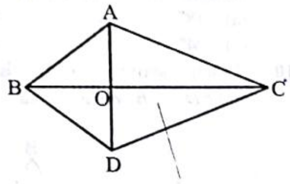


15. Let $\Delta ABC \sim \Delta DEF$ and their area be respectively 64 cm^2 and 121 cm^2. If $EF = 15.4$ cm, find BC.

16. In the given fig ABC and DBC are two triangles on the same base BC. If AD intersects BC at O.

Prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}.$$



Long Answer Questions :

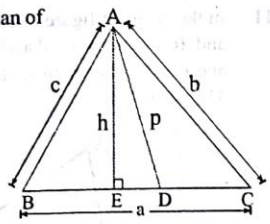
DIRECTIONS : *Give answer in four to five sentences.*

1. In a quadrilateral $ABCD$, diagonals intersect each other at O such that $\frac{AO}{OC} = \frac{BO}{OD}$. Prove that quadrilateral is a trapezium.

2. In figure, AD is the median of ΔABC and $AE \perp BC$.

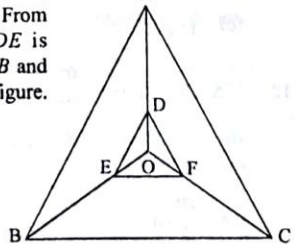
Prove that

$$b^2 + c^2 = 2p^2 + \frac{1}{2}a^2.$$



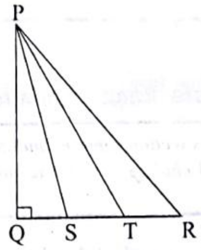
3. Prove that ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

4. Any point O, inside ΔABC, is joined to its vertices. From a point D on AO, DE is drawn so that $DE \parallel AB$ and $EF \parallel BC$ as shown in figure. Prove that $DF \parallel AC$.



5. Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals.

6. In figure, S and T trisect the side QR of a right triangle PQR, prove that $8PT^2 = 3PR^2 + 5PS^2$.

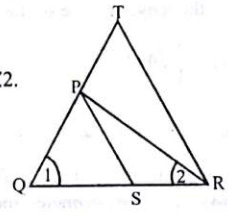


7. In adjoining figure,

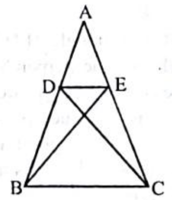
$\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$.

Prove that

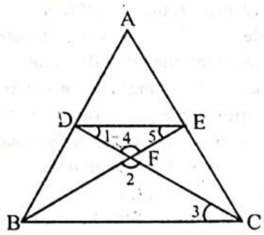
$\Delta PQS \sim \Delta TQR$



8. In adjoining figure if $\Delta ABE \cong \Delta ACD$, prove that $\Delta ADE \sim \Delta ABC$

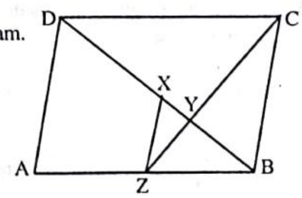


9. In below given Figure, $DE \parallel BC$ and $AD : DB = 5 : 4$. Find the ratio of areas of ΔDEF and ΔCFB.



10. In the picture, $ABCD$ is a parallelogram. AD is parallel to ZX and $\frac{AZ}{ZB}$ equals 2/3.

Then find $\frac{XY}{BD}$.



11. Through the mid-point M of the side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC at L and AD produced at E. Prove that $EL = 2BL$.

2 EXERCISE

MCQ Muliple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If in an isosceles triangle, 'a' is the length of the base and 'b' the length of one of the equal sides, then its area is –

(a) $\frac{a}{4}\sqrt{4b^2-a^2}$ (b) $\frac{b}{4}\sqrt{4b^2-a^2}$

(c) $\frac{a+b}{4}\sqrt{a^2-b^2}$ (d) $\frac{a-b}{4}\sqrt{b^2-a^2}$

2. If an equilateral triangle of area X and a square of area Y have the same perimeter, then –

(a) $X > Y$ (b) $X < Y$

(c) $X = Y$ (d) none of these

3. ABC is a triangle. If D is a point in the plane of the triangle such that the perpendicular distance from D to the three sides of the triangle are all equal, then there exist(s)–

(a) just one such point as D

(b) three such point as D

(c) four such points as D

(d) none of the above

4. ΔPSR is a triangle right angled at S. D is the mid-point of SR. If the bisector of $\angle PSR$ and perpendicular bisector of SR meet at O, then triangle ΔOSD is –

(a) isosceles (b) equilateral

(c) isosceles right angled (d) acute-angled

5. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include an angle equal to –

(a) half the sum of the base angles

(b) sum of the base angles

(c) half the difference of the base angles

(d) difference of the base angles

6. If x is the length of the median of an equilateral triangle, then its area is –

(a) x^2 (b) $\frac{\sqrt{3}}{2}x^2$

(c) $\frac{\sqrt{3}}{3}x^2$ (d) $\frac{1}{2}x^2$

7. In a triangle ΔABC, points P, Q and R are the mid-points of the sides AB, BC and CA respectively. If the area of the triangle ABC is 20 sq. units, then area of the triangle PQR equal to

(a) 10 sq. units (b) $5\sqrt{3}$ sq. units

(c) 5 sq. units (d) 5.5 sq. units

8. The area of a right angled triangle is 40 sq. cm. and its perimeter is 40 cm. The length of its hypotenuse is –

(a) 16 cm. (b) 18 cm.

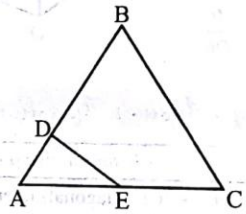
(c) 17 cm. (d) Data sufficient

9. An isoceles triangle has a 10 inch base and two 13 inch sides. What other value can the base have and still yield a triangle with the same area –

(a) 18″ (b) 19″

(c) 24″ (d) 27″

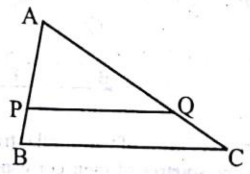
10. If each side of triangle ABC is of length 4 and if AD is 1 cm and $ED \perp AB$. What is area of region $BCDE$ –



(a) $8\sqrt{3}$ cm^2 (b) $4\sqrt{3}$ cm^2

(c) $4.5\sqrt{3}$ cm^2 (d) $3.5\sqrt{3}$ cm^2

11. In the adjacent figure, P and Q are points on the sides AB and AC respectively of a triangle ABC. PQ is parallel to BC and divides the triangle ABC into 2 parts, equal in area. The ratio of $PA : AB$ =



(a) $1:1$ (b) $(\sqrt{2}-1):\sqrt{2}$

(c) $1:\sqrt{2}$ (d) $(\sqrt{2}-1):1$

12. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(ABC)}{ar(PQR)} = \frac{9}{4}$, $AB = 18$ cm and $BC = 15$ cm; then PR is equal to

(a) 10 cm (b) 12 cm

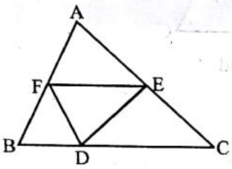
(c) $\frac{20}{3}$ cm (d) 8 cm

13. It is given that $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{3}$. Then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

(a) 9 (b) 3

(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

14. In triangle ΔABC, D, E, F are points of trisection of BC, AC and AB respectively. Which of the following statement is not true ?



(a) Area $\Delta EDC = 2/9$ area ΔABC

(b) Area $\Delta FBD = 2/7$ area ΔFDC

(c) Area $\Delta DEF = 2/9$ area ΔABC

(d) Area $(\Delta EDC + \Delta DBF + \Delta AFE) = 2$ area ΔDEF

15. The area of a right angled isosceles triangle whose hypotenuse is equal to 270 m is-

(a) $19000\, m^2$ (b) $18225\, m^2$

(c) $17256\, m^2$ (d) $18325\, m^2$

16. The perimeters of two similar triangles ABC and PQR are respectively 38 cm and 24 cm. If $PQ = 10$ cm, then $AB =$

(a) 10 cm (b) 20 cm

(c) 25 cm (d) 15 cm

17. A certain right angled triangle has its area numerically equal to its perimeter. The length of its each side is an even integer. What is the perimeter?

(a) 24 units (b) 36 units

(c) 32 units (d) 30 units

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which among the following is/are not correct?

(a) The ratios of the areas of two similar triangles is equal to the ratio of their corresponding sides.

(b) The areas of two similar triangles are in the ratio of the corresponding altitudes.

(c) The ratio of area of two similar triangles are in the ratio of the corresponding medians.

(d) If the areas of two similar triangles are equal, then the triangles are congruent.

2. Which among the following is/are correct?

(I) If the altitudes of two similar triangles are in the ratio 2 : 1, then the ratio of their areas is 4 : 1.

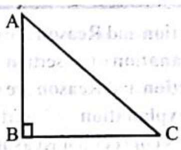
(II) $PQ \parallel BC$ and $AP : PB = 1 : 2$. Then, $\frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{1}{4}$

(III) The areas of two similar triangles are respectively $9 cm^2$ and $16 cm^2$. The ratio of their corresponding sides is 3 : 4.

(a) I (b) II

(c) III (d) None of these

3. In a right angled triangle ΔABC, length of two sides are 8cm and 6cm, then which among the given statements is/are correct?



(a) Length of greatest side is 10cm

(b) $\angle ACB > 90°$

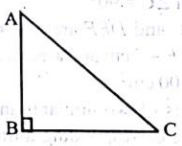
(c) $\angle BAC < 90°$

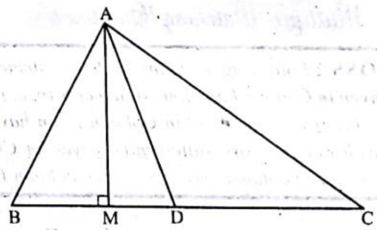
(d) Pythagoras theorem is not applicable here.

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

In Figure, AD is a median of a triangle ABC and $AM \perp BC$.





1. $AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 =$

(a) AC^2 (b) AB^2

(c) BC^2 (d) none of these

2. $AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 =$

(a) AC^2 (b) AB^2

(c) BC^2 (d) none of these

3. $2AD^2 + \frac{1}{2}BC^2 =$

(a) $AC^2 + BC^2$ (b) $AB^2 + BC^2$

(c) $AC^2 + AB^2$ (d) none of these

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** If in a ΔABC, a line $DE \parallel BC$, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$

   Reason : If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.
2. **Assertion :** ABC is an isosceles, right triangle, right angled at C. Then $AB^2 = 3\,AC^2$

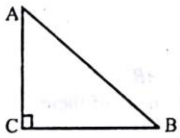
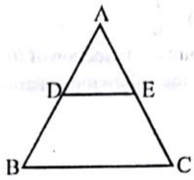
   Reason : In an isosceles triangle ABC if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C = 90°$
3. **Assertion :** ABC and DEF are two similar triangles such that $BC = 4$ cm, $EF = 5$ cm and area of $\Delta ABC = 64$ cm^2, then area of $\Delta DEF = 100$ cm^2.

   Reason : The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

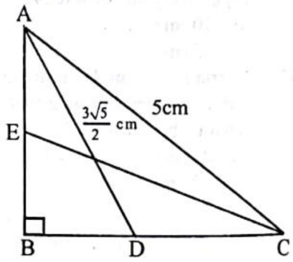
1.

	Column-I	Column-II
(a)	 ABC is an isosceles right angled triangle. $AB^2 = ?$	(p) $36:49$
		(q) $AB^2 = 2AC^2$
(b)	$\Delta ABC \sim \Delta DEF$, such that $AB = 1.2$ cm and $DE = 1.4$ cm $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = ?$	(r) $AB^2 = AC^2 + BC^2$
(c)	$\Delta ABC \sim \Delta APQ$ and $\frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{36}{49}$ $\frac{BC}{PQ} = ?$	(s) $6:7$
(d)	 If $DE \parallel BC$ and $\frac{AD}{DB} = \frac{6}{7}$ then, $\frac{AE}{EC} = ?$	(t) $72:98$

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Prove that the ratio of corresponding sides of two similar triangles is the same as the ratio of their corresponding angle bisectors.
2. In the given figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



3. Two poles of height 'a' metres and 'b' metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.
4. In an equilateral triangle with side a, Prove that

   (i) altitude $= \frac{\sqrt{3}}{2}a$ (ii) area $= \frac{\sqrt{3}}{4}a^2$
5. ABC is a right triangle with $\angle ABC = 90°$, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$, Prove that

   (i) $DM^2 = DN \times MC$ (ii) $DN^2 = DM \times AN$
6. If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then prove that the two triangles are similar.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1.	similar	2.	similar	3.	equilateral
4.	equal, ratio	5.	same	6.	third
7.	not	8.	equal	9.	trapezium
10.	third	11.	parallel		

TRUE / FALSE

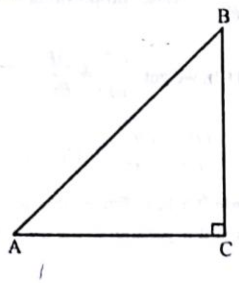
1.	True	2.	True	3.	True
4.	True	5.	True	6.	True
7.	True	8.	True	9.	True
10.	True	11.	True		

MATCH THE FOLLOWING :

1. (A) → q (B) → p (C) → s (D) → r
2. (A) → p (B) → q (C) → r (D) → s

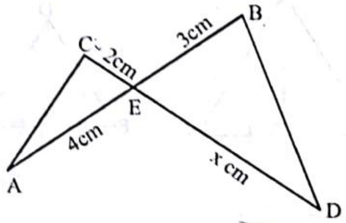
VERY SHORT ANSWER QUESTIONS :

1. $PQ \parallel RS$ (Given)
   So, $\angle P = \angle S$ (Alternate angles)
   and $\angle Q = \angle R$
   Also, $\angle POQ = \angle SOR$ (Vertically opposite angles)
   Therefore, $\Delta POQ \sim \Delta SOR$ (*AAA* similarity criterion)
2. The height of the tree is 10 meters.
3. Thus, length of the ladder is 6.5 m.
4. In ΔABC: $BC^2 = AB^2 + AC^2$ $[\angle A = 90°]$ ....(1)
   In ΔAXY: $XY^2 = AX^2 + AY^2$ $[\angle A = 90°]$........(2)
   $\therefore$ $BC^2 + XY^2 = AB^2 + AC^2 + AX^2 + AY^2$
   [Adding (1) and (2)]
   $= (AB^2 + AY^2) + (AC^2 + AX^2)$ [By grouping]
   $= BY^2 + CX^2$ [In ΔABY & ΔACX]
   $\therefore$ $BC^2 + YX^2 = BY^2 + CX^2$
5. **Proof :** $\because$ ΔABC is an isosceles right angled triangle.
   $\therefore$ $AC = BC$



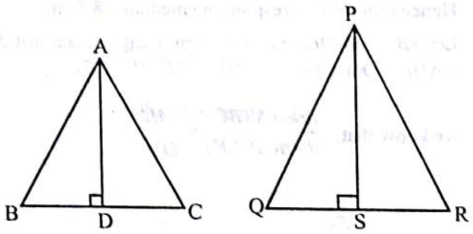
Using Pythagoras' theorem, we have
$AB^2 = AC^2 + BC^2$
$AB^2 = AC^2 + AC^2$ ....(1)
$[\because AC = BC$ (Given)]
$AB^2 = 2AC^2$

6. $\Delta ACE \sim \Delta BDE$ (Given)



$\therefore$ $\frac{AE}{DE} = \frac{CE}{BE}$ $\therefore$ $\frac{4}{x} = \frac{3}{3}$

$\therefore$ $2x = 12 \Rightarrow x = 6$ cm.

7. 

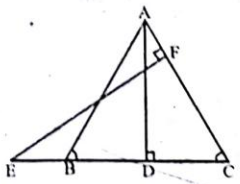
Proof : We know that if two triangles are similar then their areas are proportional to the squares of the corresponding sides,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2} \Rightarrow \frac{9}{16} = \frac{(4.5)^2}{QR^2}$$

$$\Rightarrow 3QR = \frac{4.5 \times 4}{3}$$

QR = 6 cm.

8. In ΔABC, we have, $AB = AC$



$\therefore$ $\angle B = \angle C$ (angles opposite to equal sides)

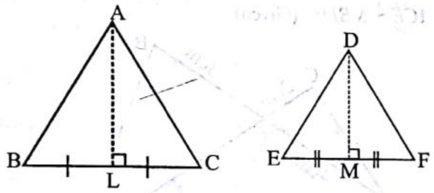
In ΔABD and ΔECF, $\angle B = \angle C$

$\angle ADB = \angle EFC$ (each 90° as $AD \perp BC, EF \perp AC$)

$\therefore$ $\Delta ABD \sim \Delta ECF$ (By AA) Q.E.D.

9. Let ABC and DEF be two triangles such that $\Delta ABC \sim \Delta DEF$.
Let AL, DM be their medians respectively

$$\therefore \frac{area(\Delta ABC)}{area(\Delta DEF)} = \frac{AC^2}{DF^2} \Rightarrow \frac{121}{64} = \frac{(12.1)^2}{DM^2}$$



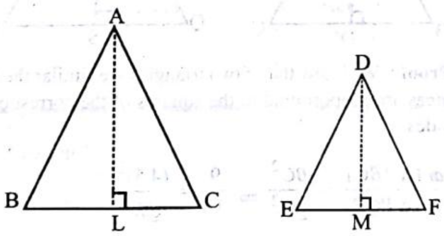
$$DM^2 = \frac{64 \times (12.1)^2}{121}$$

$$\Rightarrow DM = \frac{8 \times 12.1}{11} = 8 \times 1.1 = 8.8\,\text{cm}$$

Hence required corresponding median = 8.8 cm.

10. Let ABC and DEF be two given triangles such that $\Delta ABC \sim \Delta DEF$. Draw $AL \perp BC, DM \perp EF, AL = 4.5$ cm

we know that, $\dfrac{area(\Delta ABC)}{area(\Delta DEF)} = \dfrac{AL^2}{DM^2}$

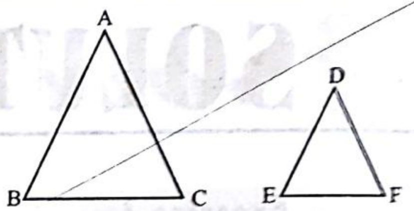


$$\Rightarrow \frac{81}{49} = \frac{(4.5)^2}{DM^2} \Rightarrow DM^2 = \frac{(4.5)^2 \times 49}{81}$$

$$\Rightarrow DM = \frac{(4.5) \times (7)}{9} = \frac{31.5}{9} = 3.5\,\text{cm}.$$

11. Also $AB = 2DE$
$\Delta ABC \sim \Delta DEF$

Hence, $\dfrac{area(\Delta ABC)}{area(\Delta DEF)} = \dfrac{AB^2}{DE^2}$

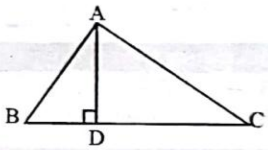


or $\dfrac{56}{area(\Delta DEF)} = \dfrac{4DE^2}{DE^2} = 4$ [$\because AB = 2DE$]

area $(\Delta DEF) = \dfrac{56}{4} = 14$ sq.cm.

12. $AB^2 = AD^2 + BD^2$ and $AC^2 = AD^2 + DC^2$

$\therefore AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2 = (BD + DC)^2$ [$\because AD^2 = BD.DC$]
$= BC^2$



$\therefore ABC$ is rt triangle, $\Rightarrow \angle BAC = 90°$
(By converse of Pythagoras theorem) Q.E.D.

13. Since $\Delta ACB \sim \Delta APQ$

$$\therefore \frac{AC}{AP} = \frac{CB}{PQ} \Rightarrow \frac{AC}{2.8} = \frac{8}{4} = 2$$

$$\Rightarrow AC = 5.6\,\text{cm}$$

SHORT ANSWER QUESTIONS :

1. In ΔOAB, $DE \parallel AB$

$\Rightarrow \dfrac{OD}{AD} = \dfrac{OE}{EB}$ [Basic proportionality theorem] ....(1)

Again in ΔOBC, $EF \parallel BC$

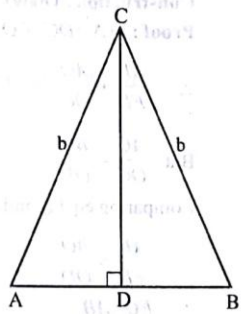
$\Rightarrow \dfrac{OE}{EB} = \dfrac{OF}{FC}$ [Basic proportionality theorem] ....(2)

From (1) and (2), we get, $\dfrac{OD}{AD} = \dfrac{OF}{FC}$

As in ΔOAC, $\dfrac{OD}{AD} = \dfrac{OF}{FC} \Rightarrow DF \parallel AC$

[$\because$ In a triangle if a line divides the two sides in the same proportion then it is parallel to the third side]

2. Let ABC be an isosceles triangle, where base $AB = a$ and equal sides $AC = BC = b$. Let CD be the perpendicular on AB,



So, $AD = DB = \frac{1}{2}AB = \frac{a}{2}$

Altitude, CD = height of the ΔABC is given by

$h = \sqrt{AC^2 - AD^2}$

$\Rightarrow h = \frac{1}{2}\sqrt{4b^2 - a^2}$

Area of the $\Delta ABC = \frac{1}{2}$ base × altitude

$= \frac{1}{2} \times a \times \frac{1}{2}\sqrt{4b^2 - a^2} = \frac{a}{4}\sqrt{4b^2 - a^2}$.

3. $AC = 8.25$ cm

4. $AB = AD$ and AE, AF are angle bisectors of $\angle BAC$ and $\angle DAC$ respectively.

Now in ΔABC, AE is angle bisector of $\angle BAC$

$\therefore \frac{AB}{AC} = \frac{BE}{EC}$ ... (1)

Similarly, $\frac{AD}{AC} = \frac{DF}{FC} \Rightarrow \frac{AB}{AC} = \frac{DF}{FC}$ ... (2)

$(\because AB = AD)$

Comparing eq. (1) and (2), we get $\frac{BE}{EC} = \frac{DF}{FC}$

$\therefore EF \parallel BD$ in ΔBCD.

5. $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$, $\frac{PF}{RF} = \frac{8}{9}$ $\therefore \frac{PE}{EQ} = \frac{PF}{RF}$

$\therefore$ $EF \parallel QR$

6. Since $\Delta ABC \sim \Delta DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \frac{AB}{10} = \frac{BC}{EF} = \frac{10}{6}$

$\Rightarrow AB = \frac{100}{6} = \frac{50}{3}$

7. In ΔABD, ΔADC,

$\angle ABD = \angle ADC$ (Each = 90°)

$\angle DAB = \angle ACD$ (given)

$\therefore \Delta ABD \sim \Delta ADC$ (By A.A. criterion)

$\therefore \frac{AB}{AC} = \frac{BD}{AD} = \frac{AD}{DC}$

$\therefore AD^2 = BD.DC$

8. **Given:** In ΔABC, $AB = AC$

$AD \perp BC$ and $EF \perp AC$

Proof: Since, $AB = AC$

$\therefore \angle ABC = \angle ACB$

Now in ΔABD and ΔECF

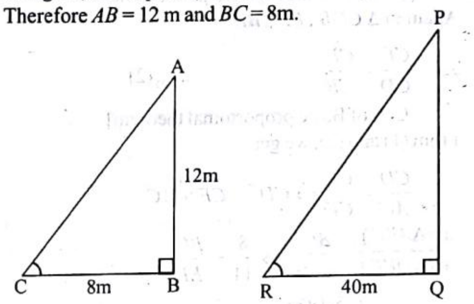
$\angle ADB = \angle EFC = 90°$

and $\angle ABC = \angle ACB$

$\Rightarrow \angle ABD = \angle FCE$

$\therefore \Delta ABD \sim \Delta EFC$

9. In figure, AB represents the stick and BC is its shadow. Therefore $AB = 12$ m and $BC = 8$m.



Again PQ is tower and QR is its shadow. Therefore $QR = 40$m

Now, $\Delta ABC \sim \Delta PQR$

$\therefore \frac{PQ}{QR} = \frac{AB}{BC} \Rightarrow \frac{PQ}{40} = \frac{12}{8} \Rightarrow PQ = 60m$

10. In ΔACB

$AB^2 = 2AC^2$ and $AC = BC$

Now in ΔABC,

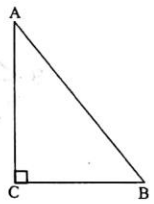
$AB^2 = 2AC^2 = AC^2 + AC^2$

$AB^2 = AC^2 + BC^2$ $(BC = AC)$

$\angle ACB = 90°$ and

ΔABC is a right triangle

(Converse of Pythagoras theorem)



11. Hence ar. $(\Delta ABC) = 25 \times 4 = 100$ sq. cm.

12. In ΔABC and ΔADC,

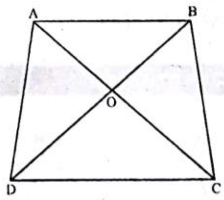
$\angle BAC = \angle ADC$ [Given]

$\angle ACB = \angle ACD$ [Common]

$\therefore \Delta BAC \sim \Delta ADC$ [AA similarity]

$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$

13. In ΔAOB and ΔCOD, $\angle ABO = \angle CDO$



[$\because AB \parallel CD$ alternate interior angles]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\therefore \Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{OC} = \frac{OB}{OD}$

14. In ΔABC, $DE \parallel AB$

$\Rightarrow \dfrac{CD}{AC} = \dfrac{CE}{BC}$ ......(1)

[Cor. of Basic proportional theorem]

Again in ΔCDB, $EF \parallel BD$

$\Rightarrow \dfrac{CF}{CD} = \dfrac{CE}{CB}$ ......(2)

[Cor. of Basic proportoinal theorem]

From (1) and (2), we get

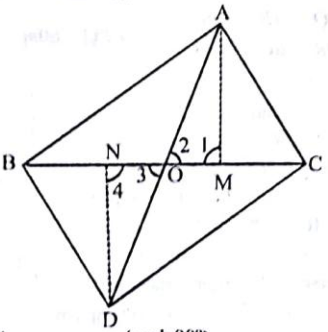
$\dfrac{CD}{AC} = \dfrac{CF}{CD} \Rightarrow CD^2 = CF \times AC$

15. $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC^2}{EF^2} \Rightarrow \dfrac{8}{11} = \dfrac{BC}{EF}$

$\Rightarrow BC = \dfrac{(15.4)(8)}{11} = 1.4 \times 8 = 11.2$ cm

16. Draw $AM \perp BC$, $DN \perp BC$

In ΔAMO and ΔDNO,



$\angle 1 = \angle 4$ ......(each 90°)

$\angle 2 = \angle 3$ ......(vetically opposite $\angle s$)

$\Delta AMO \sim \Delta DNO$ .....(By A. A. rule of similarity)

$\dfrac{AO}{DO} = \dfrac{AM}{DN}$ ......(i)

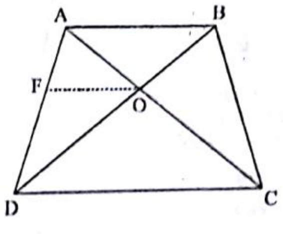
Now, $\dfrac{area(\Delta ABC)}{area(\Delta DBC)} = \dfrac{(1/2)(BC)(AM)}{(1/2)(BC)(DN)} = \dfrac{AM}{DN} = \dfrac{AO}{DO}$

......from (i)

Hence, $\dfrac{area(\Delta ABC)}{area(\Delta DBC)} = \dfrac{AO}{DO}$

LONG ANSWER QUESTIONS :

1. **Given :**



Construction : Draw $OF \parallel DC$

Proof : In ΔADC, $FO \parallel DC$,

$\therefore \dfrac{AF}{FD} = \dfrac{AO}{OC}$ ....(1) [B.P. Theorem]

But $\dfrac{AO}{OC} = \dfrac{BO}{OD}$ ....(2) [Given]

Comparing eq. (1) and eq. (2), we get

$\dfrac{AF}{FD} = \dfrac{BO}{OD}$

$\therefore$ $FO \parallel AB$ [Converse of B.P. theorem]

Also, $FO \parallel DC$ [By construction]

$\therefore$ $AB \parallel DC$

$\therefore$ $ABCD$ is a trapezium.

2. **Given :** In ΔABC, $AE \perp BC$ and $BD = DC$

To prove : $b^2 + c^2 = 2p^2 + \frac{1}{2}a^2$

Proof : Since AD is the median

$\Rightarrow BD = AC = \dfrac{a}{2}$

In right triangle ΔAED, $\angle AED = 90°$

$\therefore$ $AD^2 = AE^2 + ED^2 = h^2 + x^2$ (let $ED = x$)

$\Rightarrow p^2 = h^2 + x^2$ ....(1)

In right triangle ΔAEC, $\angle AEC = 90°$

$\therefore$ $AC^2 = AE^2 + EC^2 = AE^2 + (ED + DC)^2$

$\Rightarrow b^2 = h^2 + \left(x + \dfrac{a}{2}\right)^2 = h^2 + x^2 + \dfrac{a^2}{4} + xa$

$\Rightarrow b^2 = p^2 + \dfrac{a^2}{4} + xa$ ....(2)

Now in right triangle ΔAEB, $\angle AEB = 90°$

$\therefore$ $AB^2 = AE^2 + BE^2 = h^2 + (BD - ED)^2$

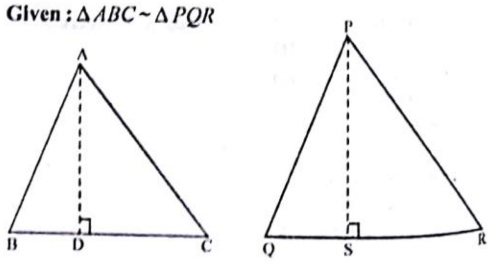
$\Rightarrow c^2 = h^2 + x^2 + \dfrac{a^2}{4} - xa = p^2 + \dfrac{a^2}{4} - xa$ ....(3)

Adding eq. (2) and (3), we get

$b^2 + c^2 = p^2 + \dfrac{a^2}{4} + xa + p^2 + \dfrac{a^2}{4} - xa$

$b^2 + c^2 = 2p^2 + \dfrac{1}{2}a^2$

3. **Given :** $\Delta ABC \sim \Delta PQR$



To prove : $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)}=\dfrac{AB^2}{PQ^2}=\dfrac{BC^2}{QR^2}=\dfrac{CA^2}{RP^2}$

Construction : We draw $AD \perp BC$ and $PS \perp QR$

Proof : We know that

Area of triangle $=\dfrac{1}{2}\times$ base $\times$ height

$\Rightarrow \dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)}=\dfrac{BC\times AD}{QR\times PS}$ .... (1)

Now, in ΔABD and ΔPQS

$\angle B=\angle Q$ [$\Delta ABC\sim\Delta PQR$]

$\angle ADB=\angle PSQ$ [90° each]

$\therefore\ \Delta ABD\sim\Delta PQS$ (AA similarity)

$\Rightarrow \dfrac{AD}{PS}=\dfrac{AB}{PQ}$ .... (2)

but $\dfrac{AB}{PQ}=\dfrac{BC}{QR}$ (since $\Delta ABC\sim\Delta PQR$)

$\therefore \dfrac{AD}{PS}=\dfrac{BC}{QR}$ .... (3)

Therefore, $\dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)}=\dfrac{BC}{QR}\times\dfrac{BC}{QR}=\dfrac{BC^2}{QR^2}$ .... (4)

[From (1) and (3)]

Since $\Delta ABC\sim\Delta PQR$

therefore $\dfrac{AB}{PQ}=\dfrac{BC}{QR}=\dfrac{CA}{RP}$

$\Rightarrow \dfrac{AB^2}{PQ^2}=\dfrac{BC^2}{QR^2}=\dfrac{CA^2}{RP^2}$ .... (5)

$\therefore \dfrac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)}=\dfrac{AB^2}{PQ^2}=\dfrac{BC^2}{QR^2}=\dfrac{CA^2}{RP^2}$

[From eq. (4) and (5)]

4. **Given :** In ΔABC, O is any point inside it. AO, OB, OC are joined. D is a point on OA. $DE \parallel AB$ meeting OB at E and $EF \parallel BC$ meeting OC at F.

To prove : $DF \parallel AF$

Proof : In ΔOAB, $DE \parallel AB$

$\Rightarrow \dfrac{OD}{AD}=\dfrac{OE}{EB}$ [Basic proportionality theorem] ....(1)

Again in ΔOBC, $EF \parallel BC$

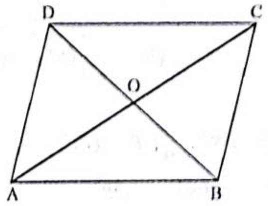
$\Rightarrow \dfrac{OE}{EB}=\dfrac{OF}{FC}$ [Basic proportionality theorem] ....(2)

From (1) and (2), we get, $\dfrac{OD}{AD}=\dfrac{OF}{FC}$

As in ΔOAC, $\dfrac{OD}{AD}=\dfrac{OF}{FC}\Rightarrow EF \parallel AC$

[$\because$ In a triangle if a line divides the two sides proportionally then it is parallel to the third side]

5.



$OC=AO=\dfrac{1}{2}AC$ and $OD=BO=\dfrac{1}{2}BD$ .... (1)

[Diagonals of a rhombus bisect each other]

We know that diagonals of a rhombus bisect each other at right angles.

In right angled ΔAOB,

$OA^2+OB^2=AB^2$ .... (2)

In right angled ΔBOC,

$OB^2+OC^2=BC^2$ .... (3)

In right angled ΔCOD,

$OC^2+OD^2=CD^2$ .... (4)

In right angled ΔDOA,

$OD^2+OA^2=AD^2$ .... (5)

Adding (2) to (5), we get

$2(OA^2+OB^2+OC^2+OD^2)$

$=AB^2+BC^2+CD^2+DA^2$

$2\left[\left(\dfrac{AC}{2}\right)^2+\left(\dfrac{BD}{2}\right)^2+\left(\dfrac{AC}{2}\right)^2+\left(\dfrac{BD}{2}\right)^2\right]$

$=AB^2+BC^2+CD^2+DA^2$ [From (1)]

$\Rightarrow AC^2+BD^2=AB^2+BC^2+CD^2+DA^2$

$\therefore AB^2+BC^2+CD^2+DA^2=AC^2+BD^2$

6. **Given :** Right triangle ΔPQR. S and T trisect QR.

To prove : $8PT^2=3PR^2+5PS^2$

Proof : $QS=ST=TR=\dfrac{1}{3}QR$ [Given] .... (1)

In right triangle ΔPQS,

$PS^2=PQ^2+QS^2$ [Phythagoras theorem] .... (2)

In right triangle ΔPQT,

$PT^2=PQ^2+QT^2$ [Phythagoras theorem] .... (3)

In right triangle ΔPQR,

$PR^2=PQ^2+QR^2$[Phythagoras theorem] .... (4)

Subtracting (3) from (2), we get

$\Rightarrow PS^2-PT^2=\left(\dfrac{1}{3}QR\right)^2-\left(\dfrac{2}{3}QR\right)^2=\dfrac{1}{3}QR^2-\dfrac{4}{9}QR^2$

[From (1)]

$\Rightarrow \quad 3PS^2 - 3PT^2 = -QR^2$ ....(5)

Subtracting (4) from (3), we get

$$PT^2 - PR^2 = QT^2 - QR^2 = \left(\frac{2}{3}QR\right)^2 - QR^2 \qquad \text{[From (1)]}$$

$\Rightarrow \quad PT^2 - PR^2 = \frac{4}{9}QR^2 - QR^2$

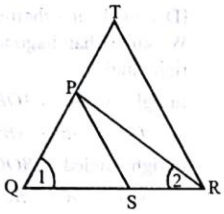
$\Rightarrow \quad 9PT^2 - 9PR^2 = -5QR^2$

Substituting for $(-QR^2)$ from (5) in (vi), we get

$\Rightarrow \quad 9PT^2 - 9PR^2 = 5(3PS^2 - 3PT^2)$

$\Rightarrow \quad 8PT^2 = 5PS^2 + 3PR^2$

7. We have $\frac{QT}{PR} = \frac{QR}{QS}$ (Given)



$\therefore \quad \frac{QT}{QR} = \frac{PR}{QS}$ ... (i)

Now $\angle 1 = \angle 2$ (Given)

$\therefore \quad PQ = PR$

(Sides opposite the equal angles) ...(ii)

$\therefore \quad \frac{QT}{QR} = \frac{PQ}{QS}$ [From (i) and (ii)]

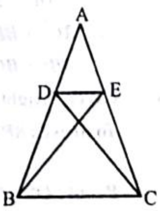
Now in triangles, ΔPQS and ΔTQR,

we have $\frac{PQ}{QS} = \frac{QT}{QR}$ from (iii)

and $\angle Q = \angle Q$ ($= \angle 1$ in each)

$\therefore \quad \Delta PQS \sim \Delta TQR$ (SAS similarity)

8. We are given that $\Delta ABE \cong \Delta ACD$



Therefore, $AB = AC$ (CPCT) ... (I)

and $AE = AD$ (CPCT) ... (II)

$\therefore \quad \frac{AB}{AD} = \frac{AC}{AE}$ [From (I) and (II)]

i.e., $\frac{AB}{AC} = \frac{AD}{AE}$ ...(III)

Now in ΔADE, $\angle A$ (i.e., $\angle DAE$) is included between sides AD and AE and in ΔABC. $\angle A$ (i.e., $\angle BAC$) is included between sides AB and AC and $\angle DAE = \angle BAC$ (Common angles)

Further $\frac{AB}{AC} = \frac{AD}{AE}$ [From (III)]

$\therefore \; \Delta ADE \sim \Delta ABC$ (SAS similarity)

9. In ΔABC, we have $DE \parallel BC$

$\Rightarrow$ Ð $ADE = \angle ABC$ and $\angle ACD = \angle ACB$

[Corresponding angles]

Thus, in triangles ΔADE and ΔABC, we have

$\angle A = \angle A$ [Common]

$\angle ADE = \angle ABC$

and $\angle AED = \angle ACB$

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AAA similarity]

$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$

We have, $\frac{AD}{DB} = \frac{5}{4}$

$\Rightarrow \quad \frac{DB}{AD} = \frac{4}{5}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{4}{5} + 1$ (By adding 1 both sides)

$\Rightarrow \quad \frac{DB + AD}{AD} = \frac{9}{5} \Rightarrow \frac{AB}{AD} = \frac{9}{5} \Rightarrow \frac{AD}{AB} = \frac{5}{9}$

$\therefore \quad \frac{DE}{BC} = \frac{5}{9}$ $(\because \Delta ADE - \Delta ABC)$

In ΔDFE and ΔCFB, we have

$\angle 1 = \angle 3$ [Alternate interior angles]

$\angle 2 = \angle 4$ [Vertically opposite angles]

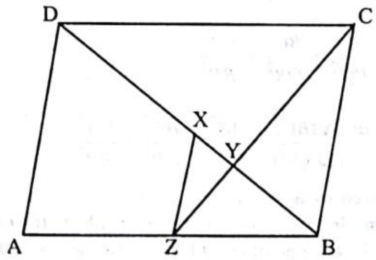
Therefore, by AA-similarity criterion, we have

$\Delta DFE \sim \Delta CFB$

$$\Rightarrow \quad \frac{\text{Area }(\Delta \text{ DFE})}{\text{Area }(\Delta \text{ CFB})} = \frac{DE^2}{BC^2} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

Hence, ratio of areas of ΔDEF and ΔCFB is 25 : 81

10.



Given that, $\frac{AZ}{ZB} = \frac{2}{3} \Rightarrow \frac{AZ}{ZB} + 1 = \frac{2}{3} + 1$

$\Rightarrow \quad \frac{AZ + ZB}{ZB} = \frac{5}{3} \Rightarrow \frac{BA}{BZ} = \frac{5}{3} \Rightarrow \frac{BZ}{BA} = \frac{3}{5}$

Now, $XZ \| AD$. ΔBXZ and ΔBDA are similar, so

$\frac{BX}{BD} = \frac{3}{5}$ ...(i)

In ΔXYZ and ΔBYC

$\angle XYZ = \angle BYC$ [opposite angles]

$\angle ZXY = \angle YBC$ [$XZ \| BC$ and BX meets them]

$\angle XZY = \angle YCB$ [$XZ \| BC$ and CZ meets them]

$\Rightarrow$ Triangle ΔXYZ and ΔBYC are similar, then

$$\frac{XY}{YB}=\frac{ZX}{BC}=\frac{3}{5}$$

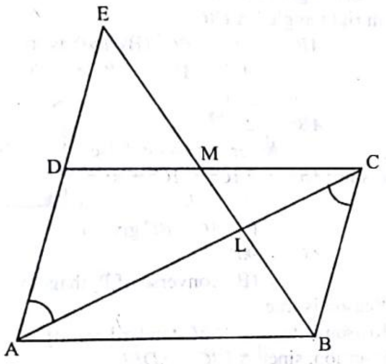
As $\frac{XY}{YB}=\frac{3}{5}\Rightarrow\frac{YB}{XY}=\frac{5}{3}$

$$\Rightarrow \frac{YB}{XY}+1=\frac{5}{3}+1\Rightarrow\frac{YB+XY}{XY}=\frac{8}{3}\Rightarrow\frac{BX}{XY}=\frac{8}{3}$$

$$\Rightarrow \frac{XY}{BX}=\frac{3}{8} \quad \text{...(ii)}$$

From (i) and (ii), $\frac{XY}{BX}\times\frac{BX}{BD}=\frac{XY}{BD}=\frac{3}{8}\times\frac{3}{5}=\frac{9}{40}$.

11. In Δ's BMC and EMD, we have



$\angle BMC = \angle EMD$ [Vertically opposite angles]
$MC = MD$ [$\because$ M is the mid-point of CD]
$\angle MCB = \angle MDE$ [Alternate angles]

So, by AAS-congruence criterion, we have

$\Delta BMC \cong \Delta EMD$
$BC = ED$

[$\because$ Corresponding parts of congruent triangles are equal]

In Δ's AEL and CBL, we have

$\angle ALE = \angle CLB$ [Vertically opposite angles]
$\angle EAL = \angle BCL$ [Alternate angles]

So, by AA-criterion of similarity, we have $\Delta AEL \sim \Delta CBL$

$$\Rightarrow \frac{AE}{BC}=\frac{EL}{BL}=\frac{AL}{CL}$$

Taking first two terms,we get

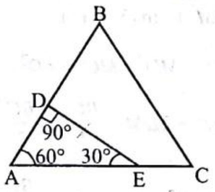
$$\frac{EL}{BL}=\frac{AE}{BC}=\frac{AD+DE}{BC}=\frac{BC+DE}{BC}=\frac{2BC}{BC}=2$$

$\Rightarrow EL=2BL.$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (a)
2. (b) If the perimeter of the polygons is the same, the polygon with greater sides has the greater area.
3. (a) A point which is equidistant from the sides of a triangle is the incentre and such a point is one and only one in the plane of the triangle.
4. (c)
5. (a) Half the sum of the base angles.
6. (c)
7. (c) As P, Q and R are the midpoints of AB, BC and AC respectively, ΔABC is divided into 4 triangles of equal areas. Therefore, Area (ΔPQR) = 20/4 = 5 sq. units.
8. (b)
9. (c) Two (5 inch × 12 inch × 13 inch) right triangles can be put together in two ways to form an isosceles triangle with equal 13 inch sides. One way involves a base of 10 inches, the other 24 inches. Naturally the area is the same in either case.
10. (d) Area of ABC will be $\frac{\sqrt{3}}{4}(4)^2=4\sqrt{3}$ cm^2



ADE is right angle triangle where $AD = 1$ cm, so we will get $DE=\sqrt{3}$ cm & $AE = 2$ cm

So area of ΔADE will be $\frac{1}{2}\times1\times\sqrt{3}=\frac{\sqrt{3}}{2}$ cm^2

So area of $BCDE = 4\sqrt{3}-\sqrt{3}/2=3.5\sqrt{3}$ cm^2

11. (b) As PQ is parallel to $BC \Rightarrow \Delta ABC = \Delta APQ$

$$\Rightarrow \frac{Area\ of\ ABC}{Area\ of\ APQ}=\frac{2}{1}$$

(Ratio of square of corresponding sides)

$\therefore$ Ratio of sides $=\frac{AB}{AP}=\frac{\sqrt{2}}{1}$

$\therefore$ Ratio of $PB = AB : AP = \sqrt{2}-1:\sqrt{2}$

12. (a)
13. (a) Since, $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{ar(\Delta PRQ)}{ar(\Delta BCA)}=\frac{AR^2}{AC^2}=\frac{QR^2}{BC^2}=\frac{9}{1}\left[\because \frac{QR}{BC}=\frac{3}{1}\right]=9$$

14. (d) In triangle ΔACD, DB = DC

$\Rightarrow \angle DBC = \angle DCB = \left(\frac{180° - 60°}{2}\right) = 60°$.

In $\Delta ABC, AB = AC$

$\Rightarrow \angle ABC = \angle ACB = \left(\frac{180° - 30°}{2}\right) = 75°$.

or $\angle EBD = 75° - 60° = 15°$

Also, in ΔDEB, $\angle BDE = 120°$

$\therefore \angle BED = 180° - (120° + 15°) = 45°$.

15. (b) Hypotenuse = 270m

$\Rightarrow$ Hypotenuse^2 = Side^2 + Side^2 = 2 Side^2

$\Rightarrow$ Side^2 = $(270)^2/2$ = 72900/2 = 36450 or side = 190.91m

$\Rightarrow$ Required Area = 1/2 × 190.91 × 190.91 = 36446.6/2 = 18225 m^2 (approx).

16. (d) 17. (a)

MORE THAN ONE CORRECT ANSWER :

1. (a, b, c) 2. (a, c) 3. (a, c)

PASSAGE BASED QUESTIONS :

1. (a) AD is the median, so D is the mid point of BC.

So, $BD = DC = \frac{1}{2}BC$ ..... (1)

In right angled ΔAMC, $AC^2 = AM^2 + MC^2$ ..... (2)

In right angled ΔAMD, $AM^2 = AD^2 - MD^2$ ..... (3)

Putting AM^2 from (3) in (2),

we get

$AC^2 = AD^2 - MD^2 + MC^2 = AD^2 - MD^2 + (MD + DC)^2$

$= AD^2 + 2DM \cdot \frac{BC}{2} + \left(\frac{BC}{2}\right)^2$

So, $AC^2 = AD^2 + BC.\,DM + \left(\frac{BC}{2}\right)^2$

2. (b) In right angled ΔABM, $AB^2 = AM^2 + BM^2$

From ΔAMD, $AM^2 = AD^2 - MD^2$

So, $AB^2 = AD^2 - MD^2 + BM^2 = AD^2 - MD^2 +$

$(BD - MD)^2 = AD^2 - MD^2 + BD^2 - 2BD.MD + MD^2$

$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$ Proved.

3. (c) From the solution of above two questions

$AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$ ...(i)

and $AB^2 = AD^2 - BC.DM + \frac{1}{2}BC^2$ ...(ii)

Adding results of (i) & (ii) we get.

$AC^2 + AB^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 + AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

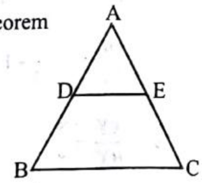
$\Rightarrow AC^2 + AB^2 = 2AD^2 + \frac{1}{2}(BC)^2$

ASSERTION & REASON :

1. (a) Reason is true. [This is Thale's Theorem]

For Assertion (A)

Since $DE \parallel BC$ $\therefore$ by Thale's Theorem



$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

$\therefore$ Assertion (A) is true.

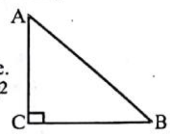
Since reason gives Assertion.

2. (d) In right angled ΔABC,

$AB^2 = AC^2 + BC^2$ (By Pythagorus Theorem)

$= AC^2 + AC^2$ [$\because BC = AC$]

$= 2AC^2$



$\therefore$ $AB^2 = 2AC^2$

$\therefore$ Assertion (A) is false.

Again since $AB^2 = 2AC^2 = AC^2 + AC^2$

$= AC^2 + BC^2$

($\because AC = BC$ given)

$\therefore$ $\angle C = 90°$

(By converse of Pythagoras Theorem)

$\therefore$ Reason is true.

3. (b) Reason is true [$\because$ of standard result]

For Assertion, since $\Delta ABC \sim \Delta DEF$

$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$

($\because$ ratio of areas of two similar Δs is equal to the ratio of the squares of corresponding sides)

$\therefore \frac{64}{\text{area}(\Delta DEF)} = \frac{16}{25} \Rightarrow \text{area}(\Delta DEF) = \frac{64 \times 25}{16}$

$= 4 \times 25 = 100\,\text{cm}^2$

$\therefore$ Assertion is true. But Reason is not the correct explanation for Assertion.

MULTIPLE MATCHING QUESTIONS :

1. (A) → q, r (B) → p, t (C) → s (D) → s

(A) $AB^2 = AC^2 + BC^2$

Since, ΔABC is an isosceles right angled triangle.

$\therefore$ $AC = BC$

Now, $AB^2 = AC^2 + AC^2 = 2AC^2$

(B) $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{(AB)^2}{(DE)^2} = \frac{(1.2)^2}{(1.4)^2} = \frac{1.44}{1.96}$

$= \frac{36}{49} = \frac{(36 \times 2)}{(49 \times 2)} = \frac{72}{98}$

(C) $\frac{\text{area}(\Delta APQ)}{\text{area}(\Delta ABC)} = \frac{(BC)^2}{(PQ)^2} = \frac{36}{49}$

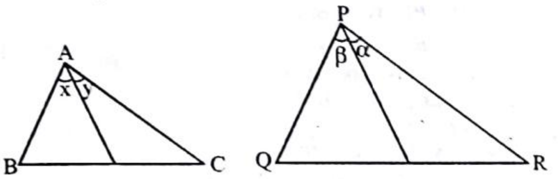
$\frac{BC}{PQ} = \frac{6}{7}$

(D) $\because DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC} = \frac{6}{7}$

HOTS SUBJECTIVE QUESTIONS :

1.



$\Delta ABC \sim \Delta PQR$. (given) $\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(1)

Let AD and PS be angle bisectors in ΔABC and ΔPQR respectively.

$\angle x = \angle y$ in ΔABC and $\angle\beta = \angle\alpha$ in ΔPQR (given).

Now, $\angle B = \angle Q$ (corresponding angles of similar triangles)

and $\angle A = \angle P$

$\therefore \frac{1}{2}\angle A = \frac{1}{2}\angle P \Rightarrow \angle x = \angle\beta$

Now in ΔABD and ΔPQS,

$\angle BAD = \angle QPS$ (from eq. 1) and $\angle B = \angle Q$

$\therefore \Delta ABD \sim \Delta PQS$ (By AA Similarity)

$\therefore \frac{AB}{PQ} = \frac{AD}{PS}$ ...(2)

From equation (1) and (2), $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AD}{PS}$

2. In right ΔABD, $AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = AB^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow AD^2 = AB^2 + \frac{1}{4}BC^2$ ...(1)

Similarly, in right ΔEBC, $EC^2 = BC^2 + \frac{1}{4}AB^2$ ...(2)

Adding (1) and (2), we get $AD^2 + EC^2 = \frac{5}{4}AB^2 + \frac{5}{4}BC^2$

$\Rightarrow \left(\frac{3\sqrt{5}}{2}\right)^2 + EC^2 = \frac{5}{4}(5)^2 \Rightarrow EC^2 = \frac{125}{4} - \frac{45}{4}$

$\Rightarrow EC^2 = \frac{80}{4} = 20 \text{ cm} \Rightarrow CE = 2\sqrt{5}$.

3. Let AB and CD be two poles of heights 'a' metres and 'b' metres respectively. Poles are p metres apart

$\Rightarrow AC = $ p metres.

Let lines AD and BC meet at O such that $OL \perp AC$ and $OL = h$ metres.

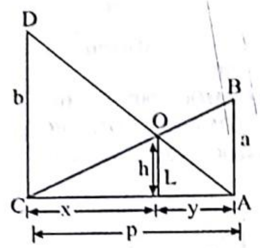
Let $CL = x$ and $LA = y$. Then, $x + y = p$.

In ΔABC and ΔLOC, we have

$\angle CAB = \angle CLO$ [Each is 90°]

$\angle BCA = \angle OCL$ [Common]

$\therefore \Delta CAB \sim \Delta CLO$ [AA-criterion of similarity]



$\Rightarrow \frac{CA}{CL} = \frac{AB}{LO}$.

$\Rightarrow \frac{p}{x} = \frac{a}{h} \Rightarrow x = \frac{ph}{a}$ ... (1)

In ΔALO and ΔACD, we have

$\angle ALO = \angle ACD$ [Each equal to 90°]

$\angle DAC = \angle OAL$ [Common]

$\therefore \Delta LAO \sim \Delta CAD$ [By AA-criterion of similarity]

$\Rightarrow \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{y}{p} = \frac{h}{b}$

$\Rightarrow y = \frac{ph}{b}$ [$\because AC = x + y = p$] ...(2)

From (1) and (2), we have $x + y = \frac{ph}{a} + \frac{ph}{b}$

$\Rightarrow p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ [$\because x + y = p$]

$\Rightarrow 1 = h\left(\frac{a+b}{ab}\right) \Rightarrow h = \frac{ab}{a+b}$ metres

4. ΔABC is an equilateral with side a, then

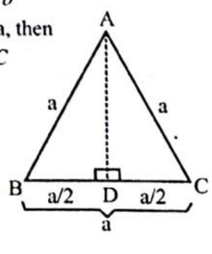
$AB = AC = BC = a$ Draw $AD \perp BC$

In ΔADB and ΔADC

$AB = AC$, $\angle ADB = \angle ADC = 90°$

and $\angle B = \angle C = 60°$

$\therefore \Delta ADB \cong \Delta ADC$ ...... (By ASA)



$\therefore \quad BD = DC = \frac{a}{2}$

Now In ΔADB,

$$AB^2 = AD^2 + BD^2 \text{ (Using Pythagoras)}$$

$$\Rightarrow \quad AD = \sqrt{AB^2 - BD^2} = \sqrt{a^2 - \frac{a^2}{4}} \Rightarrow AD = \frac{\sqrt{3}}{2}a$$

Now area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a$

$$= \frac{\sqrt{3}}{4}a^2 \text{ sq. unit. } \textbf{H.P.}$$

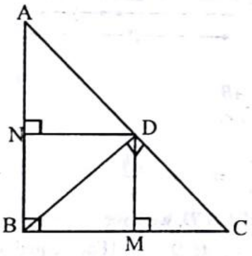
5. Since $BD \perp AC$ ......(Given)

$\therefore \quad \angle BDC = 90°$

$\therefore \quad \angle BDM + \angle MDC = 90°$ ......(i)

In ΔDMC, $\angle DMC = 90°$ [$DM \perp BC$ (given)]

$\therefore \quad \angle C + \angle MDC = 90°$ ......(ii)



From (i) and (ii)

$$\angle BDM + \angle MDC = \angle C + \angle MDC \Rightarrow \angle BDM = \angle C$$

Now in ΔDBM and Δ

$\angle BDM = \angle C$ ....(proved above)

$\angle BMD = \angle MDC$ ......(each 90°)

$\Delta BMD \sim \Delta MDC$.....(By $A.A.$ rule of similarity)

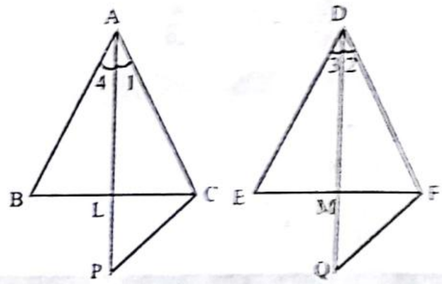
or $\frac{DM}{BM} = \frac{MC}{DM} \Rightarrow DM^2 = BM \times MC$

or $DM^2 = DN \times MC$ ......[$\because BM = DN$],

Similarly $\Delta NDA \sim \Delta NBD$, or $\frac{DN}{BN} = \frac{AN}{DN}$

$\Rightarrow \quad DN^2 = BN \times AN = DM \times AN$ .....[$\because BN = DM$].

6. Produce AL to P such that $LP = AL$.
Join CP and produce DM to Q such that $MQ = DM$ Join FQ.



In ΔALB and ΔALC

$BL = CL$ [$\because$ AL is the median]

$AL = PL$ (By Const.)

$\therefore \quad \angle ALB = \angle CLP$

(vertically opposite angles)

$\therefore \quad \Delta ALB \cong \Delta CLP$ (By SAS)

$\therefore \quad AB = PC$ ....(i)

Now, in ΔDME and ΔMFQ

$EM = MF$ ($\because$ DM is the Median)

$DM = MQ$ (By Const.)

$\angle DME = \angle FMQ$ (vertically opposite angles)

$\therefore \quad \Delta DME \cong \Delta FQM$ (By SAS)

$DE = QF$ ....(ii)

Now, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AL}{DM}$ (Given)

$\therefore \quad \frac{PC}{QF} = \frac{AC}{DF} = \frac{2AL}{2DM}$

$\Rightarrow \quad \frac{PC}{QF} = \frac{AC}{DF} = \frac{AP}{DQ}$

[$\because 2AL = AP$ and $2DM = DQ$]

In ΔABC and ΔDEF,

$\therefore \Delta APC \sim \Delta DQF$ (By AA)

$\therefore \angle 1 = \angle 2$

Similarly $\angle 3 = \angle 4$

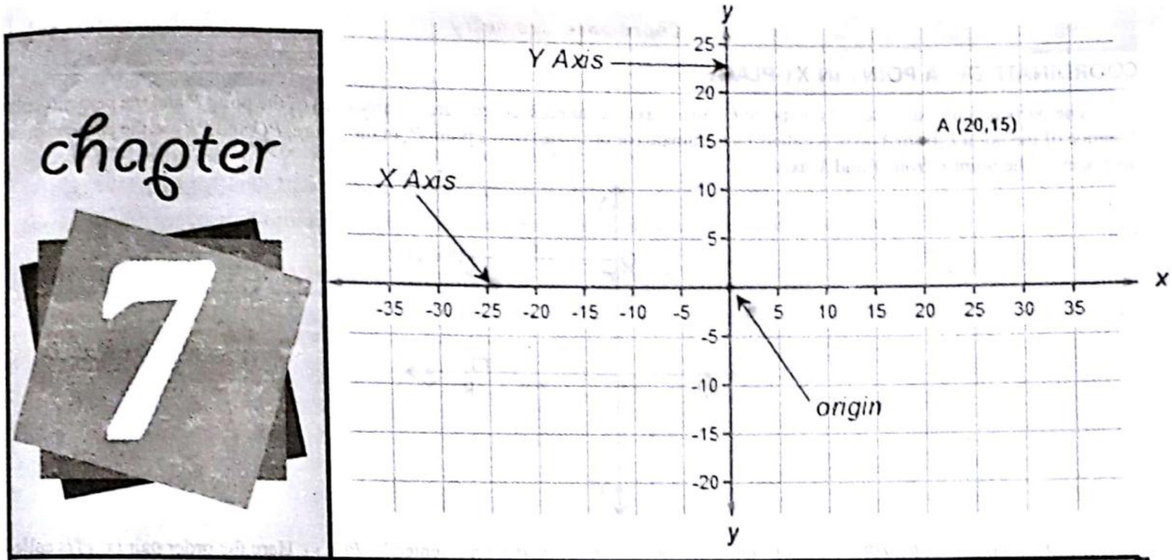
$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \angle A = \angle D$ ....(3)

$\angle A = \angle D$ (Just proved above)

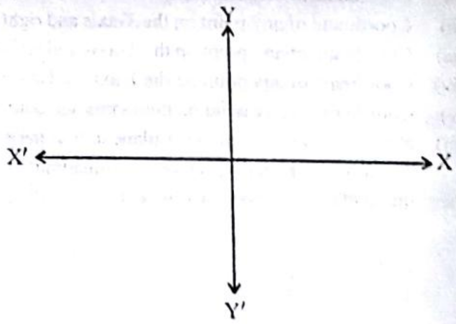
and $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)

$\therefore \Delta ABC \sim \Delta DEF$ (By SAS) Q.E.D.



COORDINATE GEOMETRY

Introduction



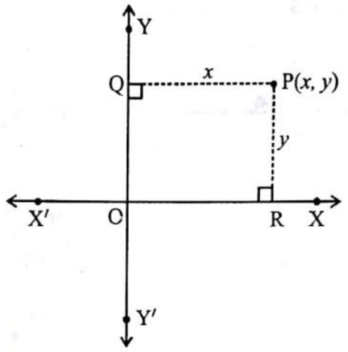
In earlier classes, you have studied to locate the position of a point in a plane with respect to a fixed point in the plane. This fixed point is called origin and represented by 0 (zero).

To find the position of a point in a plane, you draw two mutually perpendicular straight lines passing through the origin. One of these two straight lines is horizontal and other is vertical. The horizontal line passing through the origin (O) is called X-axis represented by XX' and the vertical line passing through the origin (O) is called Y-axis represented by YY'. The plane in which you draw X and Y-axis is called XOY-plane or simple XY-plane.

In this chapter you will study to find the co-ordinate of a point which is actual the location of a point in a plane from the reference point origin (O). Distance formula, section formula, Area of a triangle, Slope of a line and equation of a straight line in various forms.

COORDINATE OF A POINT IN XY-PLANE :

The perpendicular distance of any point P from Y-axis is called X-coordinate (or abscissa) of the point P and the perpendicular distance of the point P from X-axis is called Y-coordinate (or ordinate) of the point P. In the figure, PQ and PR are the perpendicular distances of the point P from Y and X-axis.



If $PQ = x$ units $(= OR)$, $PR = y$ units $(= OQ)$, the, position of point P is represented by $P(x, y)$. Here the order pair (x, y) is called the coordinates of point P. Hence, $P(x, y)$ is a point in the X, Y-plane which is x unit distant from Y-axis and y units distant from X-axis. In the order pair (x, y), the first entry 'x' is the X-coordinate and the second entry 'y' is the Y-coordinate of the point. (x, y) is called an order pair because there is a pair of numbers x and y, in which the first entry x is always X-coordinate and the second entry y is always Y-coordinate of the point.

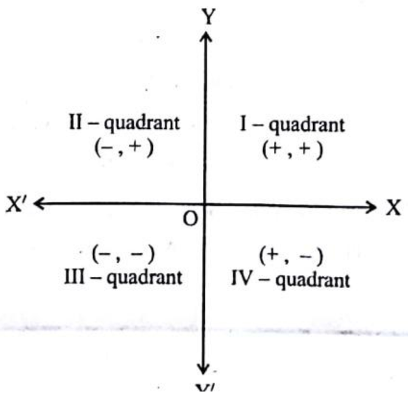
SIGN CONVENTIONS IN THE XY-PLANE :

(a) all the distances are measured from origin(0).
(b) all the distances measured along or parallel to X-axis and right side of origin are taken as $+ve$.
(c) all the distances measured along or parallel to X-axis and left side of origin are taken as $-ve$.
(d) all the distances measured along or parallel to Y-axis and above the origin are taken as $+ve$.
(e) all the distances measure along or parallel to Y-axis and below the origin are taken as $-ve$.

Acoording to the above sign conventions :

(i) Coordinate of origin is $(0, 0)$
(ii) Coordinate of any point on the X-axis and right side of origin is of the form $(x, 0)$, where $x > 0$.
(iii) Coordinate of any point on the X-axis and left side of origin is of the form $(-x, 0)$, where $x > 0$.
(iv) Coordinate of any point on the Y-axis and above the origin is of the form $(0, y)$, where $y > 0$.
(v) Coordinate of any point on the Y-axis and below the origin is of the form $(0, -y)$, where $y > 0$.
(vi) X and Y-axis divide the XOY plane in four parts. Each part is called a quadrant.

The four quadrants are written as I-quadrant (XOY), II-quadrant (YOX'), III- quadrant $(X'OY')$ and (iv) quadrant $(Y'OX)$. Eah of these quadrants shows the specific quadrant of the XOY plane as shown below :



NOTE:

(a) Any of the four quadrants does not includes any part of X or Y-axis.

(b) In the first quadrant both X and Y-coordinates of any point are $+ve$.

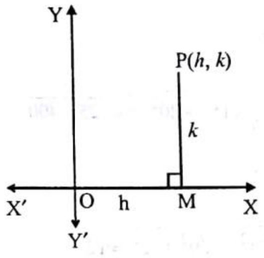
(c) In second quadrant, X-coordinate is $-ve$ and Y-coordinate is $+ve$.

(d) In third quadrant, both X and Y-coordinates of any point are $-ve$.

(e) In fourth quadrant, X-coordinate is $+ve$ and Y-coordinate is $-ve$ as shown in the above diagram.

PLOTTING OF A POINT WHOSE COORDINATES ARE KNOWN :

The point can be plotted by measuring its proper distances from both the axes. Thus, any point (h, k) can be plotted as follows :



(i) Measure OM equal to h along the X-axis.

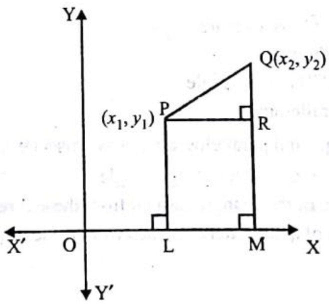
(ii) Now measure MP perpendicular to OM equal to K.

In this chapter, we shall study to find the distance between two given points, section fromula, mid-point formula, slope of a line, angles between two straight lines and equation of a line in different forms etc.

DISTANCE BETWEEN TWO POINTS :

Distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the plane is the length of the line segment PQ.

From P and Q draw PL and QM respectively perpendiculars on the X-axis and PR perpendicular on QM.



Then, $OL = x_1$, $OM = x_2$, $PL = y_1$ and $QM = y_2$

$\therefore\ PR = LM = OM - OL = x_2 - x_1$

$QR = QM - RM = QM - PL = y_2 - y_1$

Since PRQ is a right angled triangle,

$\therefore\ PQ^2 = PR^2 + QR^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$(By the Pythagoras Theorem)

$\therefore\ PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

i.e., Distance between any two points = $\sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$

Corollary : The distance of the point (x_1, y_1) from the origin (0, 0) is

$$\sqrt{(x_1-0)^2+(y_1-0)^2} = \sqrt{x_1^2+y_1^2}$$

Let us consider some examples to illustrate.

ILLUSTRATION 7.1

Find the distance between each of the following pair of points :
(a) $P(6, 8)$ and $Q(-9, -12)$ (b) $A(-6, -1)$ and $B(-6, 11)$

SOLUTION :

(a) Here the points are $P(6, 8)$ and $Q(-9, -12)$.
By using distance formula, we have

$$PQ = \sqrt{(-9-6)^2+(-12-8)^2} = \sqrt{15^2+20^2} = \sqrt{225+400} = \sqrt{625} = 25$$

Hence, $PQ = 25$ units.

(b) Here the points are $A(-6, -1)$ and $B(-6, 11)$
By using distance formula, we have

$$AB = \sqrt{\{-6-(-6)\}^2+\{11-(-1)\}^2} = \sqrt{0^2+12^2} = 12$$

Hence, $AB = 12$ units.

APPLICATIONS OF DISTANCE FORMULA :

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. First we find the length of AB, BC, and CA then we shall find that the point are
(a) Collinear, if the sum of two shorter distances is equal to the longest distance
(b) Vertices of an equilateral triangle if $AB = BC = CA$
(c) Vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$
(d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A, B, C and D;
(a) $AB = BC = CD = DA$; $AC = BD \Rightarrow ABCD$ is a square
(b) $AB = BC = CD = DA$; $\Rightarrow ABCD$ is a rhombus
(c) $AB = CD$, $BC = DA$; $AC = BD \Rightarrow ABCD$ is a rectangle
(d) $AB = CD$, $BC = DA$; $\Rightarrow ABCD$ is a parallelogram

(iii) (a) Diagonal of square, rhombus, rectangle and parallelogram always bisect each other
(b) Diagonal of rhombus and square bisect each other at right angle.
(c) Three given points are collinear if area of the triangle formed from these three points is zero.
(d) Four given points are collinear, if area of quadrilateral formed from these four points is zero.

SECTION FORMULA :

To find the co-ordinates of a point, which divides the line segment joining two points internally in a given ratio :

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the two given points and P be a point on AB which divides it in the given ratio m : n i.e., $AP : PB$ = m : n We have to find the co-ordinates of P. Let P = (x, y).

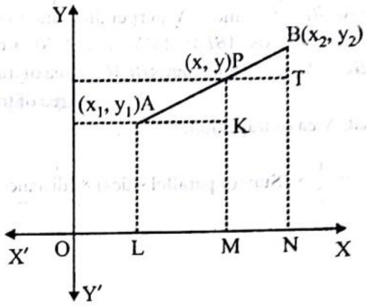
Draw the perpendiculars AL, PM, BN on OX, and, AK, PT on PM and BN respectively. Then, from similar triangles AKP and PTB,

we have, $\dfrac{AP}{PB} = \dfrac{AK}{PT} = \dfrac{PK}{BT}$ .... (i)

Now, $AK = LM = OM - OL = x - x_1$
$PT = MN = ON - OM = x_2 - x$

$PK = MP - MK = MP - LA = y - y_1$
$BT = NB - NT = NB - MP = y_2 - y$

From (i), we have, $\dfrac{m}{n} = \dfrac{x - x_1}{x_2 - x} = \dfrac{y - y_1}{y_2 - y}$



The first two relations give, $\dfrac{m}{n} = \dfrac{x - x_1}{x_2 - x}$

or $\quad mx_2 - mx = nx - nx_1$
or $\quad x(m+n) = mx_2 + nx_1$
or $\quad x = \dfrac{mx_2 + nx_1}{m+n}$

Similarly, from the relation $\dfrac{AP}{PB} = \dfrac{PK}{BT}$, we get $\dfrac{m}{n} = \dfrac{y - y_1}{y_2 - y}$ which gives on simplification.

$$y = \frac{my_2 + ny_1}{m+n}$$

Hence, $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$ ....(1)

Hence co-ordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Mid-point formula :
The co-ordinates of the mid-point of the line segment joining two points (x_1, y_1) and (x_2, y_2) can be obtained by taking $m = n$ in the section formula above. Putting $m = n$ in (1) above, we have

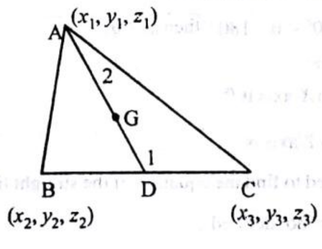
$$x = \frac{nx_2 + nx_1}{m+n} = \frac{x_2 + x_1}{2} \text{ and } y = \frac{ny_2 + ny_1}{m+n} = \frac{y_2 + y_1}{2}$$

Hence, co-ordinates of the mid-point joining two points (x_1, y_1) and (x_2, y_2) is $\left(\dfrac{x_2 + x_1}{2}, \dfrac{y_2 + y_1}{2}\right)$

CENTROID OF A TRIANGLE :

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of any triangle, then
The centroid is the point of intersection of the medians (Line segment joining the mid point of a side and its opposite vertex is called a median of the triangle) Centroid divides the median in the ratio of 2 : 1

Co-ordinates of centroid G $= \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$



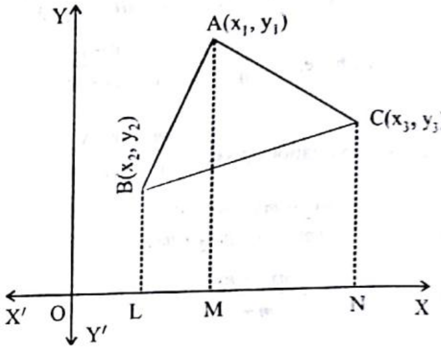
AREA OF A TRIANGLE :

Let ABC be any triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Draw BL, AM and CN perpendiculars from B, A and C respectively on the X-axis. $ABLM$, $AMNC$ and $BLNC$ are all trapeziums.

Area of ΔABC = Area of trapezium $ABLM$ + Area of trapezium $AMNC$ − Area of trapezium $BLNC$

We know that, Area of trapezium

$= \frac{1}{2} \times$ (Sum of parallel sides) × (distance between them)



Therefore, Area of $\Delta ABC = \frac{1}{2}(BL+AM)(LM)+\frac{1}{2}(AM+CN)(MN)-\frac{1}{2}(BL+CN)(LN)$

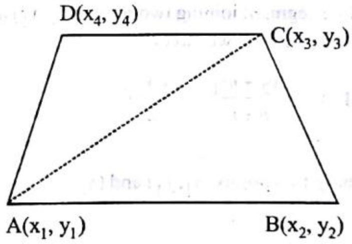
$$= \frac{1}{2}(y_2+y_1)(x_1-x_2)+\frac{1}{2}(y_1+y_3)(x_3-x_1)-\frac{1}{2}(y_2+y_3)(x_3-x_2)$$

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

AREA OF A QUADRILATERAL :

Let the vertices of quadrilateral $ABCD$ are $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$

So, Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD



ANGLE OF INCLINATION AND SLOPE OF A STRAIGHT LINE :

The angle θ formed by a straight line l with the positive direction of X-axis in anticlockwise is called the INCLINATION of the line l.

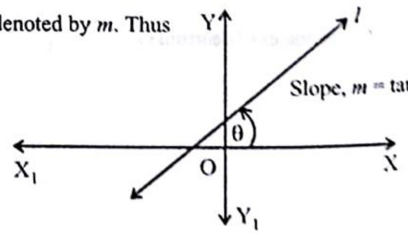
Obviously $0° \le \theta < 180°$.

If θ is the inclination then $\tan\theta$ is defined as the slope of the straight line l and denoted by m. Thus

Slope of the line l, $m = \tan\theta$

Clearly, the slope of any line parallel to Y-axis is not defined.

Again, if $0° < \theta < 90°$, then $m > 0$ and if $90° < \theta < 180°$ then $m < 0$.



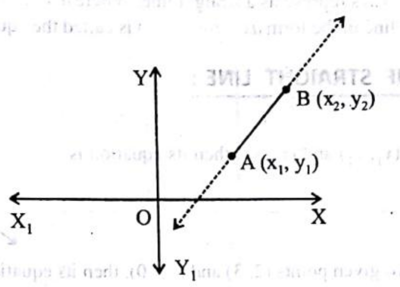
(a) Slope of any two parallel lines are same.

(b) Slope of X-axis and any line parallel to X-axis is 0.

(c) Slope of Y-axis and any line parallel to Y-axis is $\frac{1}{0}$.

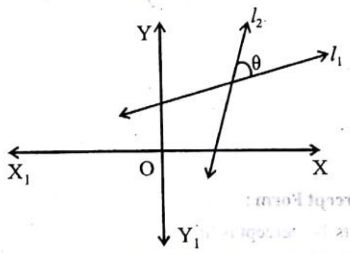
NOTE : When slope of a straight line is used to find the equation of the straight line, then slope of Y-axis or any line parallel to Y-axis is taken as '$\frac{1}{0}$' only, not as '∞', 'infinite' or 'not defined'.

Slope of a Line joining two points :



If a non-vertical line passes through two distinct points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the slope of this line AB is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

ANGLE BETWEEN TWO STRAIGHT LINES :

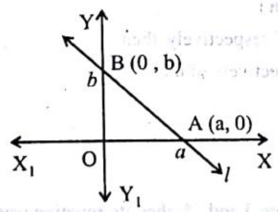


If the slope of two straight lines be m_1 and m_2, then the acute angle (θ) between the lines is given by $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

The two lines are parallel if, $m_1 = m_2$.

The two lines are perpendicular if, $m_1 m_2 = -1$

INTERCEPTS OF A LINE ON THE COORDINATE AXES :



Suppose that a line l cuts the coordinate axes at the points A and B respectively. Let the coordinates of A and B are (a, 0) and (0, b). Then

X - intercept $= OA = a$

Y - intercept $= OB = b$

EQUATIONS OF STRAIGHT LINE :

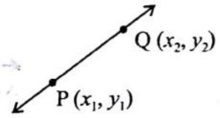
A linear equation of the form $ax + by + c = 0$ always represents a straight line. Where a, b, c are real numbers and both a and b can not be zero simultaneously. Equation of a straight line in the form $ax + by + c = 0$ is called the equation of a straight line in general form.

VARIOUS FORMS OF EQUATION OF STRAIGHT LINE :

(i) Equation of a line in Two Points Form :

If a line passes through two given point (x_1, y_1) and (x_2, y_2) then its equation is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



For example : If a line passes through two given points (2, 3) and (–3, 0), then its equation using the above formula is

$$y - 3 = \frac{0-3}{-3-2}(x-2) \Rightarrow y - 3 = \frac{3}{5}(x-2) \Rightarrow 5y - 15 = 3x - 6 \Rightarrow 3x - 5y + 9 = 0$$

(ii) Equation of a line in Point-Slope Form :

If *a* line passes through *a* point (x_1, y_1) and has slope m, then its equation is

$y - y_1 = m(x - x_1)$

For example : If a line passes through point (2, –3) has slope $\frac{1}{2}$, then its equation by using the above formula is

$$y - (-3) = \frac{1}{2}(x-2)$$

$$\Rightarrow y + 3 = \frac{1}{2}(x-2)$$

$$\Rightarrow 2y + 6 = x - 2$$

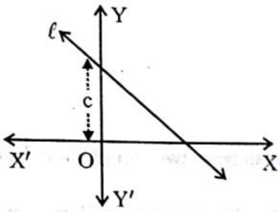
$$\Rightarrow x - 2y - 8 = 0$$

(iii) Equation of a straight line in Slope Intercept Form :

If a line ℓ intersect the Y-axis at 'c' then its Y-intercept is 'c'.

If slope of this line ℓ is m, then its euqation in slope intercept form is

$y = mx + c$



For example : If a line having slope 3, intersect the Y-axis at 2 distance above the origin, then its equation using the above formula is

$y = 3x + 2$

And if a line having slope 1 intersects the Y-axis at 2 distance belwo the Y-axis, then its equation using the above formula is

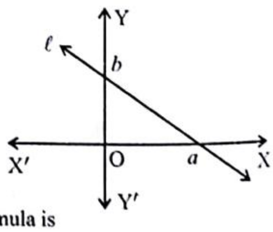
$y = 1, x + (-2)$

$\Rightarrow y = x - 2 \Rightarrow x - y - 2 = 0$

(iv) Equation of *a* straight line in Intercept Form :

If a line ℓ intersect X and Y-axis at 'a' and 'b' respectively, then 'a' and 'b' are called X and Y-intercepts respectively of the line. Equation of this line is

$$\frac{x}{a} + \frac{y}{b} = 1$$



For example : If X and Y-intercept of a line are 3 and –5, then its equation using the above formula is

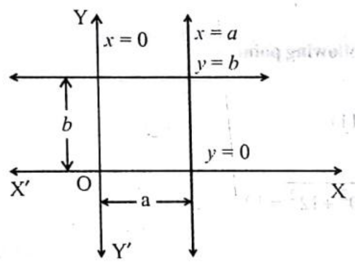
$$\frac{x}{3} + \frac{y}{-5} = 1 \Rightarrow \frac{-5x + 3y}{-15} = 1$$

$$\Rightarrow -5x + 3y = -15 \Rightarrow 5x - 3y - 15 = 0$$

NOTE:

(a) You can use any form (or formula) to find the equation of a straight line according to the information given about the line. But we preffer to write the final equation after simplication either in General Form $ax + by + c = 0$ or Slope Intercept Form $y = mx + c$

(b) The equation of X-axis is $\boldsymbol{y = 0}$.



(c) The equation of Y-axis is $\boldsymbol{x = 0}$.

(d) The equation of a straight line parallel to X-axis at a distance 'b' from it is $\boldsymbol{y = b}$ or $\boldsymbol{y - b = 0}$

(e) The equation of a straight line parallel to Y-axis at a distance 'a' from it is $\boldsymbol{x = a \Rightarrow x - a = 0}$

MISCELLANEOUS SOLVED EXAMPLES

1. Find the distance between each of the following points :
$A(-6,-1)$ and $B(-6,11)$

Sol. Here the points are $A(-6,-1)$ and $B(-6, 11)$

By using distance formula, we have

$$AB=\sqrt{\{-6-(-6)\}^2+\{11-(-1)\}^2}=\sqrt{0^2+12^2}=12$$

Hence, $AB = 12$ units.

2. The distance between two points (0, 0) and (x, 3) is 5. Find x.

Sol. By using distance formula, we have the distance between (0, 0) and (x, 3) is $\sqrt{(x-0)^2+(3-0)^2}$

It is given that $\sqrt{(x-0)^2+(3-0)^2}=5$

or $\sqrt{x^2+3^2}=5$

Squaring both sides, $x^2+9=25$ or $x^2=16$ or $x=\pm 4$

Hence, $x=+4$ or $x=-4$

3. Show that the points (1, 1), (3, 0) and (–1, 2) are collinear.

Sol. Let $P(1, 1)$, $Q(3, 0)$ and $R(-1, 2)$ be the given points

$$PQ=\sqrt{(3-1)^2+(0-1)^2}=\sqrt{4+1}=\sqrt{5}\text{ units},$$

$$QR=\sqrt{(-1-3)^2+(2-0)^2}=\sqrt{16+4}=2\sqrt{5}\text{ units}$$

$$RP=\sqrt{(-1-1)^2+(2-1)^2}=\sqrt{4+1}=\sqrt{5}\text{ units}$$

Now, $PQ+RP=(\sqrt{5}+\sqrt{5})\ units=2\sqrt{5}\ units=QR$

$\therefore$ P, Q, and R are collinear points.

4. Find the coordinates of a point on Y-axis which is equidistant from the points (13, 2) and (12, –3).

Sol. Let $P(0, y)$ be the required point and the given points be $A(12, -3)$ and $B(13, 2)$.

Then $PA = PB$ (given)

$$\sqrt{(12-0)^2+(-3-y)^2}=\sqrt{(13-0)^2+(2-y)^2}$$

$$\Rightarrow \sqrt{144+(y+3)^2}=\sqrt{169+(2-y)^2}$$

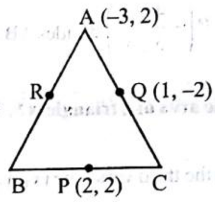
Taking square on both sides, we get

$144+9+y^2+6y=169+4+y^2-4y$

$\Rightarrow 10y=20 \Rightarrow y=2$

$\therefore$ The required point on Y-axis is (0, 2).

5. ABC is a triangle in which P, Q, R are the mid-points of BC, CA, AB respectively. The coordinates are $A(-3, 2)$, $Q(1, -2)$ and $P(2, 2)$. Find $\overline{RQ}$.



Sol. Let the coordinates of C be (x, y).

$\Rightarrow \left(\frac{-3+x}{2}, \frac{2+y}{2}\right) = (1, -2)$

$\Rightarrow \frac{-3+x}{2} = 1, \frac{2+y}{2} = -2 \Rightarrow x = 5, y = -6$

Let the coordinates of B be (a, b).

$\Rightarrow \left(\frac{a+5}{2}, \frac{b-6}{2}\right) = (2, 2) \Rightarrow a = -1, b = 10$

Coordinates of $R = \left(\frac{-1-3}{2}, \frac{10+2}{2}\right) = (-2, 6)$

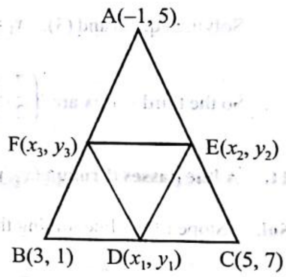
$\overline{RQ} = \sqrt{(1+2)^2 + (-2-6)^2} \Rightarrow \sqrt{9+64} = \sqrt{73}$

6. Find the ratio in which the join of $(-4, 3)$ and $(5, -2)$ is divided by (i) X-axis (ii) Y-axis.

Sol. (i) X-axis divides the join of (x_1, y_1) and (x_2, y_2) in the ratio of $-y_1 : y_2 = -3 : -2 = 3 : 2$.

(ii) Y-axis divides, in the ratio of $-x_1 : x_2 \Rightarrow 4 : 5$.

7. The coordinates of A, B and C are $(-1, 5)$, $(3, 1)$ and $(5, 7)$ respectively, D, E and F are the middle points of BC, CA and AB respectively. Calculate the area of the triangle DEF.



Sol. Mid-point D $(x_1, y_1) = \left(\frac{3+5}{2}, \frac{1+7}{2}\right) = (4, 4)$

Mid-point E $(x_2, y_2) = \left(\frac{-1+5}{2}, \frac{5+7}{2}\right) = (2, 6)$

Mid-point F $(x_3, y_3) = \left(\frac{-1+3}{2}, \frac{5+1}{2}\right) = (1, 3)$

Now, using the formula, area of triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$\Rightarrow$ Area of $\Delta DEF = \frac{1}{2}|4(6-3) + 2(3-4) + 1(4-6)| = 4$ square units.

Hence, the area of ΔDEF is 4 square units.

8. What kind of triangle is formed by $A(1, 2)$, $B(4, 3)$ and $C(5, 6)$?

Sol. $AB^2 = (4-1)^2 + (3-2)^2 = 9 + 1 = 10$

$BC^2 = (5-4)^2 + (6-3)^2 = 1 + 9 = 10$

$CA^2 = (5-1)^2 + (6-2)^2 = 16 + 16 = 32$

$AB^2 = BC^2 \Rightarrow$ it is isosceles.

$CA^2 > AB^2 + BC^2$ since $32 > 10 + 10 \Rightarrow \angle B$ is obtuse

Hence, ABC is an obtuse isosceles Δ.

9. Find the co-ordinates of a point which divides the line segment joining each of the following points in the given ratio : $(-1, 4)$ and $(0, -3)$ in the ratio 1 : 4 internally.

Sol. Let $A(-1, 4)$ and $B(0, -3)$ be the given points and let $P(x, y)$ divide AB in the ratio 1 : 4 internally

Using section formula, we have

$$x = \frac{1\times 0 + 4\times(-1)}{1+4} = -\frac{4}{5} \quad \text{and} \quad y = \frac{1\times(-3)+4\times 4}{1+4} = \frac{13}{5}$$

$\therefore \; P\left(-\frac{4}{5}, \frac{13}{5}\right)$ divides AB in the ratio 1 : 4 internally.

10. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

Sol. Let the third vertex be (x_3, y_3), area of triangle $= \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$

As $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = 2$, Area of $\Delta = 5$

$\Rightarrow \; 5 = \frac{1}{2}|2(-2 - y_1) + 3(y_3 - 1) + x_3(1+2)|$

$\Rightarrow \; 10 = |3x_3 + y_3 - 7| \Rightarrow 3x_3 + y_3 = \pm 19$

Taking positive sign, $3x_3 + y_3 - 7 = 10 \Rightarrow 3x_3 + y_3 = 17$ .... (1)

Taking negative sign, $3x_3 + y_3 - 7 = -10 \Rightarrow 3x_3 + y_3 = -3$ .... (2)

Given that (x_3, y_3) lies on $y = x + 3$

So, $-x_3 + y_3 = 3$ .... (3)

Solving eq. (1) and (3), $x_3 = \frac{7}{2}$, $y_3 = \frac{13}{2}$

Solving eq. (2) and (3), $x_3 = \frac{-3}{2}$, $y_3 = \frac{3}{2}$.

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

11. A line passes through (x_1, y_1) and (h, k). If the slope of the line be m, show that $k - y_1 = m(h - x_1)$.

Sol. Slope of the line joining the points $A(x_1, y_1)$ and $B(h, k)$

$$= \frac{k - y_1}{h - x_1} = m \qquad \text{(Given)}$$

$\therefore$ Cross multiplying, $k - y_1 = m(h - x_1)$

12. If three points $(h, 0)$, (a, b) and $(0, k)$ lies on a line, show that $\frac{a}{h} + \frac{b}{k} = 1$.

Sol. The given points are $A(h, 0)$, $B(a, b)$, $C(0, k)$, they lie on the same plane.

$\therefore$ Slope of AB = Slope of BC

$\therefore$ Slope of $AB = \frac{b - 0}{a - h} = \frac{b}{a - h}$; Slope of BC $= \frac{k - b}{0 - a} = \frac{k - b}{-a}$

$\therefore \; \frac{b}{a - h} = \frac{k - b}{-a}$ or by cross multiplication

$-ab = (a - h)(k - b)$ or $-ab = ak - ab - hk + hb$

or $0 = ak - hk + hb$ or $ak + hb = hk$

Dividing by hk, $\frac{ak}{hk} + \frac{hb}{hk} = 1$ or $\frac{a}{h} + \frac{b}{k} = 1$

13. Find the equation of the line passing through $(2, 2\sqrt{3})$ and inclined with the X-axis at an angle of 60°.

Sol. We have $x_1 = 2, y_1 = 2\sqrt{3}$

$m = \tan 60° = \sqrt{3}$

Equation of the line required is, $y - 2\sqrt{3} = \sqrt{3}(x-2) \Rightarrow y - \sqrt{3}x = 0$

14. Find the equation of the line passing through the points (–1, 1) and (2, –4).

Sol. The line passes through the points $A(-1, 1)$ $B(2, -4)$

Equation of the line passing through (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

We have $x_1 = -1, y_1 = 1, x_2 = 2, y_2 = -4$

$\therefore$ Equation of AB is $y - 1 = \frac{-4-1}{2+1}(x+1)$

$\Rightarrow y - 1 = \frac{-5}{3}(x+1) \quad \Rightarrow \quad 3(y-1) = -5(x+1)$

$3y - 3 = -5x - 5 \Rightarrow 5x + 3y - 3 + 5 = 0 \Rightarrow 5x + 3y + 2 = 0$

15. Find the equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Sol. Let the intercepts along X-axis and Y-axis be a and b

$a + b = 9$

or $b = 9 - a$ ...(i)

The point $A(2, 2)$ lies on the line

$\therefore$ Equation of the line intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$ ...(ii)

$\Rightarrow \frac{x}{a} + \frac{y}{9-a} = 1$ [Using (i)]

Since, point $A(2, 2)$ lies on it

$\frac{2}{a} + \frac{2}{9-a} = 1$

$\Rightarrow \frac{2(9-a) + 2a}{a(9-a)} = 1$

$\Rightarrow 18 - 2a + 2a = a(9-a)$

$18 = 9a - a^2$

$\Rightarrow a^2 - 9a + 18 = 0 \Rightarrow (a-3)(a-6) = 0$

$a = 3$ or $a = 6$

$b = 9 - a$ $\quad$ $b = 9 - a$

$b = 9 - 3 = 6$ $\quad$ $b = 9 - 6 = 3$

$\Rightarrow$ line is, $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{6} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{6} = 1$

$2x + y = 6$ and $x + 2y = 6$

Hence, the required equation of straight lines are, $2x + y = 6$ and $x + 2y = 6$

16. Find equation of the line parallel to the line $3x-4y+2=0$ and passing through the point $(-2,3)$

Sol. The given line is $3x-4y+2=0$

Slope of the line $=\frac{3}{4}$

Let eqn. of the parallel line to the given line $3x-4y+2$ is $3x-4y+\lambda=0$

$\therefore$ Slope of the parallel line $=\frac{3}{4}$

Therefore the eqn. of parallel line through $(-2, 3)$ is

$$y-3=\frac{3}{4}(x+2)$$

or $4y-12=3x+6$

or $3x-4y+12+6=0$ or $3x-4y+18=0$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Points $(3, 2), (-2, -3)$ and $(2, 3)$ form a .... triangle.
2. If $x-y=2$ then point (x, y) is equidistant from $(7, 1)$ and (.........)
3. Distance between $(2, 3)$ and $(4, 1)$ is ....
4. Points $(1, 5), (2, 3)$ and $(-2, -11)$ are ....
5. $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an .... triangle.
6. Point on the X-axis which is equidistant from $(2, -5)$ and $(-2, 9)$. ....
7. Point $(-4, 6)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio ....
8. $(1, 2), (4, y), (x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, x and y are ....
9. Area of a rhombus if its vertices are $(3, 0), (4, 5), (-1, 4)$ and $(-2, -1)$ taken in order is ....
10. Area of a triangle formed by the points $A(5, 2), B(4, 7)$ and $C(7, -4)$ ....
11. Relation between x and y if the points $(x, y), (1, 2)$ and $(7, 0)$ are collinear is ....
12. Y-axis divides the join of (x_1, y_1) and (x_2, y_2) in the ratio of ....
13. The area of the triangle enclosed by the axes and $\frac{x}{a}+\frac{y}{b}=1$ is ....
14. The distance of the point (x_1, y_1) from the origin ....
15. The .... of a moving point is the path traced out by it under some geometrical conditions.
16. y-intercept of the line with equation, $y=3x+6$ is ....
17. Slope of the line perpendicular to the line with equation, $y=6x+7$ is ....

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}$
2. The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2-m_2x_1}{m_1+m_2}, \frac{m_1y_2-m_2y_1}{m_1+m_2}\right)$
3. The mid-point of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$.
4. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is the numerical value of the expression $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

5. Points (1, 7), (4, 2), (−1, −1) and (−4, 4) are the vertices of a square.
6. Coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2 : 3 is (1, 3)
7. Ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6) is 3 : 7
8. Area of the triangle formed by the points $P(-1.5, 3)$, $Q(6, -2)$ and $R(-3, 4)$ is 0.
9. The ratio in which the point (3, 5) divides the join of (1, 3) and (4, 6) is 2 : 1
10. The distance of the point (5, 3) from the X-axis is 5 units
11. The slope of the line perpendicular to $5x+3y+1=0$ is $\frac{4}{5}$.
12. The point of intersection of the lines $x = 2$ and $y = 3$ is (2, 3)
13. The distance of a point (2, 3) from Y-axis is y-units.

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Column II gives distance between pair of points given in column I match them correctly.

Column I	Column II
(A) $(-5,7),(-1,3)$	(p) $\sqrt{17}$
(B) $(5,6),(1,3)$	(q) $\sqrt{8}$
(C) $(\sqrt{3}+1,1),(0,\sqrt{3})$	(r) $\sqrt{6}$
(D) $(0,0)(-\sqrt{3},\sqrt{3})$	(s) $4\sqrt{2}$

2. Column II gives the coordinates of the point P that divides the line segment joining the points given in column I, match them correctly.

Column I	Column II
(A) $A(-1,3)$ and $B(-5,6)$ internally in the ratio 1 : 2	(p) $(7,3)$
(B) $A(-2,1)$ and $B(1,4)$ internally in the ratio 2 : 1	(q) $(0,3)$
(C) $A(-1,7)$ and $B(4,-3)$ internally in the ratio 2 : 3	(r) $(1,3)$
(D) $A(4,-3)$ and $B(8,5)$ internally in the ratio 3 : 1	(s) $(1,0)$

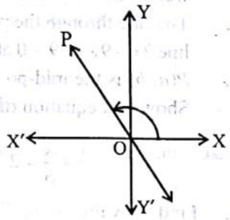
3. Column II gives the area of triangles whose vertices are given in column I, match them correctly.

Column I	Column II
(A) $(2,3),(-1,0),(2,-4)$	(p) 40
(B) $(-5,-1),(3,-5),(5,2)$	(q) 24
(C) $(1,-1),(-4,6),(-3,-5)$	(r) 32
(D) $(0,0),(8,0),(0,10)$	(s) 10.5

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find the area of a triangle whose vertices are $A(-8,-2)$, $B(-4,-6)$ and $C(-1,5)$
2. Find the radius of the circle whose centre is at (0, 0) and which passes through the point (−6, 8)
3. If distance between the point $(x, 2)$ and (3, 4) is 2, then find the value of x ?
4. A, B and C are three collinear points. The coordinates of A and B are (3, 4) and (7, 7) respectively and $AC = 10$ units. Find the co-ordinates of C.
5. Find the ratio in which the line $x + y = 4$ divides the line joining the points (−1, 1) and (5, 7)
6. Find the centroid of a triangle, whose vertices are (2, 1), (5, 2) and (3, 4).
7. The two vertices of a triangle are (6, 3) and (−1, 7) and its centroid is (1, 5). Find the third vertex.
8. Find the area of the triangle whose vertices are (a, a), $(a+1, a+1)$, $(a+2, a)$.
9. The point A divides the join of points (−5, 1) and (3, 5) in the ratio k : 1 and coordinates of points B and C are (1, 5) and (7, −2) respectively. If the area of ΔABC be 2 units, then find the value of k.
10. The line joining (−1, 4) and (5, y) is parallel to the line joining $\left(\frac{17}{2},-1\right)$ and $\left(\frac{5}{2},-\frac{5}{2}\right)$. Find the value of y.
11. Find the slope of the line, which makes an angle of 30° with the positive direction of Y-axis measured anticlockwise.



12. Find the equation of the line passing through (−3, 5) and perpendicular to the line through the points (2, 5) and (−3, 6).
13. Find the equation of the line, Which passes through (2, −5) and cuts off equal intercepts on both the axes.
14. Find the equation of the line perpendicular to the line $x-7y+5=0$ and having x intercept 3
15. Find the equation of the right bisector of the line segment joining the points (3, 4) and (−1, 2).
16. Find the equation of a line passing through point (5, 1) and parallel to the line $7x-2y+5=0$.
17. Write down the gradient and intercept on the Y-axis of the line $\frac{x}{3}+\frac{y}{4}=1$.

Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Show that the points $A(5, 6)$, $B(1, 5)$, $C(2, 1)$ and $D(6, 2)$ are the vertices of a square.
2. The graph of the equation $y = mx + c$ passes through the points $(1, 4)$ and $(-2, -5)$. Determine the values of m and c.
3. Find the equation of the line through the intersection of $4x - y = 2$ and $x + 2y = 5$ and perpendicular to $3x - y = 5$.
4. Detemine the ratio in which the point P (m, 6) divides the join of A (– 4, 3) and B (2, 8). Also find the value of m.
5. The lines represented by $3x + 4y = 8$ and $px + 2y = 7$ are parallel. Find the value of p.
6. The co-ordinates of the mid-point of a line segment are (2, 3). If co-ordinates of one of the end points of the line segment are (6, 5), find the co-ordinates of the other end point.
7. If A (3, 5), B (–5, – 4), C (7, 10) are the vertices of a parallelogram taken in the order, then find the co-ordinates of the fourth vertex.
8. Find a point on the X-axis, which is equidistant from the points (7, 6) and (3, 4).
9. The vertices of ΔPQR are $P(2, 1)$, $Q(-2, 3)$ and $R(4, 5)$. Find equation of the median through the vertex R.
10. The owners of milk store finds that, he can sell 980 litres of milk each week at ₹ 14 / litre and 1220 litres of milk each week at ₹ 16/ litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹ 17/ litre?
11. The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$ at right angle find the value of h.
12. $P(a, b)$ is the mid-point of a line segment between axes. Show that equation of the line is

$$\frac{x}{a} + \frac{x}{b} = 2.$$

13. Find the value of m, if the line passing through the points $A(2, -3)$ and $B(3, m + 5)$ is perpendicular to the line passing through the points $P(-2, 3)$ and $Q(-4, -5)$.
14. Find the value of k for which the lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are perpendicular to each other.
15. Find the equation to the straight line which passes through the point (1, 2) and the point of intersection of the lines. $x + 3y + 1 = 0$ and $2x + 7y + 3 = 0$
16. Find the median to the side BC of the triangle whose vertices are $A(-2, 1)$, $B(2, 3)$ and $C(4, 5)$.

Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Find the value of m for which the points with coordinates (3, 5), (m, 6) and (1/2, 15/2) are collinear.
2. Prove that the points (4, 4), (3, 5) and (–1, –1) are the vertices of a right angled triangle.
3. A line perpendicular to the lines segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : n. Find the equation of the line.
4. Find the co-ordinates of the points of trisection of the line segment joining (4, – 1) and (–2, –3).
5. In what ratio is the line segment joining the points (–2, –3) and (3, 7) divided by the Y-axis? Also, find the coordinates of the point of division.
6. Find the equation of the straight line which passes through the point of intersection of the lines $2x - y + 5 = 0$ and $5x + 3y - 4 = 0$ and is perpendicular to $x - 3y + 21 = 0$.
7. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.
8. Find the area of the triangle formed by the points joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
9. Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line $3x - 4y - 16 = 0$
10. The vertices of a triangle ABC are $A(4, 6)$, $B(1, 5)$ and $C(7,2)$. A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC.
11. Show that the points (2, 0), (–6, –2), (–4, –4) and (4, –2) form a parallelogram.

EXERCISE 2

Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. P, Q, R are three collinear points. The coordinates of P and R are (3, 4) and (11, 10) respectively and PQ is equal to 2.5 units. Coordinates of Q are –
   (a) (5, 11/2) (b) (11, 5/2)
   (c) (5, –11/2) (d) (–5, 11/2)
2. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is (–2, 4), then x and y respectively are –
   (a) – 6 and 1 (b) – 6 and 2
   (c) 6 and – 1 (d) 6 and – 2
3. The ratio in which the point (2, y) divides the join of (– 4, 3) and (6, 3) and hence the value of y –
   (a) $2:3, y=3$ (b) $3:2, y=4$
   (c) $3:2, y=3$ (d) $3:2, y=2$
4. Ratio in which the line $3x + 4y = 7$ divides the line segment joining the points (1, 2) and (–2, 1) is
   (a) 3 : 5 (b) 4 : 6
   (c) 4 : 9 (d) None of these
5. The point on the X-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$ is
   (a) (0,2) (b) (2,0)
   (c) (3,0) (d) (–2,0)
6. The area of the triangle formed by the line $5x - 3y + 15 = 0$ with coordinate axes is
   (a) 15 cm^2 (b) 5 cm^2
   (c) 8 cm^2 (d) $\frac{15}{2}$ cm^2
7. The point which divides the line joining the points A (1, 2) and $B(-1, 1)$ internally in the ratio 1 : 2 is
   (a) $\left(\frac{-1}{3}, \frac{5}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{5}{3}\right)$
   (c) (–1, 5) (d) (1, 5)
8. The centroid of the triangle whose vertices are (3, –7), (–8, 6) and (5, 10) is
   (a) (0,9) (b) (0,3)
   (c) (1,3) (d) (3,5)
9. The points $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ and $D(2, 3)$ are the vertices of a –
   (a) Parallelogram (b) Rectangle
   (c) Rhombus (d) Square
10. If the point P (p, q) is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$ then –
    (a) $ap = by$ (b) $bp = ay$
    (c) $ap + bq = 0$ (d) $bp + aq = 0$
11. If $y = ax^2 + 7x - 15$ makes an intercept of $1\frac{1}{2}$ units on X-axis, then the value of 'a' is
    (a) 7 (b) –15
    (c) 2 (d) –8
12. The angle made by the line $\sqrt{3}x - y + 3 = 0$ with the positive direction of X-axis is
    (a) 30° (b) 45°
    (c) 60° (d) 90°
13. The equation to the line passing through the intersection of $\frac{x}{a}+\frac{y}{b}=1, \frac{x}{b}+\frac{y}{a}=1$, where $ab = a + b$ and (1, 2) is
    (a) $x=1$ (b) $x=2$
    (c) $y=1$ (d) $y=2$
14. Equation of a line whose inclination is 45° and making an intercept of 3 units of X-axis is
    (a) $x+y-3=0$ (b) $x-y-3=0$
    (c) $x-y+3=0$ (d) $x+y+3=0$
15. Equation of a straight line out of the following :
    (a) $y=2x^2$ (b) $x^2+y^2=a^2$
    (c) $\frac{x}{a}+\frac{y}{b}=1$ (d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
16. The equation of the line with inclination 45° and passing through the point (–1, 2) is
    (a) $x+y+3=0$ (b) $x-y+3=0$
    (c) $x-y-3=0$ (d) $x+y-3=0$

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. The area of a triangle is 5 and its two vertices are $A(2, 1)$ and $B(3, -2)$. The third vertex lies on $y = x + 3$. Then third vertex is
   (a) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{5}{2}\right)$
   (c) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (d) (0, 0)

2. The medians AD and BE of the triangle with vertices $A(0, b)$, $B(0, 0)$, $C(a, 0)$ are mutually perpendicular if
(a) $b=\sqrt{2}a$ (b) $a=\sqrt{2}b$
(c) $b=-\sqrt{2}a$ (d) $a=-\sqrt{2}b$

3. The equation of the line parallel to $3x - 2y + 7 = 0$ and making an intercept –4 on X-axis is
(a) $3x-2y+12=0$ (b) $3x-2y-12=0$
(c) $3x+2y-12=0$ (d) $-3x+2y-12=0$

4. Which of the following points is not 10 units from the origin?
(a) $(-6, 8)$ (b) $(-4,-6)$
(c) $(-6,-8)$ (d) $(6,4)$

5. Equation of a straight line passing through the point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ and perpendicular to one of them is
(a) $x+y+3=0$ (b) $x+y-3=0$
(c) $x-3y-5=0$ (d) $x-3y+5=0$

6. The distance between which two points is 2 units?
(a) $(-2,-3)$ and $(-2,-4)$ (b) $(0,4)$ and $(0,6)$
(c) $(7,2)$ and $(6,2)$ (d) $(4,-3)$ and $(2,-3)$

7. Find the locus of a variable point whose distance from A (4, 0) is equal to its distance from B (0, 2).
(a) $6x-3y-9=0$ (b) $2x-y-3=0$
(c) $2x-y+3=0$ (d) $6x+3y+9=0$

PBQ Passage Based Questions:

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Let there be two points (4, 1) and (5, –2) in a two dimensional coordinate system. A line which passes through the above give points and intersects the coordinate axes forms a triangle.

1. The equation of the line passing through the above given points is
(a) $3x-y+13=0$ (b) $3x+y-13=0$
(c) $x+3y-13=0$ (d) $x-3y+13=0$

2. The point of intersection of the above line with both the coordinate axes is
(a) (13/3,0) and (0, 13) (b) (0, 13/3) and (13, 0)
(c) (13, 0) and (0, 1/3) (d) none of these

3. The area of the triangle so formed is
(a) $\frac{169}{3}$ sq. units (b) $\frac{169}{9}$ sq. units
(c) $\frac{169}{6}$ sq. units (d) none of these

A&R Assertion & Reason:

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** Let the vertices of a $\Delta\, ABC$ are A (–5, –2), B (7, 6) and C (5, – 4), then coordinate of circumcentre is (1, 2).
Reason : In a right angle triangle, mid-point of hypotenuse is the circumcentre of the triangle.
2. **Assertion :** If $A(2a, 4a)$ and $B(2a, 6a)$ are two vertices of a equilateral triangle ABC then the vertex C is given by $(2a+a\sqrt{3}, 5a)$.
Reason : In equilateral triangle all the coordinates of three vertices can be rational.
3. **Assertion :** The equation of the straight line which passes through the point (2, –3) and the point of the intersection of the lines $x+y+4=0$ and $3x-y-8=0$ is $2x-y-7=0$
Reason : Product of slopes of two perpendicular straight lines is -1.

MMQ Multiple Matching Questions:

DIRECTIONS : *Following question has statements (A, B, C, D...) given in Column I and statements (p, q, r, s....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

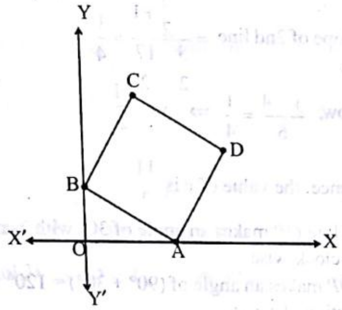
Column-I		Column-II	
(A)	Equation of a straight line	(p)	$y=mx+c$
(B)	Slope intercept form	(q)	$\frac{x}{a}+\frac{y}{b}=1$
(C)	Two point form	(r)	$m_1=m_2$
(D)	Parallel lines	(s)	$m_1 . m_2=-1$
(E)	Perpendicular lines	(t)	$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

HOTS Subjective Questions:

DIRECTIONS : *Answer the following questions.*

1. $ABCD$ is a quadrilateral fromed by the points A (–1, –1), B (–1, 4), C(5, 4) and D(5, –1). P, Q, R and S are the mid-point of AB, BC, CD and DA respectively. What type of quadrilateral $PQRS$?

2. The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.
3. In the diagram $ABCD$ is a square. Find the coordinate of C and D if the coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.



4. $A(0, 1), B(-3, -1), C(1, -1)$ and $D(4, y)$ form a parallelogram. Find the slopes of its diagonals and the point of intersection of the diagonals.
5. The hypotenuse of a right isosceles triangle has its ends at the points (1, 3) and (–4, 1). Find the equation of the legs (perpendicular sides of the triangle).
6. Find the equations of the medians of a triangle formed by the lines $x+y-6=0, x-3y-2=0$ and $5x-3y+2=0$.
7. Find the equation of the line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1.	right angle	2.	(3,5)	3.	$2\sqrt{2}$
4.	Non-collinear	5.	isosceles	6.	(–7,0)
7.	2 : 7	8.	(6,3)	9.	24. sq. units
10.	2 sq. units	11.	$x+3y=7$		

12. $-x_1 : x_2$ [Remember the formula or work out by taking two arbitrary points]

13. $\frac{1}{2}|ab|$ [The three vertices of the Δ are (0, 0), (a, 0), (0, b) etc.]

14. $\sqrt{x_1^2+y_1^2}$ 15. locus 16. 6

17. (–1/6)

TRUE / FALSE :

1.	False	2.	False	3.	True
4.	True	5.	True	6.	True
7.	False	8.	True	9.	True
10.	False	11.	False	12.	True
13.	False				

MATCH THE FOLLOWING :

1. (a) → s; (b) → p; (c) → q; (d) → r
2. (a) → s; (b) → q; (c) → r; (d) → p
3. (a) → s; (b) → r; (c) → q; (d) → p

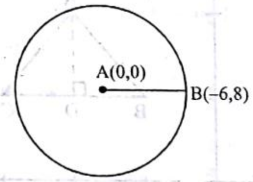
VERY SHORT ANSWER QUESTIONS :

1. Using the formula, area of triangle

$$=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

Area = 28 sq. units.

2. Let $A(0, 0)$ and $B(-6, 8)$ be the given points.
Now, radius of the cirlce is same as the distance of the line segment AB.



$\therefore$ $AB=\sqrt{(-6-0)^2+(8-0)^2}=\sqrt{36+64}=\sqrt{100}=10$

Hence radius of the circle is 10 units.

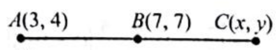
3. $2=\sqrt{(x-3)^2+(2-4)^2} \Rightarrow 2=\sqrt{(x-3)^2+4}$

Squaring both sides

$4=(x-3)^2+4 \Rightarrow x-3=0 \Rightarrow x=3$

4. $AB=\sqrt{(7-3)^2+(7-4)^2}=5$

$AC=10$



Since A, B and C are collinear

$BC=AC-AB=5$

$\Rightarrow B$ is the mid-point of AC.

If the coordinate of C are (x, y), then

$$\frac{x+3}{2}=7 \text{ and } \frac{y+4}{2}=7$$

$\Rightarrow x=11$ and $y=10$

Hence, the coordinates of C are $(11, 10)$.

5. Ratio $= -\left(\frac{-1+1-4}{5+7-4}\right)=\frac{1}{2}$

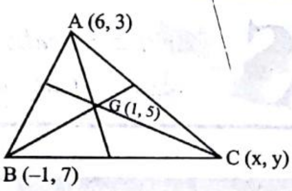
6. $x=\frac{2+5+3}{3}=\frac{10}{3}$ and $y=\frac{1+2+4}{3}=\frac{7}{3}$

7. Let ABC be a triangle whose vertices are

$A=(6, 3)$, $B=(-1, 7)$, $C=(x, y)$

and centroid $G=(1, 5)$

Then using the formula, for coordinates of centroid

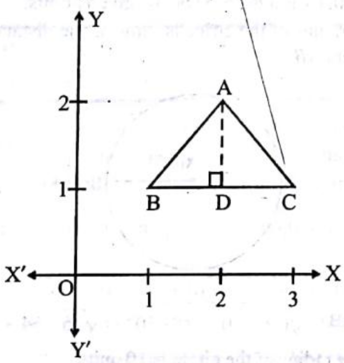


$$1=\frac{6+(-1)+x}{3} \text{ and } 5=\frac{3+7+y}{3}$$

$\Rightarrow x=-2$ and $y=5$

Hence, the third vertex is $C=(-2, 5)$

8. To make it easy, assume $a=1$



Therefore, the vertices are $(1, 1), (2, 2), (3, 1)$

$$\text{Area of } \Delta ABC=\frac{1}{2}\times AD\times BC=\frac{1}{2}\times 1\times 2=1$$

9. $A\equiv\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$, Area of $\Delta ABC=2$ units

$$\Rightarrow \frac{1}{2}\left[\left(\frac{3k-5}{k+1}\right)(5+2)+1\left(-2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}-5\right)\right]$$

$=\pm 2$

$\Rightarrow 14k-66=\pm 4(k+1) \Rightarrow k=7$ or $31/9$

10. Since the two lines are parallel, their slopes are equal.

$$\text{Slope of 1st line}=\frac{y-4}{5-(-1)}=\frac{y-4}{6}$$

$$\text{Slope of 2nd line}=\frac{-\frac{5}{2}+1}{\frac{5}{2}-\frac{17}{2}}=\frac{1}{4}$$

Now, $\frac{y-4}{6}=\frac{1}{4} \Rightarrow y=\frac{11}{2}$

Hence, the value of y is $\frac{11}{2}$.

11. The line OP makes an angle of 30° with Y-axis measured anti-clock-wise

$\therefore$ OP makes an angle of $(90°+30°)=120°$ with positive direction of X-axis

$\therefore$ Slope of $OP=\tan 120°$

$=\tan(180°-60°)=-\tan 60°=-\sqrt{3}$

12. $y-5=5(x+3) \Rightarrow 5x-y+20=0$

13. Here the intercepts on the both axes are equal, So, $a=b$.

Using the intercept form $\frac{x}{a}+\frac{y}{b}=1$

The line passes through $(2, -5)$,

$\Rightarrow \frac{2}{a}+\frac{-5}{a}=1 \Rightarrow a=-3$

The required equation is $\frac{x}{-3}+\frac{y}{-3}=1$

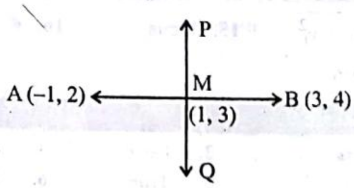
$\Rightarrow x+y+3=0$

14. $7x+y-21=0$

15. Slope of the line joining the points $A(-1, 2)$ and $B(3, 4)$

$$=\frac{4-2}{3+1}=\frac{2}{4}=\frac{1}{2}$$

PQ is the right bisector of AB



$\therefore$ slope of $PQ=-2$

middle point of AB is $\left(\frac{-1+3}{2}, \frac{2+4}{2}\right)$ i.e. $(1, 3)$

Right bisector passes through $M(1, 3)$

Equation of right bisector PQ is $y-3=-2(x-1)=-2x+2$

$\Rightarrow 2x+y-3-2=0 \Rightarrow 2x+y-5=0$

16. The required equation of the line is $7x-2y-33=0$.

17. $m=-4/3$, $c=4$

SHORT ANSWER QUESTIONS :

1. $AB = BC = CD = AD$

$\therefore\ AC^2 = AB^2 + BC^2 \Rightarrow \angle ABC = 90°$

$\therefore$ $ABCD$ is a square.

2. $m = 3,\ c = 1$

3. Point of intersection of the first two lines is got by solving them.

$4x - y = 2$ ...(1)

$x + 2y = 5$ ...(2)

$(1) \times 2 \Rightarrow 8x - 2y = 4$ ...(3)

Adding eq. (2) and (3), $9x = 9 \Rightarrow x = 1$

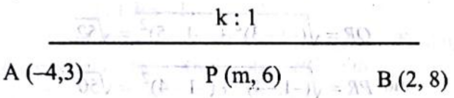
From (1), $4(1) - y = 2 \Rightarrow y = 2$

'm' of $3x - y = 5$ is 3   'm' of perpendicular line $= \dfrac{-1}{3}$

Reqd. eqn. is $(y-2) = \dfrac{-1}{3}(x-1) \Rightarrow 3y - 6 = -x + 1$

$\Rightarrow x + 3y = 7$

4. $A(-4, 3), B(2, 8)$ and $P(m, 6)$



Let P divides the join of AB in the ratio of k : 1

$\therefore$ y coordinate of P $= \dfrac{k \times 8 + 1 \times 3}{k+1}$

$\Rightarrow\ 6 = \dfrac{8k+3}{k+1} \Rightarrow 6k + 6 = 8k + 3 \Rightarrow 2k = 3 \Rightarrow k = 3/2$

$\therefore$ P divides the join of AB in the ratio of 3 : 2.

x coordiante of $P = \dfrac{3 \times 2 + 2 \times (-4)}{3+2}$

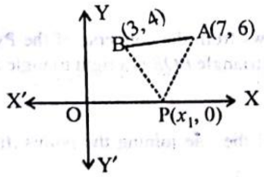
$\Rightarrow m = \dfrac{6-8}{5} \Rightarrow m = -2/5$

5. $p = 3/2$.

6. (–2, 1) is the co-ordinates of the other end point.

7. Co-ordinates of fourth vertex $D = (15, 19)$

8. Let the point on the X-axis be $(x_1, 0)$. The other two points A and B are $A(7, 6)$, $B(3, 4)$ we have $PA = PB$ or $PA^2 = PB^2$



or $(x_1 - 7)^2 + 6^2 = (x_1 - 3)^2 + 4^2$

or $x_1^2 - 14x_1 + 49 + 36 = x_1^2 - 6x_1 + 9 + 16$

or $-14x_1 + 85 = -6x_1 + 25$

or $8x_1 = 60$ or $x_1 = \dfrac{60}{8}$

$\therefore\ x_1 = \dfrac{15}{2}$

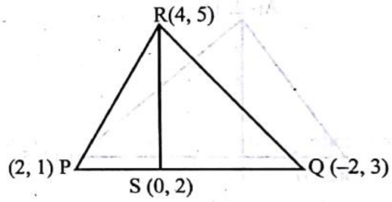
$\therefore$ The point P on X-axis equidistant from A and B is $\left(\dfrac{15}{2}, 0\right)$

9. The vertices P and Q are (2, 1) and (–2, 3) respectively.

The middle point is $\left(\dfrac{2-2}{2}, \dfrac{1+3}{2}\right)$ or (0, 2)

$\therefore$ Equation of the median RS, where R is (4, 5) and S is the point (0, 2) is

$y - 5 = \dfrac{2-5}{0-4}(x-4)$ or $y - 5 = \dfrac{-3}{-4}(x-4)$



or $4(y-5) = 3(x-4)$ or $4y - 20 = 3x - 12$

i.e. $3x - 4y = -8$

$\therefore$ Equation of median RS is $3x - 4y + 8 = 0$

10. Let y litre milk is sold at ₹ x/litre x and y have linear relationship i.e.,

$y = a + bx$ i.e. it is a straight line

Now, $y_1 = 980$ litre, $x_1 =$ ₹ 14/ litre, $y_2 = 1220$ litre, $x_2 =$ ₹ 16/ litre

Slope of the line $= \dfrac{1220 - 980}{16 - 14} = \dfrac{240}{2} = 120$

$\therefore$ Equation of the line, $y - 980 = 120(x - 14)$

when $x = 17, y = 980 + 120(17 - 14)$

$= 980 + 120 \times 3 = 980 + 360$

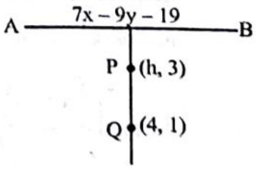
$y = 1340$

Hence $x = 1340$ litre milk may be sold at ₹ 17/litre

11. Slope of the line PQ passing through $P(h, 3)$ and $Q(4, 1)$ is $\dfrac{2}{h-4}$

Slope of the line AB,

$7x - 9y - 19 = 0$ *is* $\dfrac{7}{9}$



The lines AB and PQ are perpendicular to each other

$m_1 m_2 = -1$

or $\frac{2}{h-4}\times\frac{7}{9}=-1$ $\therefore$ $14=-9(h-4)$

or $9h=36-14$ or $9h=22$ or $h=\frac{22}{9}$

12. $\frac{x}{a}+\frac{x}{b}=2$ 13. $m=\frac{-33}{4}$ 14. $k=-5/2$

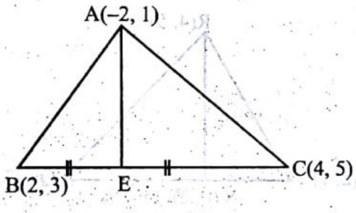
15. Hence, equation can be found out by using two point form.

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$\Rightarrow 3x+y-5=0.$

16. Let E be the mid-point of side BC.

$$\therefore E=\left(\frac{2+4}{2},\frac{3+5}{2}\right)=(3,4)$$



Equation of line AE is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

i.e., $y-1=\frac{4-1}{3-(-2)}(x-1)$

$$y-1=\frac{3}{5}(x-1)$$

$5y-5=3x-3$

$\therefore$ The required equation of the median is

$3x-5y+2=0$

LONG ANSWER QUESTIONS :

1. If points are collinear, then one point divides the other two in some ratio.

Let point (m, 6) divides the joint of (3, 5) and $\left(\frac{1}{2},\frac{15}{2}\right)$ in the ratio k : 1 Then

$$(m,6)=\left(\frac{\frac{k}{2}+3}{k+1},\frac{\frac{15k}{2}+3}{k+1}\right)\Rightarrow m=\frac{\frac{k}{2}+3}{k+1} \quad ....(1)$$

and $6=\frac{\frac{15k}{2}+3}{k+1}$ ....(2)

From (2), we get

$$6k+6=\frac{15k}{2}+3\Rightarrow 6k-\frac{15k}{2}+5=-1\Rightarrow k=\frac{2}{3}$$

Substituting, $k=\frac{2}{3}$ in (1), we get

$$m=\frac{\frac{1}{2}\times\frac{2}{3}+3}{\frac{2}{3}+1}=\frac{\frac{1}{3}+3}{\frac{2}{3}+1}=\frac{\frac{10}{3}}{\frac{5}{3}}=2$$

$\therefore$ For m = 2, points are collinear.

2. Let the points (4, 4), (3, 5) and (−1, −1) be denoted by P, Q and R, respectively.

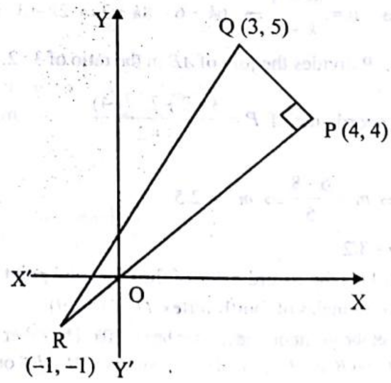
Now $PQ=\sqrt{(3-4)^2+(5-4)^2}=\sqrt{2}$

$QR=\sqrt{(-1-3)^2+(-1-5)^2}=\sqrt{52}$

and $PR=\sqrt{(-1-4)^2+(-1-4)^2}=\sqrt{50}$

Therefore, $PQ^2=2$, $QR^2=52$ and $PR^2=50$

We observe that the sum of square of two sides, PQ and PR, is equal t the square of the third side QR i.e., $QR^2=PR^2+PQ^2$



If follows from the converse of the Pythagoras theorem that the triangle PQR is a right triangle and the right angle is at P.

3. Slope of the line joining the points $A(1, 0)$ and $B(2, 3)$

$$=\frac{3-0}{2-1}=\frac{3}{1}=3$$

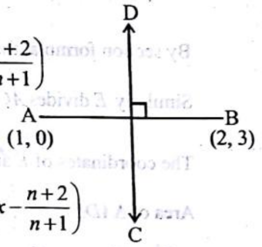
$\therefore$ Slope of the CD, perpendicular to $AB=-\frac{1}{3}$

[The lines are perpendicular to y, $m_1.m_2=-1$]
The point P divides AB in the ratio $1:n$

$\therefore$ Coordinates of P are,

$$\left(\frac{1\times2+1\times n}{1+n},\frac{1\times3+0\times n}{1+n}\right) or \left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

Equation of the line CD, which is perpendicular to AB passes through P is



$$y-\frac{3}{n+1}=-\frac{1}{3}\left(x-\frac{n+2}{n+1}\right)$$

Using the formula,

$$y-y_1=m(x-x_1)$$

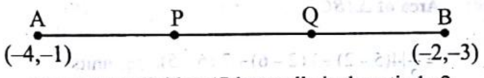
$$3(n+1)y-9=-(n+1)\left(x-\frac{n+2}{n+1}\right)$$

$$=-(n+1)x+(n+2)$$

$$\Rightarrow \quad (n+1)x+3(n+1)y=n+2+9$$

$$\Rightarrow \quad (n+1)x+3(n+1)y=n+11$$

4. Let P and Q are points of trisection (points dividing in three equal parts) of AB, i.e. $AP=PQ=QB$.



The point P divides AB internally in the ratio $1:2$.
Therefore, the coordinates of P will be given by :

$$x=\frac{m_2x_1+m_1x_2}{m_1+m_2}=\frac{2\times4+1(-2)}{1+2}=2$$

$$y=\frac{m_2y_1+m_1y_2}{m_1+m_2}=\frac{2\times(-1)+1(-3)}{1+2}=\frac{-5}{3}$$

Therefore, the co-ordinate of P are $\left(2,-\frac{5}{3}\right)$

For the coordinate of Q, $m_2=2$ and $m_1=1$,

$$x=\frac{m_2x_1+m_1x_2}{m_1+m_2}=\frac{1(4)+2(-2)}{1+2}=0$$

$$y=\frac{m_2y_1+m_1y_2}{m_1+m_2}=\frac{1(-1)+2(-3)}{1+2}=-\frac{7}{3}$$

$\therefore$ Point $Q=\left(0,-\frac{7}{3}\right)$

5. Let Y-axis divides the join of $(-2,-3)$ and $(3,7)$ in the ratio $k:1$.

$\therefore$ Point of division is $\left(\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right)$ ....(1)

As this point lies on Y-axis, therefore, X-coordinates = 0

$$\Rightarrow \quad \frac{3k-2}{k+1}=0 \Rightarrow 3k-2=0 \Rightarrow k=\frac{2}{3}$$

$\Rightarrow$ Ratio is $\frac{2}{3}:1$, i.e. $2:3$

Substituting in (1), we get

Point of division is $\left(0,\frac{\frac{14}{3}-3}{\frac{2}{3}+1}\right)=(0,1)$

6. The equation of a line passing through the point of intersection of the two given lines is

$$(2x-y+5)+k(5x+3y-4)=0 \qquad ...(i)$$

Slope of line (i) is $m_1=-\frac{2+5k}{-1+3k}=\frac{2+5k}{1-3k}$

Slope of line $x-3y+21=0$ is $m_2=\frac{1}{3}$

Since above two lines are perpendicular

$\Rightarrow m_1\times m_2=-1$

$$\Rightarrow \frac{2+5k}{1-3k}\times\frac{1}{3}=-1 \Rightarrow k=\frac{5}{4}$$

Putting $k=\frac{5}{4}$ in the equation (i), the required equation of the line is $3x+y=0$.

7. Let the third vertex be (x_3,y_3). Area of triangle , whose vertices are (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$$=\frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]|$$

As $x_1=2, y_1=1, x_2=3, y_2=-2$

Area of $\Delta=5$ (given)

$$\Rightarrow 5=\frac{1}{2}|2(-2-y_3)+3(y_3-1)+x_3(1+2)|$$

$$\Rightarrow 10=|3x_3+y_3-7|$$

Taking positive sign, $3x_3+y_3-7=10$

$\Rightarrow 3x_3+y_3=17$ .... (1)

Taking negative sign $3x_3+y_3-7=-10$

$\Rightarrow 3x_3+y_3=-3$ .... (2)

Given that (x_3,y_3) lies on $y=x+3$

So, $-x_3+y_3=3$ .... (3)

Solving eq. (1) and (3), $x_3=\frac{7}{2}$, $y_3=\frac{13}{2}$

Solving eq. (2) and (3), $x_3=\frac{-3}{2}$, $y_3=\frac{3}{2}$.

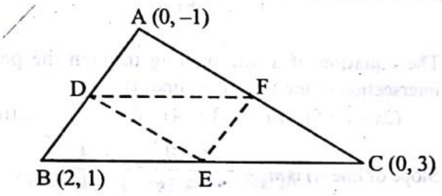
So, the third vertex are $\left(\frac{7}{2},\frac{13}{2}\right)$ or $\left(\frac{-3}{2},\frac{3}{2}\right)$

8. Let $A(0,-1)$, $B(2,1)$ and $C(0,3)$ be the vertices of ΔABC. Let D, E and F be the mid-points of sides AB, BC and AC.

$\therefore$ The coordinates of D, E and F are $\left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$,

$\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ and $\left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$

i.e., $D(1, 0)$, $E(1, 2)$ and $F(0, 1)$ respectively.



Area of ΔABC, according to formulae,

$=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

[where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are coordinates of vertices]

$=\frac{1}{2}[(0)(1-3)+(2)(3+1)+(0)(-1-1)]$

$=\frac{1}{2}(0+8+0)=4$ sq.units

Area of $\Delta DEF=\frac{1}{2}[1(2-1)+1(1-0)+(0)(0-2)]=1$

$\therefore \frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC}=\frac{1}{4}$

Hence the required ratio is 1 : 4.

9. The equation of the given line is $3x-4y-16=0$. Let the equation of a line perpendicular to the given line is $4x+3y+k=0$, where k is a constant .If this line passes through the point $(-1, 3)$, then

$-4+9+k=0 \Rightarrow k=-5$

$\therefore$ The equation of a line passing through the point $(-1, 3)$ and perpendicular to the given line is

$4x+3y-5=0$

$\therefore$ The required point of the foot of the perpendicular is the point of the intersection of the lines

$3x-4y-16=0$ ... (i) and

$4x+3y-5=0$ ... (ii)

Solving (i) and (ii) by cross-multiplication, we have

$\frac{x}{20+48}=\frac{y}{-64+15}=\frac{1}{9+16} \Rightarrow$

$\frac{x}{68}=\frac{y}{-49}=\frac{1}{25} \Rightarrow x=\frac{68}{25}, y=-\frac{49}{25}$

$\therefore$ The required point is $\left(\frac{68}{25}, \frac{-49}{25}\right)$

10. $\frac{AD}{AB}=\frac{1}{4}$

$\Rightarrow 4AD=AD+BD \Rightarrow 3AD=DB$

$\Rightarrow \frac{AD}{DB}=\frac{1}{3}$

D divides AB in the ratio 1 : 3.

By section formula, the coordinates of D are $\left(\frac{13}{4}, \frac{23}{4}\right)$.

Similarly, E divides AC in the ratio 1 : 3.

The coordinates of E are $\left(\frac{19}{4}, 5\right)$.

Area of ΔADE

$=\frac{1}{2}\left|4\left(\frac{23}{4}-5\right)+\frac{13}{4}(5-6)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right|$ sq. units

$=\frac{1}{2}\left|3-\frac{13}{4}+\frac{19}{4}\right|$ sq. units $=\frac{15}{32}$ sq. units

Area of ΔABC

$=\frac{1}{2}|4(5-2)+1(2-6)+7(6-5)|$ sq. units

$=\frac{1}{2}|12-4+7|$ sq. units $=\frac{15}{2}$ sq. units

$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC}=\frac{15}{32}\times\frac{2}{15}=\frac{1}{16}$

11. Let the given points be $A(2,0)$, $B(-6, -2)$, $C(-4, -4)$ and $D(4,-2)$

Then, $AB=\sqrt{(-6-2)^2+(-2-0)^2}=\sqrt{68}$ units.

$BC=\sqrt{(-4+6)^2+(-4+2)^2}=\sqrt{8}$ units.

$CD=\sqrt{(4+4)^2+(-2+4)^2}=\sqrt{68}$ units.

$DA=\sqrt{(4-2)^2+(-2-0)^2}=\sqrt{8}$ units.

$AC=\sqrt{(-4-2)^2+(4-0)^2}=\sqrt{52}$ units.

$BD=\sqrt{(4+6)^2+(-2+2)^2}=10$ units.

Clearly,

$AB=CD$, $BC=DA$ and $AC \neq BD$.

i.e., the opposite sides of the quadrilateral are equal and diagonals are not equal. Hence, the given points form a parallelogram.

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (a)

2. (a) Since $C(y,-1)$ is the mid-point of $P(4,x)$ and $Q(-2,4)$.

We have, $\frac{4-2}{2}=y$ and $\frac{4+x}{2}=-1$

$\therefore\ y=1$ and $x=-6$

3. (c) Let the required ratio be $k:1$

Then, $2=\frac{6k-4(1)}{k+1}$ or $k=\frac{3}{2}$

$\therefore$ The required ratio is $\frac{3}{2}1$ or $3:2$

Also, $y=\frac{3(3)+2(3)}{3+2}=3$

4. (c) $\frac{3(1)+4(2)-7}{3(-2)+4(1)-7}=-\frac{4}{-9}=\frac{4}{9}$

5. (b) **Hint** Let P $(x, 0)$ be a point on X-axis such that $AP=BP$

$\Rightarrow\ AP^2=BP^2$

$\Rightarrow\ (x+2)^2+(0-3)^2=(x-5)^2+(0+4)^2$

$\Rightarrow\ x^2+4x+4+9 \Rightarrow x^2-10x+25+16$

$\Rightarrow\ 14x=28 \Rightarrow x=2$

6. (d) **7.** (b)

8. (b) **[Hint.** Centroid is $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

i.e. $\left(\frac{3+(-8)+5}{3},\frac{-7+6+10}{3}\right)$

i.e. $\left(\frac{0}{3},\frac{9}{3}\right)$ *i.e.* $(0,3)$]

9. (b) **10.** (b) **11.** (c)
12. (c) **13.** (a) **14.** (b)
15. (c) **16.** (b)

MORE THAN ONE CORRECT :

1. (a, c) **2.** (b, d) **3.** (a, d)
4. (b, d) **5.** (b, d) **6.** (b, d)
7. (a, b)

Let the coordinates of P be (x, y).

$AP^2=(x-4)^2+(y-0)^2=x^2+y^2-8x+16$

and $BP^2=(x-0)^2+(y-2)^2=x^2+y^2-4y+4$.

Since $AP^2=BP^2$

$\Rightarrow x^2+y^2-8x+16=x^2+y^2-4y+4$ from which

$8x-4y-12=0$ or, $2x-y-3=0$

PASSAGE BASED QUESTIONS :

1. (b) Equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{x-x_1}{y-y_1}=\frac{x_2-x_1}{y_2-y_1}$

Required equation is $\frac{x-4}{y-1}=\frac{5-4}{-2-1}$

$\Rightarrow\ -3x+12=y-1$

Required equation is $3x+y-13=0$

2. (a) We get the point on x-axis by putting $y=0$ in the equation

$\Rightarrow\ 3x-13=0$ or $x=\frac{13}{3}$

$\therefore$ Point is $\left(\frac{13}{3},0\right)$

And, we get the point on y-axis by putting $x=0$,

$\therefore\ y=13$

$\therefore$ Point is $(0, 13)$

3. (c) Area of triangle is $\frac{1}{2}\times$base$\times$height

$=\frac{1}{2}\times\frac{13}{3}\times 13=\frac{169}{6}$ sq. units

ASSERTION & REASON :

1. (a) ABC is a right triangle, right angled at C as

$(m_{AC})(m_{BC})=\left(\frac{-4+2}{5+5}\right)\left(\frac{-4-6}{5-7}\right)=-1$

Hence circumcentre is mid point of $AB\equiv(1,2)$.

2. (c) Let $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ are all rational coordinates

$$\text{ar}(\Delta ABC)=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\frac{\sqrt{3}}{4}\ [(x_1-x_2)^2+(y_1-y_2)^2]$$

LHS = rational, RHS = irrational

Hence $(x_1,y_1)(x_2,y_2)$ & (x_3,y_3) cannot be all rational

3. (b) Any line through the intersection of $x+y+4=0$ & $3x-y-8=0$ is $(x+y+4)+\lambda(3x-y-8)=0$ since it passes through $(2,-3)$ so $\lambda=-3$ hence required equation is $2x-y-7=0$.

MULTIPLE MATCHING QUESTIONS :

1. (a) – p, q, t; (b) – p; (c) – t; (d) – r; (E) – s

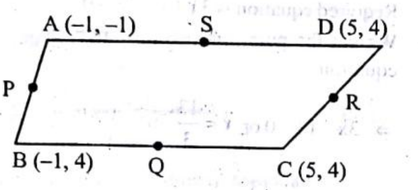
HOTS SUBJECTIVE QUESTIONS :

1. P is the mid point of AB

$\therefore$ Coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Q is the mid-point of BC

$\therefore$ Coordinates of Q are $\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$



R is the mid-point of CD

$\therefore$ Co-ordinate of R are $\left(\frac{5+5}{2}, \frac{-1+4}{2}\right) = \left(5, \frac{3}{2}\right)$

S is the mid-point of DA

$\therefore$ Co-ordinate of S are $\left(\frac{5-1}{2}, \frac{-1-1}{2}\right) = (2, -1)$

$\therefore$ $P\left(-1, \frac{3}{2}\right)$, $Q(2, 4)$, $R\left(5, \frac{3}{2}\right)$ and $S(2, -1)$ are the mid-points of AB, BC, CD and DA respectively.

Now

$$PQ = \sqrt{(2+1)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(2-5)^2 + \left(-1\frac{-3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(-1-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \sqrt{\frac{61}{2}}$$

Also diagonal PR

$$= \sqrt{(5+1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36+0} = \sqrt{36} = 6$$

and diagonal

$$SQ = \sqrt{(2-2)^2 + (4+1)^2} = \sqrt{0+(5)^2} = \sqrt{25} = 5$$

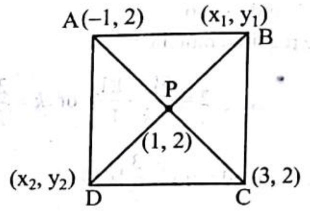
$\therefore$ Diagonal $PR \neq$ Diagonal QS

2. Let the vertices of a square be A, B, C, D and $A = (-1, 2)$ and $C = (3, 2)$. Let $B = (x_1, y_1)$ and $D = (x_2, y_2)$

In a square, all sides are equal, so, $AB = BC = CD = DA$ and both the diagonals are equal, so, $AC = BD$.

Diagonals of a square bisect each other.

Let diagonals bisect each other at P. So, P is the mid point of AC, So, co-ordinates of $P = \left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$



$AB = BC \Rightarrow AB^2 = BC^2$

$\Rightarrow (x_1+1)^2 + (y_1-2)^2 = (x_1-3)^2 + (y_1-2)^2$

$\Rightarrow x_1^2 + 2x_1 + 1 = x_1^2 - 6x_1 + 9 \Rightarrow 8x_1 = 8 \Rightarrow x_1 = 1$

Since P is also the mid point of BD ; $\frac{x_1+x_2}{2} = 1$

$\Rightarrow x_1 + x_2 = 2$ or $1 + x_2 = 2 \Rightarrow x_2 = 1$

$AC^2 = AD^2 + DC^2$

$(3+1)^2 + (2-2)^2 = (x_2+1)^2 + (y_2-2)^2 + (x_2-3)^2 + (y_2-2)^2$

$\Rightarrow 2(y_2-2)^2 + (1+1)^2 + (1-3)^2 = 4^2$

$\Rightarrow 2(y_2-2)^2 + 4 + 4 = 16$

$\Rightarrow 2(y_2-2)^2 = 8 \Rightarrow (y_2-2)^2 = 4 \Rightarrow y_2 - 2 \pm 2$

$\Rightarrow y_2 = 4$, or 0

Since P is mid point of B and D also

$\Rightarrow \frac{y_1+y_2}{2} = 2 \Rightarrow y_1 + y_2 = 4 \Rightarrow$ if $y_2 = 0, y_1 = 4$ and if $y_2 = 4, y_1 = 0$

So, $y_1 = 0$ amd $y_2 = 4$ or $y_1 = 4$ and $y_2 = 0$

Hence, coordinates of the other two vertices are $(1, 0)$ and $(1, 4)$.

3. As given, coordinates of A is $(a, 0)$ and B is $(0, b)$. $OA = a$ and $OB = b$.

Let coordinates of $C \equiv (x_1, y_1)$ and $D \equiv (x_2, y_2)$. Draw perpendicular from C on Y-axis which meets at E and $CE \perp OE$. Draw perpendicular from D to X-axis which meets at F and $DF \perp OF$.

$\angle ABO + \angle ABC + \angle CBE = 180°$

$\Rightarrow \angle ABO + 90° + \angle CBE = 180°$

$\Rightarrow \angle ABO + \angle CBE = 90°$ ...(1)

In ΔBEC, $\angle EBC = 90°$ and $\angle CBE + \angle ECB + \angle CEB = 180°$

$\Rightarrow \angle CBE + 90° + \angle CEB = 180°$ [$\because \angle CEB = 90°$]

$\Rightarrow \angle CBE + \angle ECB = 90°$ ...(2)

From (1) and (2) $\angle ABO = \angle ECB$ and so, $\angle OAB = \angle CBE$.

$$AB = BC = CD = DA$$

[These are sides of square $ABCD$.]

$$\sin\angle OAB = \frac{b}{AB} = \sin\angle EBC = \frac{EC}{BC} = \frac{x_1}{BC}$$

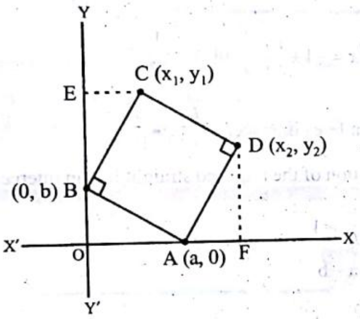
$$\Rightarrow \frac{b}{AB} = \frac{x_1}{BC} \Rightarrow x_1 = b \quad [Since\ AB = BC]$$

$$\cos\angle OAB = \frac{OA}{AB} = \frac{a}{AB}$$

$$= \cos\angle EBC = \frac{EB}{BC} = \frac{y_1 - b}{BC}$$

$$\Rightarrow \frac{a}{AB} = \frac{y_1 - b}{BC} \Rightarrow a = y_1 - b$$

$$\Rightarrow y = a + b \qquad [Since\ AB = BC]$$



So, $C \equiv (a, a+b)$

Similarly, in Δs AFD and AOB

$$\angle DAF = \angle OBA$$

$$\sin\angle DAF = \frac{DF}{AD} = \frac{y_2}{AD} = \sin\angle OBA = \frac{a}{AB}$$

$$\Rightarrow \frac{y_2}{AD} = \frac{a}{AB} \Rightarrow y_2 = a \quad [Since\ AD = AB]$$

Also, $\cos\angle DAF = \dfrac{AF}{AD} = \dfrac{OF - OA}{AD} = \dfrac{x_2 - a}{AD}$

$$= \cos\angle OBA = \frac{OB}{AB} = \frac{b}{AB}$$

$$\Rightarrow \frac{x_2 - a}{AD} = \frac{b}{AB} \Rightarrow x_2 = a + b \qquad [Since\ AD = AB]$$

So, $D \equiv (a+b, a)$

4. Slope of line joining $A(0, 1)$ and $B(-3, -1) = \dfrac{-1-1}{-3-0} = \dfrac{2}{3}$

Slope of line joining $C(1, -1)$ and $D(4, 4) = \dfrac{y+1}{4-1} = \dfrac{y+1}{3}$

Since, $\dfrac{y+1}{3} = \dfrac{2}{3} = y = 1$

Slope of diagonal $AC = \dfrac{-1-1}{1-0} = -2$

Slope of diagonal $BD = \dfrac{1+1}{4+3} = \dfrac{2}{7}$

Equation of $AC \Rightarrow y - 1 = -2(x - 0)$ (passing through A)

$$\Rightarrow y + 2x = 1 \quad ...(1)$$

Equation of $BD \Rightarrow y + 1 = (x + 3)$ (passing through B)

$$\Rightarrow 7y + 7 = 2x + 6 \Rightarrow 7y - 2x = -1$$

$$\Rightarrow 2x - 7y = 1 \quad ...(2)$$

Solving (1) and (2), we get their intersection

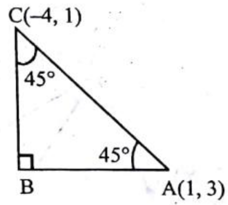
$$2x + y = 1$$
$$2x - 7y = 1$$

$$8y = 0 \Rightarrow y = 0 \text{ and } x = \frac{1}{2}$$

Hence, point of intersection is $\left(\dfrac{1}{2}, 0\right)$

5. Let ABC be the right triangle with diagonal AC.
Let m be the slope of a line making 45° angle with AC.
We have,

Slope of $AC = \dfrac{1-3}{-4-1} = \dfrac{2}{5}$



Now, $\tan 45° = \left|\dfrac{m - \frac{2}{5}}{1 + \frac{2m}{5}}\right|$

$$\Rightarrow 2m + 5 = \pm(5m - 2)$$

$$\Rightarrow 2m + 5 = 5m - 2 \text{ or, } 2m + 5 = -(5m - 2)$$

$$\Rightarrow m = \frac{7}{3} \text{ or } m = -\frac{3}{7}$$

Thus, the lines making 45° angle with AC have slopes $\dfrac{7}{3}$ or $-\dfrac{3}{7}$

So, the possible equations of AB are :

$$y - 3 = \frac{7}{3}(x - 1) \text{ and } y - 3 = -\frac{3}{7}(x - 1)$$

$$\Rightarrow 7x - 3y + 2 = 0 \text{ and } 3x + 7y - 24 = 0$$

The possible equations of BC are :

$$y-1=\frac{7}{3}(x+4) \text{ and } y-1=-\frac{3}{7}(x+4)$$

$\Rightarrow$ $7x-3y+31=0$ and $3x+7y+5=0$

Hence, the equations of the sides are :

$7x-3y+2=0$ and $3x+7y+5=0$

or $7x-3y+31=0$ and $3x+7y-24=0$

6. The given equations are :

$x+y-6=0$ ... (i) $x-3y-2=0$ ...(ii) and

$5x-3y+2=0$ ...(iii)

Suppose equations (i), (ii) and (iii) represent the sides, AB, BC and CA respectively of triangle ABC.

Solving (i) and (ii), we get : $x=5$ and $y=1$.

Thus, AB and BC intersect at $B(5, 1)$.

Solving (ii) and (iii), we get : $x=-1$ and $y=-1$.

Thus, BC and CA intersect at $C(-1, -1)$.

Solving (i) and (iii), we get : $x=2$ and $y=4$.

Thus, AB and CA intersect at $A(2, 4)$.

Thus, the coordinates of the vertices A, B and C of triangle ABC are (2, 4), (5, 1) and (–1, –1) respectively.

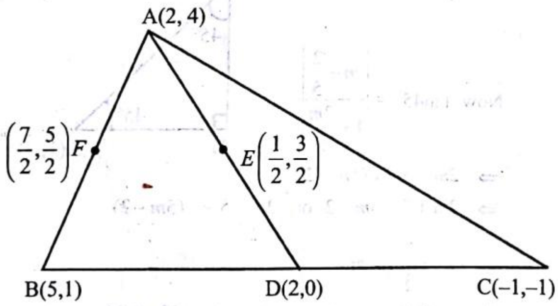
Let D, E and F be the mid-points of sides BC, CA and AB respectively. Then, the coordinates of D, E and F are :

$$D\left(\frac{5-1}{2},\frac{1-1}{2}\right)=(2,0);\ E\left(\frac{2-1}{2},\frac{4-1}{2}\right)=\left(\frac{1}{2},\frac{3}{2}\right) \text{ and}$$

$$F\left(\frac{2+5}{2},\frac{4+1}{2}\right)=\left(\frac{7}{2},\frac{5}{2}\right) \text{ respectively.}$$

The equation of median AD is

$$y-4=\frac{0-4}{2-2}(x-2)$$



$$\Rightarrow \quad x-2=\frac{2-2}{0-4}(y-4)$$

$\Rightarrow$ $x-2=0 \Rightarrow x=2$

The equation of median BE is

$$y-1=-\frac{1}{9}(x-5) \Rightarrow x+9y-14=0$$

The equation of CF is $y+1=\dfrac{\frac{5}{2}+1}{\frac{7}{2}+1}(x+1)$

$$\Rightarrow \quad y+1=\frac{7}{9}(x+1) \Rightarrow 7x-9y-2=0$$

Hence, the equations of the medians of the triangle are $x=2$, $x+9y-14=0$ and $7x-9y-2=0$

7. The given lines are

$2x-3y=-1$ ...(i)

$4x+7y=3$ ...(ii)

Multiplying eqn. (i) by 2,

$4x-6y=-2$ ...(iii)

Subtracting (iii) from (ii)

$13y=5$ $\therefore y=\frac{5}{13}$

Putting the value of y in (i) $2x-\frac{3\times5}{13}=-1$

or $2x=-1+\frac{15}{13}$ or $x=\frac{1}{13}$

Given lines intersect at $\left(\frac{1}{13},\frac{5}{13}\right)$

Equation of the required straight line in intercept form:

$$\frac{x}{a}+\frac{y}{b}=1$$

Here a = b

$$\frac{x}{a}+\frac{y}{b}=1 \quad ...(1)$$

Since the line (1) passes through the point $\left(\frac{1}{13},\frac{5}{13}\right)$

$$\therefore \frac{1}{13a}+\frac{15}{13a}=1$$

$$\Rightarrow \frac{1}{13}+\frac{15}{13}=a$$

$$\Rightarrow \frac{16}{13}=a$$

Now putting the value of 'a' in equation (1), we get

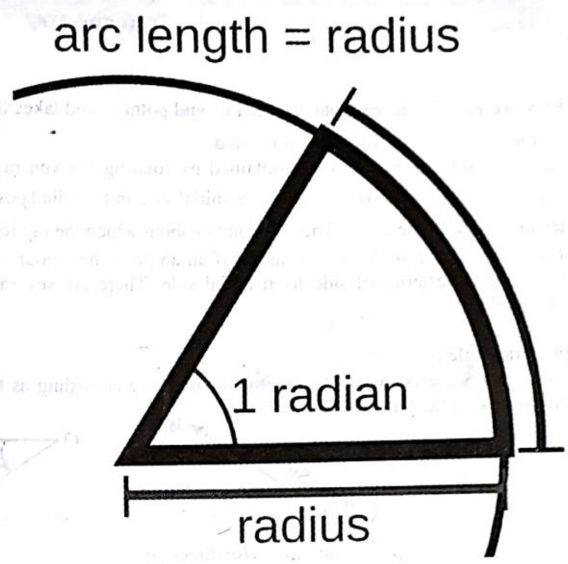
$$\frac{x}{\frac{16}{13}}+\frac{13y}{\frac{16}{13}}=1$$

$$\Rightarrow x+y=\frac{16}{13}$$

$\Rightarrow 13x+13y-16=0$

This is the required equation.

chapter 8



TRIGONOMETRY

Introduction

In Greek 'Trigonon' means a triangle. 'Metron' means a measure. The combination of these two words gives us the word 'Trigonometry'. Trigonometry is the branch of mathematics that deals with the relations between the sides and angles of triangles.

The first introduction to this topic was done by Hipparcus in 140 *B.C.*, when he hinted at the possibility of finding distances and heights of inaccessible objects. In 150 *A.D.* Tolemy again raised the same possibility and suggested the use of a right triangle for the same. But it was Aryabhatta (476 *A.D.*) whose introduction to the name "Jaya" lead to the name "sine" of an acute angle of a right triangle. The subject was completed by Bhaskaracharya (1114 *A.D.*) while writing his work on Goladhayay. In that, he used the words Jaya, Kotijya and "sparshjya" which are presently used for sine, cosine and tangent (of an angle). But it goes to the credit of Neelkanth Somstuvan (1500 *A.D.*) who developed the science and used terms like elevation, depression and gave examples of some problems on heights and distance.

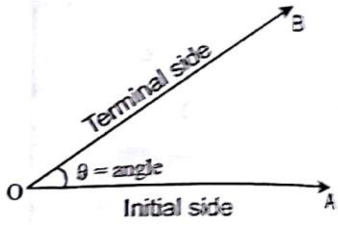
Historically, it was developed for astronomy and geography, but scientists have been using it for centuries for other purposes, too. Besides other fields of mathematics, trigonometry is used in physics, engineering, and chemistry.

Of course, trigonometry is used throughout mathematics, and since mathematics is applied throughout the natural and social sciences, trigonometry has many applications.

ANGLE :

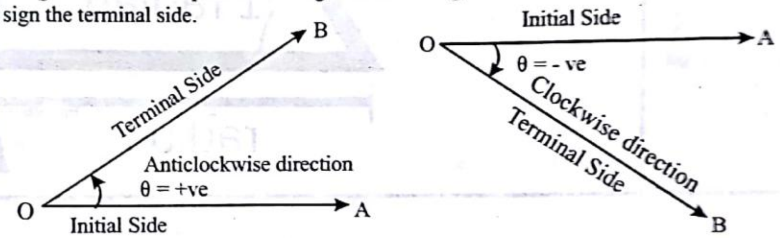
Consider a ray $\overrightarrow{OA}$. If this ray rotates about its end point O and takes the position OB, then the angle $\angle AOB$ has been generated.

An angle is considered as the figure obtained by rotating a given ray about its end-point. The initial position OA is called the initial side and the final position OB is called terminal side of the angle. The end point O about which the ray rotates is called the vertex of the angle. The measure of an angle is the amount of rotation performed to get the terminal side from initial side. There are several units for measuring angles.



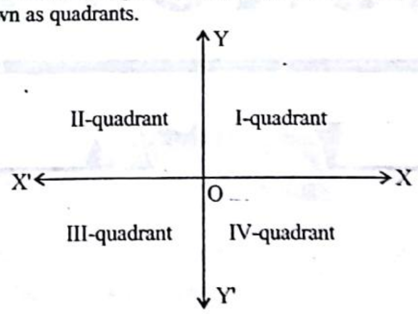
Sense of sign of an angle :

The sense of an angle is said to be positive or negative according as the initial side rotates in anticlockwise or clockwise direction to get sign the terminal side.



SOME USEFUL TERMS :

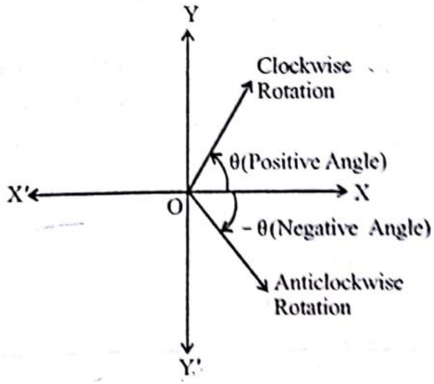
Quadrants : Let $X'OX$ and YOY' be two lines at right angles in the plane of the paper. These lines divide the plane of the paper into four equal parts which are known as quadrants.



The lines $X'OX$ and YOY' are known as x-axis and y-axis respectively. These two lines taken together are known as the coordinate axes. The regions XOY, YOX', $X'OY'$ and $Y'OX$ are known as first, second, third and fourth quadrant respectively.

NOTE : Any part of X or Y-axis does not lies in any quadrant I.

Angle In Standard Position : An angle is said to be in standard position if its vertex coincides with the origin O and the initial side coincides with OX i.e. the positive direction of x-axis.



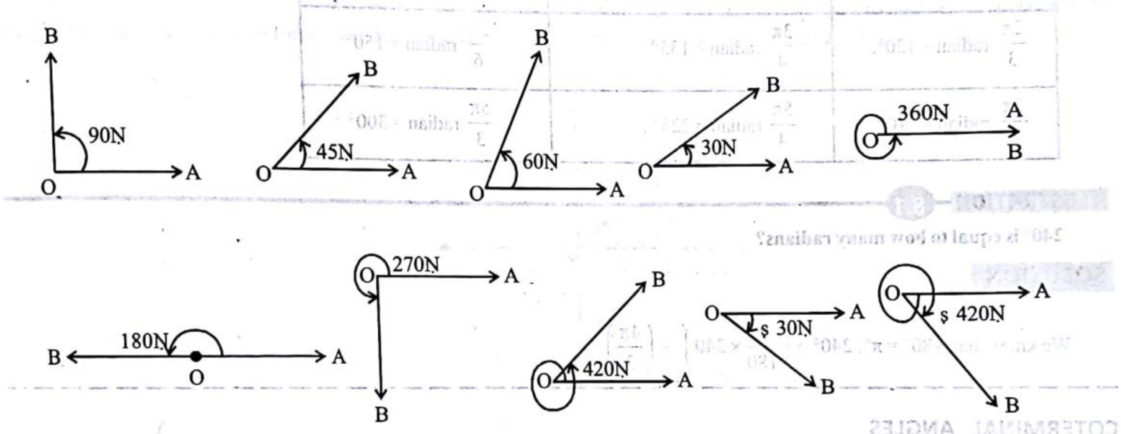
SYSTEMS OF MEASUREMENT OF ANGLES :

Sexagesimal or English system :

The principal unit in this system is degree (N). One right angle is divided into 90 equal part and each part is called one degree (1N). One degree is divided into 60 equal parts and each part is called one minute one minute is denoted by (1'). One minute is divided into 60 equal parts and each part is called one second (1").

$\therefore$ One right angle = 90N   1N= 60'   1' = 60"

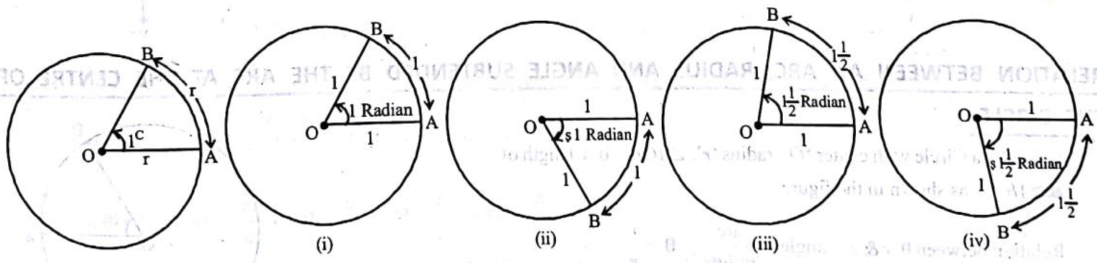
Some of the angles whose measures are 90N 45N 60N 30N 360N 180N 270N 420N ş30N ş420N are shown in Figure.



Circular system :

One radian, written as 1^c, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

Consider a circle of radius r having centre at *O*. Consider an arc *AB* of the circle whose length is equal to the radius *r* of the circle. Then by the definition the measure of $\angle AOP$ is 1 radian (1^c).



In the figure, *OA* is the initial side and *OB* is the terminal side. The figures show the angles whose measures are 1 radian, ş 1 radian, $1\frac{1}{2}$ radian and $-1\frac{1}{2}$ radian.

Relation Between Systems of Measurement of Angles: $\frac{D}{90} = \frac{2C}{\pi}$

Here *D* and *C* are the angles in degree and radians.

Remember: 1N= $\frac{\pi}{180}$ radians; 1 rad = $\frac{180}{\pi}$ degrees

Some Important Conversion :

π radian = 180°,	One radian = $\left(\frac{180}{\pi}\right)^{\circ}$,	$\frac{p}{6}$ radian = 30°
$\frac{\pi}{4}$ radian = 45°,	$\frac{\pi}{3}$ radian = 60°,	$\frac{\pi}{2}$ radian = 90°,
$\frac{2\pi}{3}$ radian = 120°,	$\frac{3\pi}{4}$ radian = 135°,	$\frac{5\pi}{6}$ radian = 150°
$\frac{7\pi}{6}$ radian = 210°,	$\frac{5\pi}{4}$ radian = 225°,	$\frac{5\pi}{3}$ radian = 300°

ILLUSTRATION 8.1

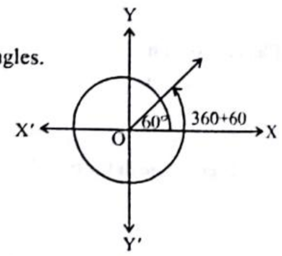
240° is equal to how many radians?

SOLUTION :

We know that, $180° = \pi^c$; $240° = \left(\frac{\pi}{180}\times 240\right)^c = \left(\frac{4\pi}{3}\right)^c$

COTERMINAL ANGLES

The angles that differ by either 360° or the integral multiples of 360° are called coterminal angles.
Example : 60°, 360° + 60° = 420°, 2 × 360° + 60° = 780° are coterminal angles.

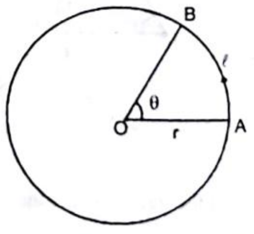


NOTE :

1. If θ is an angle then its coterminal angle is in the form of $(n\,360 + \theta)$
2. The terminal side of coterminal angles in their standard position coincides.

RELATION BETWEEN AN ARC, RADIUS AND ANGLE SUBTENDED BY THE ARC AT THE CENTRE OF THE CIRCLE :

Consider a Circle with center 'O', radius 'r', $\angle AOB = \theta$ & length of arc $AB = \ell$ as shown in the figure.



Relation between θ, r & s : angle = $\frac{\text{arc}}{\text{radius}}$; $\theta = \frac{\ell}{r}$

Here θ is always in radian and units of ℓ and r are always same

ILLUSTRATION 8.2

Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°.

SOLUTION :

Let ℓ be the length of an arc subtending an angle θ at the centre of a circle of radius r.

$\theta = \frac{\ell}{r}$

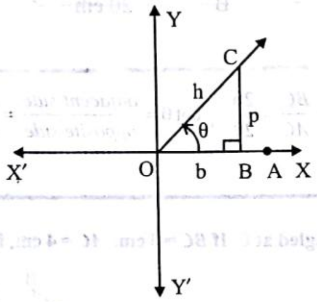
Here, $r = 5$ cm, and $\theta = 15° = \left(15\times\frac{\pi}{180}\right)^c$; $\theta = \left(\frac{\pi}{12}\right)^c$; $\theta = \frac{\ell}{r} \Rightarrow \frac{\pi}{12} = \frac{\ell}{5}$ or $\ell = \frac{5\pi}{12}$ cm

TRIGONOMETRIC RATIOS :

Let XOX' and YOY' be horizontal and vertical axes of respectively. Let A be a point on OX. Let the ray OA start rotating in the plane XY in an anti-clockwise direction from the initial position OA about the point O till it reaches its final position OC after some interval of time. (See Fig.). Thus, an angle COA is formed with x-axis. Let $\angle COA = \theta$. (θ is a Greek letter, and we read it as "theta"). Draw $CB \perp OX$. Now clearly ΔCBO is a right angled triangled.

In right ΔCBO, OC is the hyponenus. For angle $\theta = \angle COA$, BC and OB are called side opposite to angle θ and adjucent side of angle θ respectively.

Let $CB = p$, $OB = b$ and $OC = h$. We define the different ratios between hypotenus, side opposite to angle θ and adjucent side of angle θ as trigonometric ratios for angle θ. Horizontal axis $X'OX$ is called X-axis and vertical axis YOY' is called Y-axis.



These trigonometrical ratios are :

Sine of $\theta = \dfrac{\text{Side opposite to angle } \theta}{\text{Hypotenuse}} = \dfrac{CB}{OC} = \dfrac{p}{h}$; Cosine of $\theta = \dfrac{\text{Adjacent side to angle } \theta}{\text{Hypotenuse}} = \dfrac{OB}{OC} = \dfrac{b}{h}$

Tangent of $\theta = \dfrac{\text{Side opposite to angle } \theta}{\text{Adjacent side to angle } \theta} = \dfrac{CB}{OB} = \dfrac{p}{b}$; Cotangent of $\theta = \dfrac{\text{Adjacent side to angle } \theta}{\text{Side opposite to angle } \theta} = \dfrac{OB}{CB} = \dfrac{b}{p}$

Secant of $\theta = \dfrac{\textit{Hypotenuse}}{\text{Adjacent side to angle } \theta} = \dfrac{OC}{OB} = \dfrac{h}{b}$; Cosecant of $\theta = \dfrac{\textit{Hypotenuse}}{\text{Side opposite to angle } \theta} = \dfrac{OC}{CB} = \dfrac{h}{p}$

Sine of θ is abbreviated as $\sin\theta$, Cosine of θ is abbreviated as $\cos\theta$, Tangent of θ is abbreviated as $\tan\theta$
Cotangent of θ is abbreviated as $\cot\theta$, Secant of θ is abbreviated as $\sec\theta$ and Cosecant of θ is abbreviated as Cosec θ

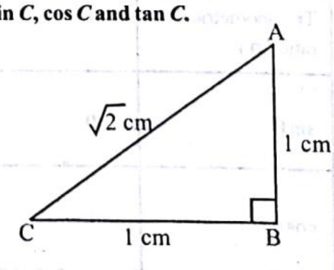
(i) Throughout the study of trigonometry we shall be using only abbreviated form of these trigonometric ratios.
Thus,

$$\sin\theta = \frac{p}{h}, \cos\theta = \frac{b}{h}; \tan\theta = \frac{p}{b}, \cot\theta = \frac{b}{p}; \sec\theta = \frac{h}{b}, \text{cosec}\theta = \frac{h}{p}$$

(ii) $\sin\theta$ is an abbreviation for "sine of angle θ" and not the product of sin and θ.

ILLUSTRATION 8.3

In figure, ΔABC has a right angle at B. If $AB = BC = 1$cm and $AC = \sqrt{2}$cm, find $\sin C$, $\cos C$ and $\tan C$.



SOLUTION :

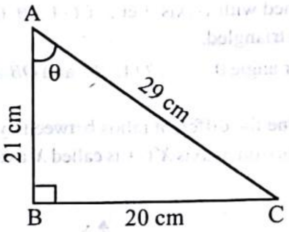
In right angled ΔABC

$$\sin C = \frac{\textit{opposite} \text{ side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{1}{\sqrt{2}}, \quad \cos C = \frac{\textit{adjacent} \text{ side}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

and $\tan C = \dfrac{\textit{opposite} \text{ side}}{\text{adjacent side}} = \dfrac{AB}{BC} = \dfrac{1}{1} = 1$

ILLUSTRATION 8.4

In Figure ABC is a right angled triangle. If $AB = 21$ cm, $BC = 20$ cm and $CA = 29$ cm and $\angle A = \theta$, find $\sin\theta$, $\cot\theta$ and $\sec\theta$.

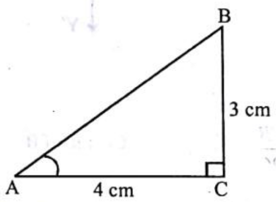


SOLUTION :

We know that, $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{20}{29}$; $\cot\theta = \dfrac{\text{adjacent side}}{\text{opposite side}} = \dfrac{AB}{BC} = \dfrac{21}{20}$; $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}} = \dfrac{AC}{AB} = \dfrac{29}{21}$

ILLUSTRATION 8.5

In Figure ΔABC is a triangle right angled at C. If $BC = 3$ cm, $AC = 4$ cm, find the values of $\cot A$, $\sec A$ and $\operatorname{cosec} A$.



SOLUTION :

ΔABC is a right angled triangle,

$\therefore\ AB^2 = BC^2 + AC^2$ [By Pythagoras theorem]

$= (3)^2 + (4)^2 = 9 + 16 = 25 = (5)^2$

$\Rightarrow AB = 5$ cm

$\therefore\ \cot A = \dfrac{AC}{BC} = \dfrac{4}{3}$, $\sec A = \dfrac{AB}{AC} = \dfrac{5}{4}$ and $\operatorname{cosec} A = \dfrac{AB}{BC} = \dfrac{5}{3}$

VALUE OF TRIGONOMETRIC RATIOS FOR SOME SPECIFIC ANGLES :

The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90° are quite often used in solving problems in our day-to-day life. Thus the following table is very useful.

IMPORTANT TABLE

Trigonometrical ratio (θ)	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec ?	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined

ILLUSTRATION 8.6

Find the value of $\tan^2 60° - \sin^2 30°$.

SOLUTION:

We know that $\tan 60° = \sqrt{3}$ and $\sin 30° = 1/2$

$$\therefore \tan^2 60° - \sin^2 30° = (\sqrt{3})^2 - \left(\frac{1}{2}\right)^2 = 3 - \frac{1}{4} = \frac{11}{4}$$

ILLUSTRATION 8.7

Find the value of $\tan^2 60° \operatorname{cosec}^2 45° + \sec^2 45° \sin 30°$

SOLUTION:

We know that

$\tan 60° = \sqrt{3}$; $\operatorname{cosec} 45° = \sqrt{2}$

$\sec 45° = \sqrt{2}$; $\sin 30° = 1/2$

$$\therefore \tan^2 60° \operatorname{cosec}^2 45° + \sec^2 45° \sin 30° = (\sqrt{3})^2(\sqrt{2})^2 + (\sqrt{2})^2 \frac{1}{2} = 3\times2+2\times\frac{1}{2} = 6+1 = 7$$

BASIC FORMULAE OR TRIGONOMETRIC IDENTITY :

(i) $\sin\theta . \operatorname{cosec}\theta = 1$ or

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ *or* $\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \theta \neq n\pi$. Here n is an integer.

(ii) $\cos\theta . \sec\theta = 1$ or $\cos\theta = \frac{1}{\sec\theta}$ *or* $\sec\theta = \frac{1}{\cos\theta}, \theta \neq (2n+1)\frac{\pi}{2}$. Here n is an integer.

(iii) $\tan\theta . \cot\theta = 1$ or $\cot\theta = \frac{1}{\tan\theta}$ *or* $\tan\theta = \frac{1}{\cot\theta}, \theta \neq n\frac{\pi}{2}$

(iv) $\sin^2\theta + \cos^2\theta = 1$ or $\cos^2\theta = 1 - \sin^2\theta$ or $\sin^2\theta = 1 - \cos^2\theta$

(v) $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta = 1 + \tan^2\theta$ or $\tan^2\theta = \sec^2\theta - 1$

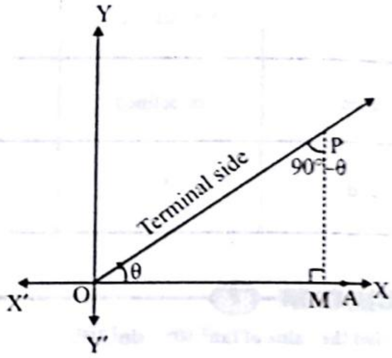
(vi) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ or $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ or $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

(vii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\theta \neq (2n+1)\frac{\pi}{2}$. Here n is an integer.

(viii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$; $\theta \neq n\pi$

TRIGONOMETRIC RATIOS FOR COMPLEMENTARY ANGLES :

Let XOX' and YOY' be horizontal and vertical axes respectively. Horizonal axis is called the X-axis and vertical axis is called Y-axis. Let A be any point on OX. Let a ray OA rotate in an anti-clockwise direction and trace an angle θ from its initial position (X-axis) in any interval of time. Let $\angle POM = \theta$



Draw $PM \perp OX$. ΔPMO is a right triangle

Also, $\angle POM + \angle OPM + \angle PMO = 180°$

$\angle POM + \angle OPM + 90° = 180°$

$\therefore$ $\angle POM + \angle OPM = 90°$

$\Rightarrow$ $\angle OPM = 90° - \theta$

i.e. $\angle OPM$ and $\angle POM$ are complementary angles.

In right-angled ΔPMO, we know that

$$\sin\theta = \frac{PM}{OP},\ \cos\theta = \frac{OM}{OP} \text{ and } \tan\theta = \frac{PM}{OM},$$

$$\operatorname{cosec}\theta = \frac{OP}{PM},\ \sec\theta = \frac{OP}{OM} \text{ and } \cot\theta = \frac{OM}{PM}$$

For angle $(90° - \theta)$,

$$\sin(90° - \theta) = \frac{OM}{OP} = \cos\theta,\ \cos(90° - \theta) = \frac{PM}{OP} = \sin\theta,\ \tan(90° - \theta) = \frac{OM}{PM} = \cot\theta,$$

$$\cot(90° - \theta) = \frac{PM}{OM} = \tan\theta,\ \operatorname{cosec}(90° - \theta) = \frac{OP}{OM} = \sec\theta \text{ and } \sec(90° - \theta) = \frac{OP}{PM} = \operatorname{cosec}\theta$$

TRIGNOMETRIC RATIOS OF SUM AND DIFFERENCE OF ANGLES :

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(ii) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(iii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(iv) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$,

(vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(vii) $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$,

(viii) $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

FORMULAS TO TRANSFORM SUM OR DIFFERENCE INTO PRODUCT :

(i) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(ii) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(iii) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(iv) $\cos C - \cos D = 2\sin\dfrac{C+D}{2}\sin\dfrac{D-C}{2}$

TRIGONOMETRIC FUNCTIONS OF MULTIPLE AND SUBMULTIPLE ANGLES :

(i) $\sin 2\theta = 2\sin\theta.\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(iv) $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

(v) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(vi) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

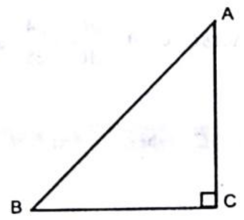
(vii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

MISCELLANEOUS SOLVED EXAMPLES

1. The difference between two acute angle of a right angle triangle is $\left(\frac{p}{9}\right)$. Then the angles in degree.

Sol. In triangle ABC, let $\angle C = 90°$

So $\angle A - \angle B = \left(\frac{\pi}{9}\right)^c = 20°$ ....(i)



Sum of all the angles in ΔABC,

$\angle A + \angle B + \angle C = 180°$

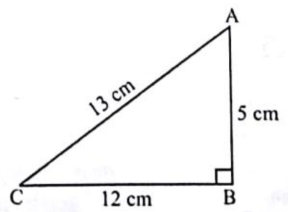
$\because \angle C = 90°$

$\angle A + \angle B = 90°$ ....(ii)

Solving (i) & (ii)

$\angle A = 55°$, $\angle B = 35°$

2. In Fig. Δ ABC is a right angled triangle, right angled at B. If $AB = 5$ cm, $AC = 13$ cm and $BC = 12$ cm, find cosec C, cot C and sec C.



Sol. Δ ABC is a right angled triangle

$\therefore$ $\text{Cosec C} = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{AC}{AB} = \frac{13}{5}$, $\text{Cot C} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{BC}{AB} = \frac{12}{5}$ and $\text{Sec C} = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{\text{AC}}{\text{BC}} = \frac{13}{12}$

3. In Fig. ΔABC is a right triangle right angled at C. If $BC = 3$ cm, $AC = 4$ cm, find the values of cot A, sec A and cosec A.

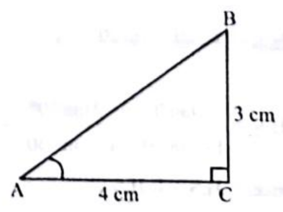
Sol. ΔABC is a right angled triangle

$\therefore$ $AB^2 = BC^2 + AC^2$ [By Pythagoras theorem]

$= (3)^2 + (4)^2 = 9 + 16 = 25 = (5)^2$

$\Rightarrow$ $AB = 5$ cm

$\therefore$ $\cot A = \frac{AC}{BC} = \frac{4}{3}$, $\sec A = \frac{AB}{AC} = \frac{5}{4}$ and $\text{cosecA} = \frac{AB}{BC} = \frac{5}{3}$



4. If $\sin\theta = \frac{7}{25}$, find the value of $\cos\theta$ and $\tan\theta$.

Sol. Draw *a* right angled triangle ABC in which $\angle B = 90$N and $\angle C = \theta$, as shown in Fig.

We know that $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{7}{25}$

Let $AB = 7$ and $AC = 25$

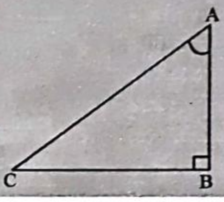
By the Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

or $(25)^2 = (7)^2 + BC^2$ or $BC^2 = 625 \,\$\, 49 = 576 = (24)^2$ $\therefore$ $BC = 24$

Now in ΔABC, $\cos\theta = \frac{BC}{AC} = \frac{24}{25}$ and $\tan\theta = \frac{AB}{BC} = \frac{7}{24}$

5. For a ΔABC, right angled at C, if $\tan A = 1$, find the value of $\cos B$.



Sol. In Fig. , ΔABC is a right triangle, right angled at C

We have $\tan A = 1 = \frac{BC}{AC}$

Let $AC = 1 = BC$ $\therefore$ $AB = \sqrt{2}$

Now, $\cos B = \frac{BC}{AB} = \frac{1}{\sqrt{2}}$. Hence $\cos B = \frac{1}{\sqrt{2}}$

6. Verify that $\frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} - \frac{2\sin 90°}{\cos 0°} = \frac{1}{2}$

Sol. We know that $\tan 45$N$= 1$, cosec 30N$=2$, $\sec 60$N$= 2$, $\cot 45$N$= 1$, $\sin 90$N$= 1$ and $\cos 0$N$= 1$

$\therefore$ L.H.S. $= \frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} - \frac{2\sin 90°}{\cos 0°} = \frac{1}{2} + \frac{2}{1} - \frac{2\times 1}{1} = \frac{1}{2} + 2 - 2 = \frac{1}{2}$ = R.H.S.

7. If $\theta = 30°$, verify that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Sol. We have $\theta = 30°$

L.H.S. $= \tan 2\theta = \tan 60° = \sqrt{3}$

$$\text{R.H.S.} = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-(1/\sqrt{3})^2} = \frac{2/\sqrt{3}}{1-(1/3)} = \frac{2\times\sqrt{3}}{\sqrt{3}\times 2} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Hence, L.H.S. = R.H.S.

8. Given that $\operatorname{cosec}\theta - \cot\theta = 5$, find the value of $\operatorname{cosec}\theta + \cot\theta =$ and $\sin\theta$.

Sol. We have, cosec²θ ş cot²θ = 1 ⇒ (cosecθ ş cotθ) (cosecθ + cotθ) = 1

$\Rightarrow 5(\operatorname{cosec}\theta + \cot\theta) = 1 \Rightarrow \operatorname{cosec}\theta + \cot\theta = \dfrac{1}{5}$

Now, cosecθ ş cotθ = 5

$$\text{Add }\frac{\operatorname{cosec}\theta + \cot\theta = \frac{1}{5}}{2\operatorname{cosec}\theta = 5 + \frac{1}{5} = \frac{26}{5}} \Rightarrow \operatorname{cosec}\theta = \frac{13}{5} \Rightarrow \sin\theta = \frac{5}{13}$$

9. Evaluate $\sin^2 40° - \cos^2 50°$.

Sol. We know that, cos (90N ş θ) = sin θ

cos 50N= cos (90 ş 40)N= sin 40N

Hence, sin²40N ş cos²50N= sin²40N ş sin²40N= 0

10. Evaluate $\dfrac{\cos 43°}{\cos 47°} + \dfrac{\sec 32°}{\operatorname{cosec} 58°}$

Sol. We know that cos (90N ş θ) = sin θ

sin 47N= sin (90N ş θ) = cos 43N

Also, cosec 58N= cosec (90N ş 32N) = cos 32N

$$\therefore \frac{\cos 43°}{\cos 47°} + \frac{\sec 32°}{\operatorname{cosec} 58°} = \frac{\cos 43°}{\cos 43°} + \frac{\sec 32°}{\sec 32°} = 1 + 1 = 2$$

11. Prove that $\dfrac{\sin(90° - \theta)}{\operatorname{cosec}(90° - \theta)} + \dfrac{\cos(90° - \theta)}{\sec(90° - \theta)} = 1$

Sol. We know that sin (90N ş θ) = cos θ , cos (90N ş θ) = sin θ , cosec (90N ş θ) = sec θ , sec (90N ş θ) = cosec θ

$$\text{L.H.S.} = \frac{\sin(90° - \theta)}{\operatorname{cosec}(90° - \theta)} + \frac{\cos(90° - \theta)}{\sec(90° - \theta)} = \frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\operatorname{cosec}\theta} = \cos^2\theta + \sin^2\theta = 1 = \text{R.H.S.}$$

12. Prove that $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$

Sol. $\text{L.H.S.} = \tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta} = \text{R.H.S}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

13. Prove that $\dfrac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$

Sol. $\text{L.H.S.} = \dfrac{1 - \sin A}{1 + \sin A} = \dfrac{1 - \sin A}{1 + \sin A} \times \dfrac{1 - \sin A}{1 - \sin A} = \dfrac{(1 - \sin A)^2}{1 - \sin^2 A} = \dfrac{(1 - \sin A)^2}{\cos^2 A}$ $[\because 1 - \sin A = \cos^2 A]$

$$= \left(\frac{1 - \sin A}{\cos A}\right)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 = (\sec A - \tan A)^2 = R.H.S.$$

14. The length of a pendulum is 80 cm. Its end describes an arc of length 16 cm. Find the angle through which it swings while making that arc.

Sol. $l = r\theta$ where l is the arc, r the radius and 'θ' the angle.

$\Rightarrow 16 = 80\theta \Rightarrow \theta = \dfrac{16}{80} = \dfrac{1^c}{5}$

15. Find the value of other five trigonometric function when $\cos x = -\frac{1}{2}$, x lies in third quadrant.

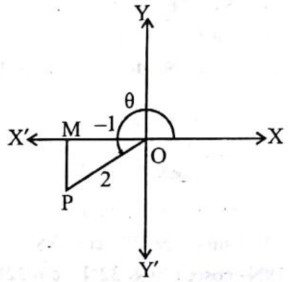
Sol. Since x lies in the 3rd quadrant

$\therefore \cos x = -\dfrac{1}{2} \Rightarrow \dfrac{OM}{OP} = -\dfrac{1}{2}$

Let $OM = -1$ and $OP = 2$

$\therefore MP = -\sqrt{OP^2 - OM^2} = -\sqrt{4-1} = -\sqrt{3}$

Now, $\text{Sin}\, x = \dfrac{MP}{OM} = \dfrac{-\sqrt{3}}{2}$



$\cos x = -\dfrac{1}{2}, \tan x = \dfrac{MP}{OM} = \dfrac{-\sqrt{3}}{2} = \sqrt{3}$

$\cot x = \dfrac{OM}{MP} = \dfrac{-1}{-\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\sec x = \dfrac{OP}{OM} = \dfrac{2}{-1} = -2\,;\ \operatorname{cosec} x = \dfrac{OP}{MP} = \dfrac{2}{-\sqrt{3}} = \dfrac{-2}{\sqrt{3}}$

16. Prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Sol. L.H.S. $= \sin x + \sin 3x + \sin 5x + \sin 7x$

$= (\sin 7x + \sin x) + (\sin 5x + \sin 3x) = 2\sin\dfrac{7x+x}{2}\cos\dfrac{7x-x}{2} + 2\sin\dfrac{5x+3x}{2}\cos\dfrac{5x-3x}{2}$

$= 2\sin 4x\ \cos 3x + 2\sin 4x \cos x\ = 2\sin 4x\,[\,\cos 3x + \cos x]$

$= 2\sin 4x\left[2\cos\dfrac{3x+x}{2}\cos\dfrac{3x-x}{2}\right] = 4\sin 4x.\cos 2x.\cos x$

$= 4\cos x \cos 2x \sin 4x = \text{RHS}.$

17. Prove that $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\frac{x}{2}\cos\frac{3x}{2}$

Sol. LHS $= \sin 3x + (\sin 2x - \sin x)$

$= 2\sin\dfrac{3x}{2}\cos\dfrac{3x}{2} + 2\cos x\dfrac{2x+x}{2}\sin\dfrac{2x-x}{2}\ = 2\sin\dfrac{3x}{2}\cos\dfrac{3x}{2} + 2\cos\dfrac{3x}{2}\sin\dfrac{x}{2}$

$= 2\cos\dfrac{3x}{2}\left[\sin\dfrac{3x}{2} + \sin\dfrac{x}{2}\right] = 2\cos\dfrac{3x}{2}\,2\sin\dfrac{\frac{3x}{2}+\frac{x}{2}}{2}\cos\dfrac{\frac{3x}{2}-\frac{x}{2}}{2}$

$= 2\cos\dfrac{3x}{2}(2\sin x.\cos\dfrac{x}{2}) = 4\cos\dfrac{3x}{2}.\sin x\cos\dfrac{x}{2} = 4\sin x\cos\dfrac{x}{2}\cos\dfrac{3x}{2}.$ = RHS

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The value of sin A or cos A never exceeds ......
2. $\sin^2 A + \cos^2 A =$ ....
3. If $\tan A = 4/3$ then $\sin A$ ....
4. In a right trianlge ABC, right angled at B, if $\tan A = 1$, $\sin A \cos A =$ ....
5. In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. $\sin A =$ ....
6. If $15 \cot A = 8$, $\sec A =$ ....
7. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. The value of $\tan P$ is ....
8. $\sin 60° \cos 30° + \sin 30° \cos 60° =$ ....
9. $\sin(\theta + 30°) \sin(\theta - 30°) = \sin^2\theta$
10. $\sin^2\theta + \sin^2(90° - \theta) =$ ....
11. $2\tan 45° + 3\cos 30° - \sin 60° =$ ....
12. $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°} =$ ....
13. $\dfrac{\sin 18°}{\cos 72°} =$ ....
14. $\cos 48° - \sin 42° =$ ....
15. $\sec A(1 - \sin A)(\sec A + \tan A) =$ ....

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The value of $\tan A$ is always less than 1.
2. $\sec A = 12/5$ for some value of angle A.
3. $\cos A$ is the abbreviation used for the cosecant of angle A.
4. $\cot A$ is the product of cot and A.
5. $\sin\theta = \dfrac{4}{3}$ for some angle θ.
6. $\sin(A + B) = \sin A + \sin B$.
7. The value of $\sin\theta$ increases as θ increases.
8. The value of $\cos\theta$ increases as θ increases.
9. $\sin\theta = \cos\theta$ for all values of θ.
10. $\cot A$ is not defined for $A = 0°$
11. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then $\angle B \neq \angle Q$.
12. If $3\cot A = 4$, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$
13. $\dfrac{\tan 65°}{\tan 25°} = 1$
14. $\tan 48° \tan 23° \tan 42° \tan 67° \neq 1$

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. In ΔABC, $\angle B = 90°$, $AB = 3$ cm and $BC = 4$ cm then match the column.

Column I	Column II
(A) $\sin C$	(p) 3/5
(B) $\cos C$	(q) 4/5
(C) $\tan A$	(r) 5/3
(D) $\sec A$	(s) 4/3

2.

Column I	Column II
(A) $\dfrac{1-\tan^2 A}{1+\tan^2 A}$	(p) $\sin 2A$
(B) $\dfrac{2\tan A}{1+\tan^2 A}$	(q) $\cos^2 A - \sin^2 A$
(C) $\tan(90° - A)$	(r) $\sin A$
(D) $\cos(90° - A)$	(s) $\cot A$

3.

Column I	Column II
(A) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$	(p) $\operatorname{cosec} A + \cot A$
(B) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$	(q) $2\sec A$
(C) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$	(r) $\sec A + \tan A$
(D) $\dfrac{\sin^2 A}{1-\cos A}$	(s) $\dfrac{1+\sec A}{\sec A}$

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. If $\cot\theta = \frac{7}{8}$, evaluate $\cot^2\theta$
2. It $\tan A = \cot B$, prove that $A + B = 90$N
3. Prove that $\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1$
4. Prove that $\frac{1+\cos^2\theta}{\sin^2\theta} = 2\operatorname{cosec}^2\theta - 1$
5. Evaluate: $\sin^2 60$N$\cos^2 45$N$\cos^2 60$N$\operatorname{cosec}^2 90$N
6. If $\tan 2A = \cot (A$ ş 18N), where $2A$ is an acute angle, find the value of A.
7. If $\tan A = \cot B$, prove that $A + B = 90$N
8. Convert $\frac{17\pi}{18}$ into degrees.
9. Find the length of an arc of a circle of 3 cm radius if the angle subtended at the centre is 30N. ($\pi = 3.14$)

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\sin\theta)(1+\sin\theta)}$
2. If $13\tan\theta = 12$, then find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$
3. If $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, find A and B.
4. Find θ if $\sin 3\theta = \cos(\theta - 6)°$, where 3θ and $(\theta-6)°$ are acute angles.
5. If $5\tan\theta = 4$, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$
6. Prove that $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$
7. If $\sin\theta = \frac{3}{5}$, find the value of $(\tan\theta + \sec\theta)^2$.
8. Find the value of $\sin^2 10$N+ $\sin^2 30$N+ $\sin^2 60$N+ $\sin^2 80$N
9. If $\tan\theta + \sin\theta = m$ and $\tan\theta$ ş $\sin\theta = n$, then prove that m^2 ş $n^2 = 4\sqrt{mn}$.
10. Express $\frac{2\tan 30°}{1+\tan^2 30°}$ in terms of sin.
11. If $\sin 3A = \cos(A$ ş 26N), where $3A$ is an acute angle, find the value of A.
12. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then prove that $2m = n(m^2$ ş $1)$.
13. Show that : tan 1Ntan 2Ntan 3Ntan 87Ntan 88Ntan 89N= 1
14. Prove that : $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$
15. Prove that $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot\theta$

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$, then find $\operatorname{cosec}\theta + \cot\theta$.
2. Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 + 2\tan^2 A$
3. Evaluate :
$$\frac{\sec\theta.\operatorname{cosec}\theta(90°-\theta) - \tan\theta\cot(90°-\theta) + \sin^2 55° + \sin^2 35°}{\tan 10°\tan 20°\tan 60°\tan 70°\tan 80°}$$
4. Prove the following identities:
   (i) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$.
   (ii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.
5. Without using tables, evaluate the following:
   (i) $\frac{\tan 40°}{\cot 50°} + (\sin^2 20° + \sin^2 70°) + \tan 5°\tan 10°\tan 30°\tan 80°\tan 85°$.
   (ii) $\frac{\sin 39°}{\cos 51°} + 2\tan 11°\tan 31°\tan 45°\tan 59°\tan 79° - 3(\sin^2 21° + \sin^2 69°)$
6. Prove that
$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta = \sec\theta\operatorname{cosec}\theta + 1$$
7. Prove : $\frac{1+\sin\theta-\cos\theta}{\cos\theta - 1 + \sin\theta} = \frac{1+\sin\theta}{\cos\theta}$
8. Evaluate
$$\cot\theta.\tan(90-\theta) - \sec(90-\theta)\operatorname{cosec}\theta + \sin^2 25° + \sin^2 65° + \sqrt{3}(\tan 5°\tan 45°\tan 85°)$$
9. Prove that:
$$\frac{\cos A}{(1 \text{ ş } \tan A)} + \frac{\sin A}{(1 \text{ ş } \cot A)} = \sin A + \cos A.$$
10. Find the values of sin 18N

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. $\cos 1^\circ . \cos 2^\circ . \cos 3^\circ$ .... $\cos 179^\circ$ is equal to –

(a) -1 (b) 0

(c) 1 (d) $1/\sqrt{2}$

2. $\sin^2\theta + \operatorname{cosec}^2\theta$ is always –

(a) greater than 1

(b) less than 1

(c) greater than or equal to 2

(d) equal to 2

3. If $\sin\theta + \cos\theta = a$ and $\dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = b$, then

(a) $b = \dfrac{2a}{a^2-1}$ (b) $a = \dfrac{2b}{b^2-1}$

(c) $ab = b^2 - 1$ (d) $a + b = 1$

4. The value of $(\sin^2 7\frac{1}{2}^\circ + \cos^2 7\frac{1}{2}^\circ) - (\sin^2 30^\circ + \cos^2 30^\circ) + (\sin^2 7^\circ + \sin^2 83^\circ)$ is equal to

(a) 3 (b) $3\frac{1}{2}$

(c) 2 (d) 1

5. If $\tan 15^\circ = 2 - \sqrt{3}$, then the value of $\cot^2 75^\circ$ is –

(a) $7+\sqrt{3}$ (b) $7-2\sqrt{3}$

(c) $7-4\sqrt{3}$ (d) $7+4\sqrt{3}$

6. The maximum value of $(3\sin\theta + 4\cos\theta)$ is –

(a) 7 (b) 5

(c) 1 (d) -1

7. If $x = p\sec\theta$ and $y = q\tan\theta$ then –

(a) $x^2 - y^2 = p^2q^2$ (b) $x^2q^2 - y^2p^2 = pq$

(c) $x^2q^2 - y^2p^2 = \dfrac{1}{p^2q^2}$ (d) $x^2q^2 - y^2p^2 = p^2q^2$

8. If $b\tan\theta = a$, the value of $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

(a) $\dfrac{a-b}{a^2+b^2}$ (b) $\dfrac{a+b}{a^2+b^2}$

(c) $\dfrac{a^2+b^2}{a^2-b^2}$ (d) $\dfrac{a^2-b^2}{a^2+b^2}$

9. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then the value of $m^2 - n^2$ is equal to –

(a) $4mn$ (b) $2\sqrt{mn}$

(c) $4\sqrt{mn}$ (d) $2\sqrt{m/n}$

10. A circular wire of radius 7 cm is cut and bend again into an arc of a circle of radius 12 cm. The angle subtended by the arc at the centre is –

(a) 50° (b) 210°

(c) 100° (d) 60°

11. If $\sin\theta = \dfrac{24}{25}$ and θ lies in the second quadrant, then $\sec\theta + \tan\theta =$

(a) -7 (b) 6

(c) 4 (d) -5

12. $\cot x - \tan x =$

(a) $\cot 2x$ (b) $2\cot^2 x$

(c) $2\cot 2x$ (d) $\cot^2 2x$

13. $\tan 9^\circ \times \tan 27^\circ \times \tan 63^\circ \times \tan 81^\circ =$

(a) 4 (b) 3

(c) 2 (d) 1

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. If $\operatorname{cosec} A + \cot A = \dfrac{11}{2}$, then $\tan A$

(a) $\dfrac{21}{22}$ (b) $\dfrac{15}{16}$

(c) $\dfrac{44}{117}$ (d) $\dfrac{88}{234}$

2. Which of the following is/are not correct?

(a) $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x . \tan y}$

(b) $\cos 2x = \cos^2 x - \sin^2 x$

(c) $\sin 2x = \dfrac{2\tan x}{1-\tan^2 x}$

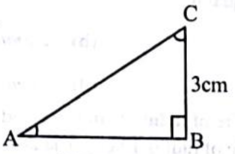
(d) $\cos 3x = 4\cos^3 x + 3\cos x$

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage-1

In ΔABC, right angled at B



$AB + AC = 9$ cm and $BC = 3$cm.

1. The value of cot C is

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$

(c) $\frac{5}{4}$ (d) none

2. The value of sec C is

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$

(c) $\frac{1}{3}$ (d) none

3. $\sin^2 C + \cos^2 C =$

(a) 0 (b) 1

(c) -1 (d) none

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect.**

(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion:** In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.
Reason: $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$.

2. **Assertion :** In a right angled triangle, if $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $\tan\theta = \sqrt{3}$

Reason: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

3. **Assertion:** In a right angled triangle, $\sin 47° = \cos 43°$
Reason: $\sin\theta = \cos(90 + \theta)$, where θ is an angle in the right angled triangle.

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r, s......) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. If $\sin A = \frac{7}{25}$, then

Column-I	Column-II
(A) Cos A	(p) 24/25
(B) tan A	(q) 7/24
(C) Cosec A	(r) 25/7
(D) Sec A	(s) 25/24
	(t) $1 - 1/25$
	(u) $1 + 1/24$

HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Evaluate

$$\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45° + \frac{2}{\sqrt{3}} \tan 17° \tan 60° \tan 73°.$$

2. Evaluate :

(i) $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$

(ii) $\operatorname{cosec}(65° + \theta) - \sec(25° - \theta) - \tan(55° - \theta) + \cot(35° + \theta)$.

3. (i) If $\cos\theta + \sqrt{3}\sin\theta = 2\sin\theta$.

Show that $\sin\theta - \sqrt{3}\cos\theta = 2\cos\theta$.

(ii) If $\cos\theta + \sec\theta = \sqrt{3}$. Prove that $\cos^3\theta + \sec^3\theta = 0$.

(iii) If $\sin\theta + \operatorname{cosec}\theta = 2$, show that $\sin^n\theta + \operatorname{cosec}^n\theta = 2$

4. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

5. Show that $3(\sin x + \cos x)^4 - 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 1$.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. 1 2. 1 3. 4/5

4. $\frac{1}{2}$ 5. 7/25 6. 17/8

7. 12/5 8. 1

9. $\frac{1}{4}$ **Hint :** [sin (θ + 30N) sin (θ \$ 30N) = sin²θ \$ sin² 30N]

10. 1 [**Hint :** sin² (90N\$ θ)] = cos²θ]

11. $\frac{7}{2}$ 12. $\frac{3(\sqrt{3}-1)}{4}$ 13. 1

14. 0 15. 1

TRUE / FALSE

1. False	2. True	3. False
4. False	5. False	6. False
7. True	8. False	9. False
10. True	11. False	12. True
13. False	14. False	

MATCH THE FOLLOWING :

1. (A) → p; (B) → q; (C) → s; (D) → r
2. (A) → q; (B) → p; (C) → s; (D) → r
3. (A) → q; (B) → p; (C) → r; (D) → s

VERY SHORT ANSWER QUESTIONS :

1. $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

2. $\tan A = \cot B \Rightarrow \tan A = \tan (90N\$ B)$
$\Rightarrow A = 90N\$ B \Rightarrow A + B = 90N$

4. L.H.S. $= \frac{1+\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta = \cot^2\theta = \operatorname{cosec}^2\theta + (\operatorname{cosec}^2\theta - 1)$

$= 2\operatorname{cosec}^2\theta - 1 =$ R.H.S.

5. Substitute the required values and simplify.

6. $\cot (A \$ 18N) = \tan \{90N\$ (A \$ 18N)\}$
So, $\tan 2A = \tan (90N\$ A + 18N) \Rightarrow 2A = 108N\$ A$
$\Rightarrow A = 36N$

7. $\cot B = \tan (90 \$ B) \Rightarrow \tan A = \tan (90N\$ B) \Rightarrow A = 90 \$ B$
$\Rightarrow A + B = 90N$

8. 170N

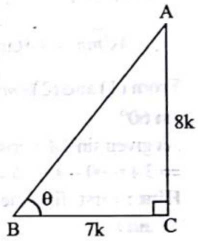
9. 1.57 cm (Hint : apply formula. Arc = rθ)

SHORT ANSWER QUESTIONS :

1. $\cot\theta = \frac{BC}{AC} = \frac{7k}{8k}$

In ΔABC, $AB^2 = (8k)^2 + (7k)^2$

$\therefore AB = k\sqrt{113}$



$\frac{(1-\cos\theta)(1+\cos\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{1-\cos^2\theta}{1-\sin^2\theta}$

$= \frac{AB^2 - BC^2}{AB^2 - AC^2} = \frac{64}{49}$

2. Given : $13\tan\theta = 12$

Now given expression is, $\frac{2\sin\theta.\cos\theta}{\cos^2\theta - \sin^2\theta}$

Dividing numerator and denominator by $\cos^2\theta$,

Given expression is $= \frac{312}{25}$

3. $A = 30N$ and $B = 15N$

4. $\sin 3\theta = \cos (\theta - 6)°$

where $(\theta - 6)°$ is an acute angle

$\therefore \theta = \frac{96}{4} = 24°$

5. Given $5\tan\theta = 4$

$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$

Dividing the numerator and denominator by $\cos\theta$, we get

Given expression is $\frac{1}{6}$

6. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \tan\theta + \cot\theta =$ R.H.S.

7. Given $\sin\theta = \frac{3}{5}$

$\Rightarrow \cos\theta = \frac{4}{5} \qquad \therefore \sec = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$

$\therefore (\tan\theta + \sec\theta)^2 = 4$

8. Consider $\sin^2 10° + \sin^2 30° + \sin^2 60° + \sin^2 80°$

$= \sin^2 10° + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \sin^2(90° - 10°) = 2$

9. By adding and subtracting both the given equations, we have $(m+n) = 2\tan\theta$, $m - n = 2\sin\theta$

$\therefore \quad m^2 - n^2 = 4\tan\theta.\sin\theta \quad ...(1)$

$4\sqrt{mn} = 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sin\theta.\tan\theta \quad ...(2)$

From (1) and (2), $m^2 - n^2 = 4\sqrt{mn}$.

10. $\sin 60°$.

11. As given $\sin 3A = \cos(A - 26°)$ or $\sin 3A = \sin\{90 - (A - 26°)\}$ $\Rightarrow 3A = 90 - A + 26 \Rightarrow 4A = 116 \Rightarrow A = 29°$

12. **Hint :** First, find the value of R.H.S. by putting the value of m and n.

13. L.H.S. $= \tan 1° \tan 2° \tan 3° \tan 87° \tan 88° \tan 89°$
$= \tan 1° \tan 2° \tan 3° \tan(90° - 3°) \tan(90° - 2°)$
$\tan(90° - 1°)$
$= (\tan 1° \cot 1°)(\tan 2° \cot 2°)(\tan 3° \cot 3°) = 1 \times 1 \times 1 = 1 =$ R.H.S.

14. **Hint :** First, find the sum of L.H.S. by taking L.C.M.

15. **Hint :** It is easy to prove that $\cot\theta - \tan\theta = 2\cot 2\theta$

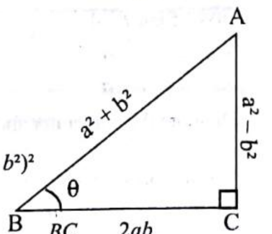
LHS $= \cot\theta - (\cot\theta - \tan\theta) + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

$= \cot\theta - 2\cot 2\theta - \tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$ etc.

LONG ANSWER QUESTIONS :

1. $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$ Since $\sin\theta = \frac{\text{perpendicular}}{\text{base}}$

$\therefore \frac{AC}{AB} = \frac{a^2 - b^2}{a^2 + b^2}$



Now in ΔABC,
$\angle B = \theta$ and $\angle C = 90°$
$(a^2 + b^2)^2 = BC^2 + (a^2 - b^2)^2$
$\therefore BC = 2ab$

$\operatorname{cosec}\theta = \frac{a^2 + b^2}{a^2 - b^2}, \quad \cot\theta = \frac{BC}{AC} = \frac{2ab}{a^2 - b^2}$

$\operatorname{cosec}\theta + \cot\theta = \frac{a^2 + b^2}{a^2 - b^2} + \frac{2ab}{a^2 - b^2} = \frac{a+b}{a-b}$

2. L.H.S. $= \frac{2\cot^2 A}{\cot^2 A} + \frac{2}{\cot^2 A} = 2 + 2\tan^2 A = R.H.S.$

3. $\frac{\sec\theta.\operatorname{cosec}\theta(90° - \theta) - \tan\theta \cot(90° - \theta) + \sin^2 55° + \sin^2 35°}{\tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}$

$= \frac{\sec\theta.\sec\theta - \tan\theta.\tan\theta + \sin^2(90° - 35°) + \sin^2 35°}{\tan 10° \tan 20°.\sqrt{3}.\tan(90° - 20°)\tan(90° - 10°)}$

[Using $\operatorname{cosec}(90° - \theta) = \sec\theta$, $\cot(90° - \theta) = \tan\theta$]

$= \frac{(\sec^2\theta - \tan^2\theta) + (\cos^2 35° + \sin^2 35°)}{\sqrt{3}\tan 10° \tan 20° \cot 20° \cot 10°}$

[Using $\sin(90° - \theta) = \cos\theta$, $\tan(90° - \theta) = \cot\theta$]

$= \frac{1+1}{\sqrt{3}.(\tan 10° \cot 10°)(\tan 20° \cot 20°)}$

[Using $\sec^2\theta - \tan^2\theta = 1$, $\sin^2\theta + \cos^2\theta = 1$]

$= \frac{2}{\sqrt{3} \times 1 \times 1} = \frac{2}{\sqrt{3}}$ [Using $\tan\theta.\cot\theta = 1$]

4. (i) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$= \frac{1 + \tan^2 A}{1 + \tan^2 A} \times \tan^2 A = \tan^2 A$ ...(i)

Again, $\tan^2 A = (\tan A)^2 = \left[\frac{(1 - \tan A)\tan A}{(1 - \tan A)}\right]^2$

$= \left(\frac{1 - \tan A}{1 - \cot A}\right)^2$ ...(ii) $[\because (a-b)^2 = (b-a)^2]$

From equation (i), and (ii)

$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

5. (i) $\frac{\tan 40°}{\cot 50°} + (\sin^2 20° + \sin^2 70°) +$
$\tan 5° \tan 10° \tan 30° \tan 80° \tan 85°$

$= \frac{\tan(90° - 50°)}{\cot 50°} + \{\sin^2 20° + \sin^2(90° - 20°)\}$

$+ \tan 5° \tan 10° \times \frac{1}{\sqrt{3}} \cot 10° \cot 5°$

$= \frac{\cot 50°}{\cot 50°} + (\sin^2 20° + \cos^2 20°)$

$+ \frac{1}{\sqrt{3}}(\tan 5° \cot 5°)(\tan 10° \cot 10°)$

$= \frac{2\sqrt{3} + 1}{\sqrt{3}}$.

(ii) $\frac{\sin 39^\circ}{\cos 51^\circ} = \frac{\sin 39^\circ}{\cos(90^\circ - 39^\circ)} = \frac{\sin 39^\circ}{\sin 39^\circ} = 1$

$\tan 11^\circ \tan 31^\circ \tan 45^\circ \tan 59^\circ \tan 79^\circ$

$= \tan 11^\circ \tan 31^\circ (1) \tan(90^\circ - 31^\circ) \tan(90^\circ - 11^\circ)$

$= (\tan 11^\circ \cot 11^\circ)(\tan 31^\circ \cot 31^\circ) = (1)(1) = 1$

$\sin^2 21^\circ + \sin^2 69^\circ = \sin^2 21^\circ + \sin^2(90^\circ - 21^\circ)$

$= \sin^2 21^\circ + \cos^2 21^\circ = 1$

$\therefore$ Given expression $= 1 + 2(1) - 3(1) = 3 - 3 = 0$.

6. LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\sin\theta}{\cos\theta.\left(\frac{\sin\theta - \cos\theta}{\sin\theta}\right)} + \frac{\cos\theta}{\sin\theta.\left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^2\theta \times \sin\theta - \cos^2\theta \times \cos\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$= \sec\theta\,\mathrm{cosec}\theta + 1$ ......(1)

RHS : $1 + \tan\theta + \cot\theta$

$= \frac{\sin\theta\cos\theta + 1}{\sin\theta\cos\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta =$ LHS

LHS = RHS Hence Proved.

7. L.H.S. : $\frac{1+\sin\theta - \cos\theta}{\cos\theta - 1 + \sin\theta}$

Dividing each term of N^r and D^r by '$\cos\theta$'.

8. $\cot\theta\tan(90-\theta) - \sec(90-\theta)\mathrm{cosec}\theta$

$+ \sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}(\tan 5^\circ \tan 45^\circ \tan 85^\circ)$

$= \cot^2\theta - \mathrm{coses}^2\theta + \sin^2 25^\circ$

$+ \cos^2 25^\circ + \sqrt{3}[\tan 5^\circ.1 \times \cot 5^\circ]$

$= -(\mathrm{coses}^2\theta - \cot^2\theta) + 1 + \sqrt{3}\left(\tan 5^\circ \times \frac{1}{\tan 5^\circ}\right)$

$= -1 + 1 + \sqrt{3} \times 1 = \sqrt{3}$

9. **Hint :** Put $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ in L.H.S. and the simply the L.H.S.

$\tan\theta = \frac{\cos A}{(1-\tan A)} + \frac{\sin A}{1-\cot A} = \cos A + \sin A =$ R.H.S

10. Let $\theta = 18^\circ$, then $2\theta = 36^\circ = 90^\circ - 3\theta$

Now $\sin 2\theta = 2\sin\theta\cos\theta$ and

$\sin(90 - 3\theta) = \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

We have

$2\sin\theta\cos\theta = \cos\theta(1 - 4\sin^2\theta)$

Hence $2\sin\theta = 1 - 4\sin^2\theta$ (as $\cos\theta \neq 0$)

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-1 \pm \sqrt{5}}{4}$

But as $\sin\theta > 0$, we have $\sin\theta = \frac{\sqrt{5}-1}{4}$

i.e., $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (b) 2. (c) 3. (a)

4. (d) $\sin 83^\circ = \cos 7^\circ$

$\therefore$ the given expression is $1 - 1 + 1 = 1$

5. (c) $\cot^2 75^\circ = (2-\sqrt{3})^3 = 7 - 4\sqrt{3}$

6. (b) $\sqrt{3^2 + 4^2} = 5$

7. (d) We know $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p}$, $\tan\theta = \frac{y}{q}$

$\therefore$ $x^2q^2 - p^2y^2 = p^2q^2$

8. (d) $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$$

9. (c)

10. (b) Given that diameter of circular wire = 14 cm.

Therefore, length of circular wire = 14π cm

$\therefore$ Required angle

$= \frac{\text{arc}}{\text{radius}} = \frac{14\text{p}}{12} = \frac{7\text{p}}{6} = \frac{7}{6}\text{p}.\frac{180^\circ}{\text{p}} = 210^\circ$

11. (a) $\sec\theta + \tan\theta = \frac{-25}{7} + \frac{-24}{7} = -7$

12. (c) $\cot x - \tan x \quad \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} = 2\cot 2x$

13. (d)

MORE THAN ONE CORRECT :

1. (c), (d) 2. (c, d)

PASSAGE BASED QUESTIONS :

1. In ΔABC,
By Pythagoras theorem,
$AC^2 = AB^2 + BC^2 \Rightarrow AB = 4$ cm.
$AC = 5$ cm.

(i) (a) $\cot C = \dfrac{BC}{AB} = \dfrac{3}{4}$

(ii) (b) $\sec C = \dfrac{AC}{BC} = \dfrac{5}{3}$

(iii) (b) $\sin C = \dfrac{4}{5}$

$\cos C = \dfrac{3}{5}$

L.H.S $= \sin^2 C + \cos^2 C = \left(\dfrac{4}{5}\right)^2 + \left(\dfrac{3}{5}\right)^2 = \dfrac{16+9}{25} = 1 =$ R.H.S

ASSERTION & REASON :

1. (a) Both Assertion and Reason are correct and Reason is the correct explanation of the assertion.

greatest side $= \sqrt{(3)^2 + (4)^2} = 5$ units.

2. (a) Both assertion and reason are correct and reason is the correct explanation of the assertion.

$\tan\theta = \dfrac{\sqrt{3}}{2} \times 2 = \sqrt{3}$.

3. (c) Assertion is true, but reason is not correct.
$\sin\theta = \cos(90 - \theta)$
$\sin 47° = \cos(90 - 47) = \cos 43°$

MULTIPLE MATCHING QUESTIONS :

1. (A) → p, t; (B) → q; (C) → r; (D) → s,u

HOTS SUBJECTIVE QUESTIONS :

1. The given expression is

$$\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°$$

$$+ \frac{2}{\sqrt{3}} \tan 17° \tan 60° \tan 73°$$

$$= \frac{\sec^2(90° - 36°) - \cot^2 36°}{\operatorname{cosec}^2(90° - 33°) - \tan^2 33°} + 2\sin^2 38° \sec^2 (90° - 38°)$$

$$-\sin^2 45° + \frac{2}{\sqrt{3}} \tan(90° - 73°) \tan 73° \tan 60°$$

$$= \frac{1}{1} + 2\sin^2 38° \times \frac{1}{\sin^2 38°} - \frac{1}{2} + \frac{2}{\sqrt{3}} \times \frac{1}{\tan 73°} \times \tan 73° \times \sqrt{3}$$

$[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1,\ \sec^2\theta - \tan^2\theta = 1]$

$$= 1 + 2 - \frac{1}{2} + 2 = 5 - \frac{1}{2} = \frac{9}{2}$$

2. (i) Let $x = \tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$
$= \tan 35° \tan 40° \tan 45° \tan(90 - 40) \tan(90 - 35°)$
$= \tan 35° \tan 40° \tan 45°.\ \cot 40° \cot 35°$
$= (\tan 35°.\ \cot 35°)(\tan 40°.\ \cot 40°).\ \tan 45° = 1$

(ii) Let $x = \operatorname{cosec}(65° + \theta) - \sec(25° - \theta) - \tan(55 - \theta) + \cot(35 + \theta)$
$= \operatorname{cosec}\{90 - (25 - \theta)\} - \sec(25 - \theta) - \tan(55 - \theta) + \cot\{90 - (55 - \theta)\} = \sec\{(25 - \theta)\} - \sec(25 - \theta) - \tan(55 - \theta) + \tan(55 - \theta) = 0.$

3. (i) Let $\cos\theta + \sqrt{3}\sin\theta = 2\sin\theta$

$\Rightarrow \cos\theta = 2\sin\theta - \sqrt{3}\sin\theta = (2 - \sqrt{3})\sin\theta$

Multiplying both sides by $2 + \sqrt{3}$, we get

$(2 + \sqrt{3})\cos\theta = (2 + \sqrt{3})(2 - \sqrt{3})\sin\theta$

$\Rightarrow (2 + \sqrt{3})\cos\theta = \{(2)^2 - (\sqrt{3})^2\}\sin\theta$

$\Rightarrow 2\cos\theta + \sqrt{3}\cos\theta = (4 - 3)\sin\theta$

$\Rightarrow 2\cos\theta + \sqrt{3}\cos\theta = \sin\theta \Rightarrow \sin\theta - \sqrt{3}\cos\theta = 2\cos\theta$

(ii) Let $\cos\theta + \sec\theta = \sqrt{3}$ ...(1)
Cubing both sides of (1), we get

$\cos^3\theta + \sec^3\theta + 3\cos\theta.\sec\theta(\cos\theta + \sec\theta) = (\sqrt{3})^3$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3 \times 1 \times \sqrt{3} = 3\sqrt{3} \Rightarrow \cos^3\theta + \sec^3\theta = 0$

(iii) Let $\sin\theta + \operatorname{cosec}\theta = 2$,

$\Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2 \Rightarrow \dfrac{\sin^2\theta + 1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0 \Rightarrow (\sin\theta - 1)^2 = 0$

$\Rightarrow \sin\theta - 1 = 0; \sin\theta = 1$

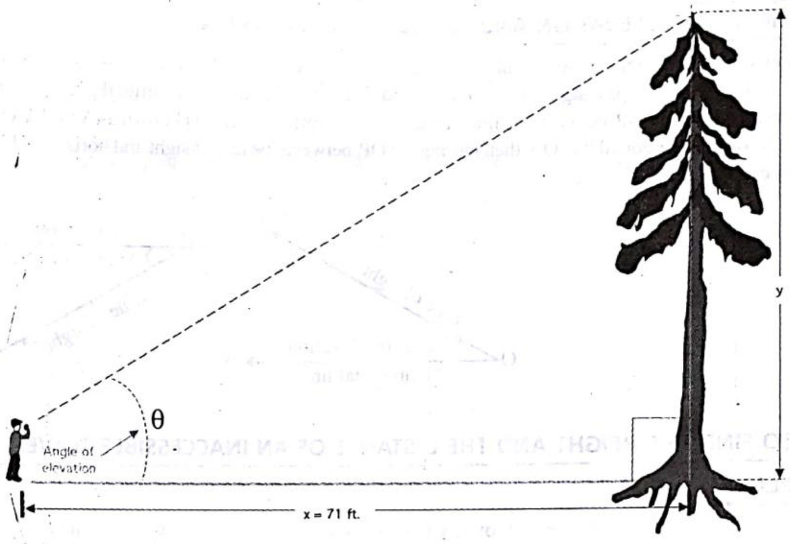
$\therefore \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = 1$

Hence, $\sin^n\theta + \operatorname{cosec}^n\theta = (1)^n + (1)^n = 1 + 1 = 2.$

4. **Hint :** Square and add both the given equations.

5. Expand the given expression by using necessary algebraic identities.

chapter 9



APPLICATIONS OF TRIGONOMETRY

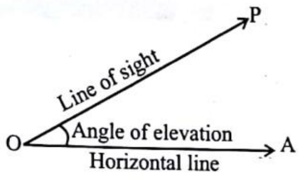
Introduction

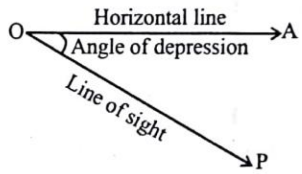
Trigonometry has an enormous variety of applications. It is used extensively in a number of academic fields, primarily mathematics, science and engineering.

Trigonometry, in ancient times, was often used in the measurement of heights and distances of objects which could not be otherwise measured. For example, trigonometry was used to find the distance of stars from the Earth. Even today, in spite of more accurate methods being available, trigonometry is often used for making quick and simple calculations regarding heights and distances of far-off objects. For this, the value of various trigonometric functions is needed. A simple example is given below to demonstrate how trigonometry can help to find the height or distance of an object.

ANGLE OF ELEVATION AND ANGLE OF DEPRESSION :

Let an observer at the point O is observing an object at the point P. The line OP is called the LINE OF SIGHT of the point P. Let OA be the horizontal line passing through O. O, A and P should be in the same vertical plane. If object P be above the line of sight OA, then the acute angle AOP, between the line of sight and the horizontal line is known as ANGLE OF ELEVATION of object P. If the object P is below the horizontal line OA then the angle AOP, between the line of sight and horizontal line is known as ANGLE OF DEPRESSION of object P.



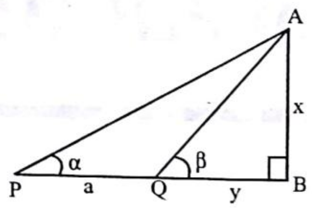


TO FIND THE HEIGHT AND THE DISTANCE OF AN INACCESSIBLE TOWER STANDING ON A HORIZONTAL PLANE :

Let AB be a tower and B be its foot. On the horizontal line through B, take two points P and Q. Measure the length PQ. Let PQ = a.

Let the angles of elevation of the top A of the tower as seen from P and Q be respectively α and β ($\beta > \alpha$) then $\angle APB = \alpha$, $\angle AQB = \beta$.

Let $AB = x$, $BQ = y$.



From right angled Δ ABP, $\tan\alpha = \dfrac{AB}{PB} = \dfrac{x}{a+y}$

$\therefore a + y = x\cot\alpha$ ....(i)

From right angled ΔABQ, $\tan\beta = \dfrac{AB}{BQ} = \dfrac{x}{y}$

$\therefore y = x\cot\beta$ ....(ii)

From equation (i) and (ii),

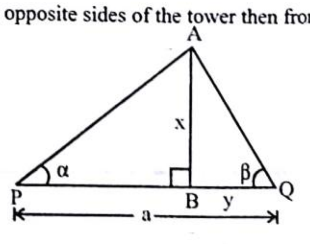
$\therefore a = x\cot\alpha - x\cot\beta$.

$$\Rightarrow x = \frac{a}{\cot\alpha - \cot\beta}$$

Also $y = x\cot\alpha - a$

$$\Rightarrow y = \frac{a\cot\alpha}{\cot\alpha - \cot\beta} - a \Rightarrow y = \frac{a\cot\alpha - a(\cot\alpha - \cot\beta)}{\cot\alpha - \cot\beta} \Rightarrow y = \frac{a\cot\beta}{\cot\alpha - \cot\beta}$$

In the above case, P and Q are on the same side of the tower. If the two points are on the opposite sides of the tower then from the adjoining figure, we get



$\tan\alpha = \dfrac{x}{PB}$ *or* $PB = x\cot\alpha$ and $\tan\beta = \dfrac{x}{BQ}$ *or* $BQ = x\cot\beta$.

$\therefore a = PB + BQ = x(\cot\alpha + \cot\beta)$

$$\therefore x = \frac{a}{\cot\alpha + \cot\beta}$$

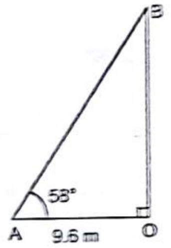
and $y = BQ = x\cot\beta$

NOTE : Here, all the lines AP, AQ, AB are in the same plane.

MISCELLANEOUS SOLVED EXAMPLES

1. The angle of elevation of a ladder leaning against a wall is 58°, and the foot of the ladder is 9.6 m from the wall. Find the length of the ladder.

Sol. Let AB be the ladder leaning against a wall OB such that $\angle OAB = 58°$ and $OA = 9.6$ m



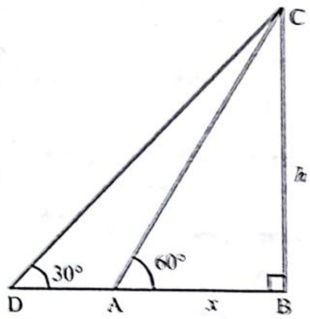
In ΔAOB, we have, $\cos 58° = \dfrac{OA}{AB}$

$\Rightarrow \quad AB = \dfrac{OA}{\cos 58°}$

$\Rightarrow AB = \dfrac{9.6}{0.5299} = 18.11$ m

2. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°; when he retreates 20m from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river.

Sol. Let AB be the width of the river and BC be the tree which makes an angle of 60° at a point A on the opposite bank. Let D be the position of the person after retreating 20 m from the bank.
Let AB = x metres and BC = h metres.
From right angled triangles ABC and DBC,



we have $\tan 60° = \dfrac{BC}{AB}$ and $\tan 30° \Rightarrow \sqrt{3} = \dfrac{h}{x}$

and $\dfrac{1}{\sqrt{3}} = \dfrac{h}{x+20} \Rightarrow h = x\sqrt{3}$

and $h = \dfrac{x+20}{\sqrt{3}} \quad \Rightarrow \quad x\sqrt{3} = \dfrac{x+20}{\sqrt{3}}$

$\Rightarrow \quad 3x = x + 20 \Rightarrow x = 10$ m

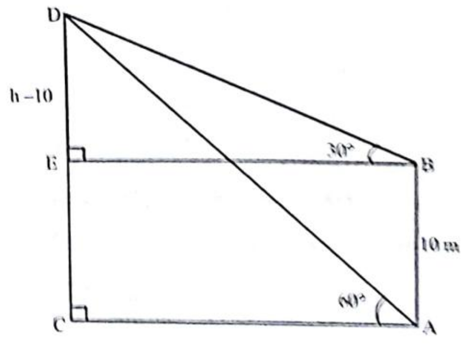
Putting $x = 10$ in $h = \sqrt{3}\,x$, we get

$h = 10\sqrt{3} = 17.32$ m

Hence, height of the tree = 17.32 m and the breadth of the river = 10 m.

3. The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° and 60° respectively. Find the height of the tower.

Sol. Let AB and CD be the pole and tower respectively.
Let CD = h
Then $\angle DAC = 60°$ and $\angle DBE = 30°$



Now $\dfrac{CD}{CA} = \tan 60° = \sqrt{3} \therefore CD = \sqrt{3}\,CA \qquad \Rightarrow \dfrac{h}{\sqrt{3}} = CA$

Again $\dfrac{DE}{BE} = \tan 30° = \dfrac{1}{\sqrt{3}}$

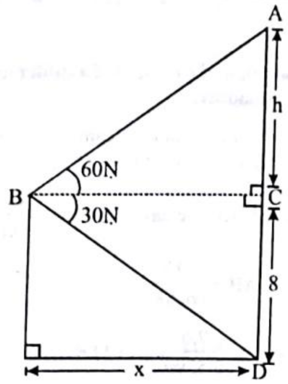
$\therefore (h-10) = \dfrac{BE}{\sqrt{3}} = \dfrac{CA}{\sqrt{3}} = \dfrac{h/\sqrt{3}}{\sqrt{3}} = \dfrac{h}{3} \quad [\because BE = CA]$

$\Rightarrow 3h - 30 = h \Rightarrow 2h = 30 \qquad \Rightarrow h = 15$

Hence, height of the tower = 15 m

4. A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Sol. Let x be the distance of hill from man and $h + 8$ be height of hill which is required.



In rt. ΔACB,

$$\tan 60° = \frac{AC}{BC} = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$$

In rt. ΔBCD,

$$\tan 30° = \frac{CD}{BC} = \frac{8}{x}$$

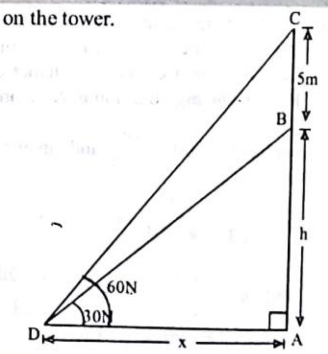
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{x} \Rightarrow x = 8\sqrt{3}$$

$\therefore$ Height of hill $= h + 8 = \sqrt{3}x + 8 = (\sqrt{3})(8\sqrt{3}) + 8 = 32\ m$

Distance of ship from hill $= x = 8\sqrt{3}\ m$

5. A vertical to stands on a horizontal plane and is surmounted by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.

Sol. Let AB be the tower of height h meter and BC be the height of flag staff surmounted on the tower.
Let the point of the plane be D at a distance m meter from the foot of the tower.



In ΔABD,

$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \quad \text{............ (1)}$$

In Δ ADC,

$$\tan 60° = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{5+h}{x} \Rightarrow x = \frac{5+h}{\sqrt{3}} \quad \text{............ (2)}$$

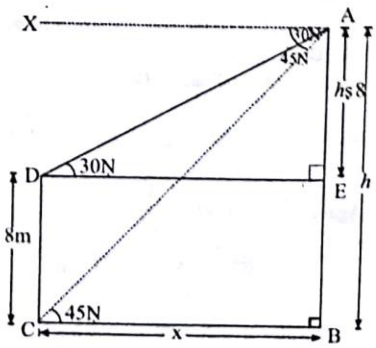
From (1) and (2), $\sqrt{3}h = \frac{5+h}{\sqrt{3}} \Rightarrow 3h = 5 + h \Rightarrow 2h = 5 \Rightarrow h = \frac{5}{2} = 2.5\text{ m}$

So, the height of tower = 2.5 m

6. The angles of depressions of the top and bottom of 8m tall building fron the top of a multistoried building are 30° and 45° respectively. Find the height of multistoried building and the distance between the two buildings.

Sol. Let AB be the multistoried building of height h and let the distance between two buildings be x meters.

$\angle XAC = \angle ACB = 45N$ (Alternate angles)
$\angle XAD = \angle ADE = 30N$ (Alternate angles)



In ΔADE, $\tan 30° = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$ $[\because CB = DE = x]$

$\Rightarrow x = \sqrt{3}(h-8)$ .... (1)

In ΔACB,

$$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow = h \quad \text{.......... (2)}$$

From (1) and (2),

$$\sqrt{3}(h-8) = h \Rightarrow \sqrt{3}h - 8\sqrt{3} = h$$

$$\Rightarrow \sqrt{3}h - h = 8\sqrt{3} \Rightarrow h(\sqrt{3}-1) = 8\sqrt{3}$$

$$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1} \Rightarrow h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$$

$\Rightarrow \quad h = 4\sqrt{3}(\sqrt{3}+1) \quad \Rightarrow \quad h = 4(3+\sqrt{3})$ metres

From (2), $x = h$

So, $x = 4(3+\sqrt{3})$ metres

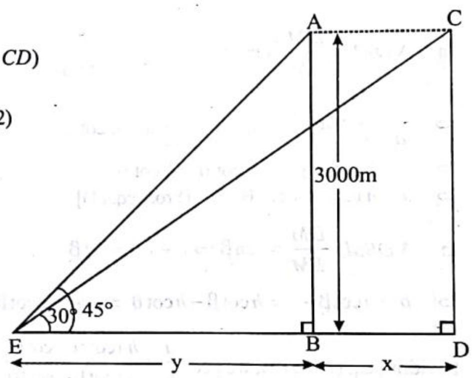
Hence, height of mulistoried building = $4\,(3+\sqrt{3})$ metres

distance betwen two building = $4\,(3+\sqrt{3})$ metres.

7. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 sec, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Sol. Let the point on the ground is E which is y metres from point B and let after 15 sec. flight it covers x metres distance

In ΔAEB, $\tan 45° = \dfrac{AB}{EB} \Rightarrow 1 = \dfrac{3000}{y} \Rightarrow y = 3000\text{m}$ .... (1)



In ΔCED, $\tan 30° = \dfrac{CD}{ED} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{3000}{x+y}$ $(\because AB = CD)$

$\Rightarrow x + y = 3000\sqrt{3}$ .... (2)

From eq. (1) and (2)

$x + 3000 = 3000\sqrt{3} \quad \Rightarrow \quad x = 3000\sqrt{3} - 3000$

$\Rightarrow x = 3000\,(\sqrt{3}-1) \quad \Rightarrow \quad x = 3000 \times (1.732 - 1)$

$\Rightarrow x = 3000 \times 0.732 \quad \Rightarrow \quad x = 2196\text{m}$

Speed of aeroplane $= \dfrac{\text{Distance covered}}{\text{Time taken}}$

$= \dfrac{2196}{15}\text{m/sec} = 146.4\text{ m/sec} = \dfrac{2196}{15} \times \dfrac{18}{5}\text{km/hr} = 527.04\text{ km/hr}$

Hence, the speed of aeroplane is 527.04 km/hr.

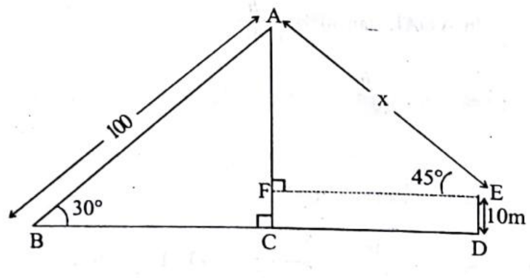
8. A boy is standing on the ground and flying a kite with 100m of string at an elevation of 30°. Another boy is standing on the roof of a 10m high building and is flying his at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Sol. Let the length of second string be x m.

In ΔABC,

$\sin 30° = \dfrac{AC}{AB}$ or $\dfrac{1}{2} = \dfrac{AC}{100} \Rightarrow AC = 50\text{m}$

In ΔAEF,



$\sin 45° = \dfrac{AF}{AE} \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{AF - FC}{x}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{50-10}{x}$ $[\because AC = 50\text{m}, FC = ED = 10\text{m}]$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{40}{x} \quad \Rightarrow \quad x = 40\sqrt{2}\text{m}$

So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$m

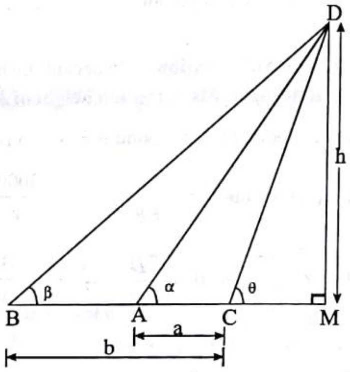
9. Two stations due south of a leaning tower, which leans towards the north, are at distances a and b from its foot. If α and β are the angles of elevations of the top of the tower from these stations prove that its inclination θ to the horizontal is given by $\cot\theta = \dfrac{b\cot\alpha - a\cot\beta}{b-a}$, if a < b

Sol. Let CD, A and B represent the leaning tower and the two given stations.

$AC = a,\ BC = b$

$\angle DCM = \theta,\ \angle DAM = \alpha,\ \angle DBM = \beta$

Let $DM = h$ and $CM = x$.



In ΔDCM, $\dfrac{DM}{CM} = \tan\theta$

$\Rightarrow \dfrac{h}{x} = \tan\theta \Rightarrow x = h\cot\theta$ ....(1)

In ΔDAM, $\dfrac{DM}{AM} = \tan\alpha$

$\Rightarrow \dfrac{h}{a+x} = \tan\alpha \Rightarrow a + x = h\cot\alpha.$

$\Rightarrow a = h\cot\alpha - x = h\cot\alpha - h\cot\theta$ ....(2)

$\Rightarrow a = h(\cot\alpha - \cot\theta)$ [From eq. (1)]

In ΔDBM, $\dfrac{DM}{BM} = \tan\beta \Rightarrow b + x = h\cot\beta$

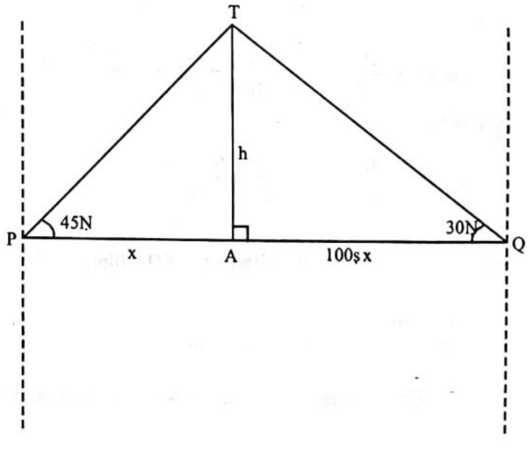
$\Rightarrow b = h\cot\beta - x = h\cot\beta - h\cot\theta \Rightarrow b = h(\cot\beta - \cot\theta)$ [From eq. (1)] ....(3)

Dividing eq. (2) by (3), we get $\dfrac{a}{b} = \dfrac{h(\cot\alpha - \cot\theta)}{h(\cot\beta - \cot\theta)}$

$\Rightarrow a\cot\beta - a\cot\theta = b\cot\alpha - b\cot\theta \Rightarrow (b-a)\cot\theta = b\cot\alpha - a\cot\beta \Rightarrow \cot\theta = \dfrac{b\cot\alpha - a\cot\beta}{b-a}$

10. There is a small island in the middle of a 100m wide river and a tall tree stands on the island. P and Q are points directly each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 30° and 45°, find the height of the tree.

Sol. Let the height of the tree be h i.e., AT = h



In Δ PAT, $\tan 45^\circ = \dfrac{h}{x} \Rightarrow 1 = \dfrac{h}{x} \Rightarrow h = x$

In Δ QAT, $\tan 30^\circ = \dfrac{h}{100-x}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{100-x}$ $[\because x = h]$

$\Rightarrow \sqrt{3}h = 100 - h$

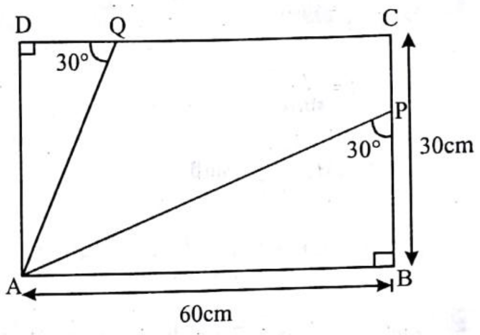
$\Rightarrow (\sqrt{3}+1)\,h = 100$

$\Rightarrow h = \dfrac{100}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = 50(\sqrt{3}-1) = 36.6$ m

$\Rightarrow$ The height of the tree is 36.6 m

11. In figure, ABCD is a rectangle in which segments AP and AQ are drawn as shown. Find the length of (AP + AQ).

Sol. In rt. ΔADQ,



$$\frac{AD}{AQ} = \sin 30°$$

$$\Rightarrow \frac{AD}{AQ} = \frac{1}{2}$$

$$\Rightarrow AQ = 2AD$$

$$\Rightarrow AQ = 2 \times 30 = 60 \text{ cm.} \quad \text{.......... (1)}$$

In rt. ΔABP, $\frac{AB}{AP} = \cos 60° \Rightarrow \frac{AB}{AP} = \frac{1}{2}$

$$\Rightarrow AP = 2AB \Rightarrow AP = 2 \times 60 = 120 \text{ cm.} \quad \text{.......... (2)}$$

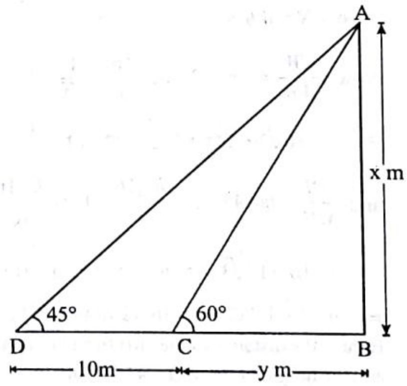
$$\therefore AP + AQ = 120 + 60 = 180 \text{ cm.}$$

12. The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 metres longer than when it is 60°. Find the height of the tower.

Sol. Let length of tower $AB = x\,m$

Angle of elevation at point $C = 60°$

Shadow of tower, $BC = ym$, $CD = 10$m



Now in rt. ΔABC, $\frac{AB}{AC} = \tan 60° \Rightarrow \frac{x}{y} = \sqrt{3}$

$$\Rightarrow \sqrt{3}y = x \Rightarrow y = \frac{x}{\sqrt{3}} \quad \text{.......... (1)}$$

In rt. ΔABD, $\frac{AB}{BD} = \tan 45°$

$$\Rightarrow \frac{x}{10+y} = 1 \Rightarrow x = 10 + y \quad \text{.......... (2)}$$

Putting the value of y in (2), we get

$$x = 10 + \frac{x}{\sqrt{3}} \Rightarrow \sqrt{3}x = 10\sqrt{3} + x$$

$$\Rightarrow \sqrt{3}x - x = 10\sqrt{3} \Rightarrow x(\sqrt{3}-1) = 10\sqrt{3} \Rightarrow x = \frac{10\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow x = \frac{10\sqrt{3}(\sqrt{3}+1)}{2}$$

$$\Rightarrow x = 5\sqrt{3}(\sqrt{3}+1) \Rightarrow x = 15 + 5\sqrt{3}$$

$$\Rightarrow x = 15 + 5(1.73) \Rightarrow x = 15 + 8.65 = 23.65\text{m}$$

13. A round balloon of radius r subtends an angle 2α at th eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon vertically above the horizontal level of eye is $\frac{r \sin B}{\sin \alpha}$.

Sol. O is the centre of the balloon. r is its radius. A is the position of the eye.

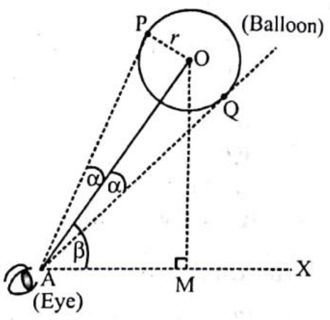
$$\angle PAQ = 2\alpha$$

$$\therefore \angle PAO = \alpha$$

$\frac{OP}{OA} = \sin\alpha$

$\Rightarrow \frac{r}{OA} = \sin\alpha$

$\Rightarrow OA = \frac{r}{\sin\alpha}$



In ΔOAM, $\frac{OM}{OA} = \sin\beta$

$\Rightarrow OM = OA \times \sin\beta = \frac{r}{\sin\alpha} \times \sin\beta \Rightarrow OM = \frac{r\sin\beta}{\sin\alpha}$

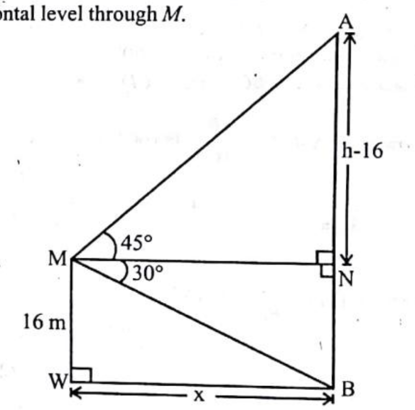
14. A man on the deck of a ship is 16m above water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Sol. Let the man be at M, 16m above water level WB. $AB = h$ m is the cliff.
Let $WB = x$ m be the distance of the ship from the cliff. MN is the horizontal level through M.

$\angle AMN = 45°$ and $\angle NMB = 30°$

$\therefore \angle MBW = 30°$

Also $MN = WB = xm$.



Now, $\frac{MW}{WB} = \tan 30° \Rightarrow \frac{16}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = 16\sqrt{3} = 16 \times 1.732 = 27.712$

and $\frac{AN}{MN} = \tan 45° \Rightarrow \frac{h-16}{x} = 1 \Rightarrow \frac{h-16}{16\sqrt{3}} = 1$

$\Rightarrow h - 16 = 16\sqrt{3} \Rightarrow h = 16\sqrt{3} + 16 = 16\left(\sqrt{3}+1\right)$

$\Rightarrow h = 16(1.732+1) = 16 \times 2.732 = 43.712$

Hence, the distance of the cliff from the ship = 27.712 m
and the height of the cliff = 43.712 m.

15. A person observed the angle of elevation of the top of a tower as 30°. He walked 50m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Sol. Suppose height of the tower $AB = x$ m and distance $BC = y$ m

In rt. ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{x}{y} = \sqrt{3} \Rightarrow y = \frac{x}{\sqrt{3}}$ .... (1)

In rt. ΔABD, $\frac{AB}{DB} = \tan 30° \Rightarrow \frac{x}{50+y} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}x = 50 + y$ .... (2)

Putting the value of y in (2), we get, $\sqrt{3}x = 50 + \frac{x}{\sqrt{3}}$

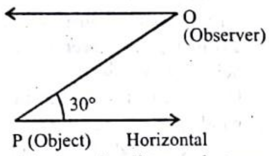
$\Rightarrow \sqrt{3}x \times \sqrt{3} = 50\sqrt{3} + x \Rightarrow 3x = 50\sqrt{3} + x$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The .... is the line drawn from the eye of an observer to the point in the object viewed by the observer.
2. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.
3. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.
4. In the adjoining figure, the positions of observer and object are marked. The angle of depression is ....



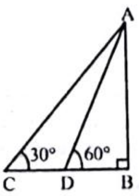
5. The height or length of an object or the distance between two distant objects can be determined with the help of ....
6. The angles of elevation and depression are............
7. The top of a building from a fixed point is observed at an angle of elevation 60° and the distance from the foot of the building to the point is 100 m. then the height of the building is...............

True / False :

DIRECTIONS : *Read the following given information and write your answer as true or false for the statements which are based on this information.*

A straight highway leads to the foot of a tower of height 50m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60°.

Now, based on the above passage, mark the given statements as true or false.

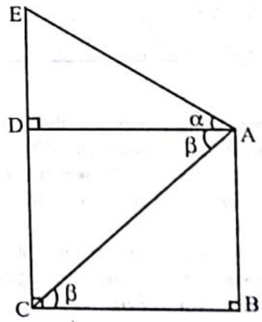


1. Distance between the cars is 57.6m.
2. First car is at a distance of 38.90 m from the tower.
3. Second car is at a distance of 86.50 m. from the tower.
4. Car at point C is at a distance of 200 m away from the top of the tower.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. From a window, h metres high above the ground, of a hous in a street, the angles of elevation and depression of the top and bottom of another house on the opposite side of the street are α and β. respectively, then match the column.



Column I	Column II
(A) *DB*	(p) $h(1+\tan\alpha\cot\beta)$
(B) *DE*	(q) $h\sin\beta$
(C) *CE*	(r) $h\tan\alpha\cot\beta$
(D) *AD*	(s) $h\cot\beta$

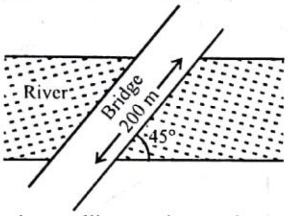
2. A ladder (length = ℓ) rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance d, so that it slides a distance h down the wall, making an angle β with the horizonta, then match the column.

Column I	Column II
(A) d	(p) $\ell(\sin\alpha-\sin\beta)$
(B) h	(q) $\ell(\cos\beta-\cos\alpha)$
(C) d/h	(r) $\dfrac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$
(D) dh/ℓ^2	(s) $\sin(\alpha+\beta)-\cos(\alpha-\beta)$

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. The string of a kite is 250m long it makes an angle of 60° with the horizontal. Find the height of the kite, assuming that there is no slackness in the string.
2. Find the angle of elevation of the sun (sun's altitude) when the length shadow of a vertical pole is equal to its height.
3. The horizontal distance between two towers is 140m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60m, find the height of the first tower.
4. The angle of elevation of the top of a tower, from a point on the ground and at a distance of 30 m from its foot, is 30°. Find the height of the tower.
5. A bridge across a river makes an angle of 45° with the river bank. If the length of the bridge across the river is 200 metres, what is the breadth of the river?

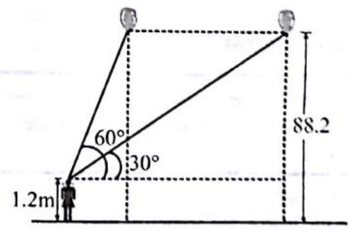


6. A, B, C are three collinear points on the ground such that B is between A and C and distance $AB = 10$ m. If the angles of elevation of the top of a vertical tower at D are, respectively 30° and 60° as seen from A and B, then find the height of the tower (in metres).
7. A vertical tree 30 m long breaks at a height of 10 m. Its two parts and the ground form a triangle. Find the angle between the ground and the broken part.
8. An electric pole is $10\sqrt{3}$ m high. If its shadow is $10\sqrt{3}$ m in length, find the angle of elevation of the sun.
9. A kite is flying at a height of 60 metres from the level ground, attached to a string inclined at 60° to the horizontal. Find the length of the string.
10. A tower stands vertically on the ground. From a point on the ground 30 metres away from the foot of the tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Two boats are are sailing in the sea on either side of a lighthouse. At a particular time the angles of depression of the two boats, as observed from the top of the lighthouse are 45° and 30° respectively. If the lighthouse is 100m high, find the distance between the two boats.
2. A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observation points 100m apart from each other on the side to his left. Find, in metres, the aproximate height from which he falls and also find, in metres, the aproximate distance of the point where he falls on the ground from the first observation point.
3. A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill is 60° and the angle of depression of the base of the hill is 30°. Calculate the distance of the hill from the ship and the height of the hill.
4. If a flag-staff of 6 metres height placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground then find the angle that the sun makes with the ground.
5. In figure, ABCD is a rectangle in which segments AP and AQ are drawn as shown. Find the length of $(AP + AQ)$.
6. The length of the shadows of a vertical pole of height h thrown by the sun's rays at three different moments are h, 2h, 3h. Find the sum of angles of elevation of the rays at these three points.
7. An observer finds that the angular elevation of a tower is A, on advancing 3 m towards, the elevation is 45° and on advancing 2m nearer, the elevation is 90°–A, then find the height of the tower.
8. Upper part of a vertical tree which is broken over by the winds just touches the ground and makes an angle of 30° with the ground. If the length of the broken part is 20 metres, then find the length of remaining part of the tree.
9. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



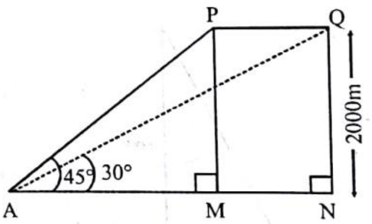
10. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. Find the height of the tower and distance of the tower from the point A.
11. From a window (60 metres high above ground) of a house in street the angles elevation and depression of the top and the foot of another house on opposite side of street 60° and 45° respectively. Show that the height of the opposite house is $60(1+\sqrt{3})$ metres.

12. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill.
13. An observer 1.5m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from his eye is 45°. What is the height of the chimney?
14. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
15. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the bank, find the width of the river.
16. Two pillars of equal height stand on either side of a roadway which is 80 m wide. At a point on the road between pillars, the elevations of the pillars are 60° and 30°. Find the height of the pillars and the position of the point.
17. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. *A* fire at a building *B* is reported by a telephone to two fire-stations. F_1 and F_2, 10 km apart from each other on a straight road. F_1 and F_2 observe that the fire is at an angle of 60° and 45° respectively to the road. Which station should send its team and how much will it have to travel ?
2. There are two churches, one on the each bank of a river, just opposite to each other. One church is 30 m high. From the top of this church, the angles of depression of top and foot of the other church are 30° and 60° respectively. Find the width of the river and height of the other church.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°. Find the height of the tower PQ and the distance XQ.
5. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the tower? Give your answer to the nearest second.
6. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud.
7. The angle of elevation of a cliff from a fixed point A is θ. After going up a distance K metres towards the top of the cliff an angle of ϕ, it is found that the angle of elevation is α. Show that the height of the cliff, in metres, is $\dfrac{K(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$.
8. In the adjoining figure, the angle of elevation of a helicopter from a point A on the ground is 45°. After 15 seconds flight, the angle of elevation changes to 30°. If the helicopter is flying at a height of 2000 m, find the speed of the helicopter.



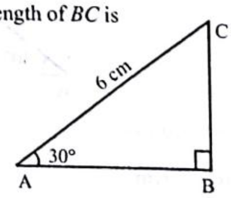
(Take $\sqrt{3} = 1.732$)
9. The height of a house subtends a right angle at the opposite window. The angle of elevation of the window from the base of the house is 60°. If the width of the road is 6 m, find the height of the house.
10. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the mult-storeyed building and the distance between the two buildings.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. In the adjoining figure, the length of BC is
(a) $2\sqrt{3}$ cm
(b) $3\sqrt{3}$ cm
(c) $4\sqrt{3}$ cm
(d) 3 cm



2. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(a) $25\sqrt{3}\,m$ (b) $50\sqrt{3}\,m$

(c) $75\sqrt{3}\,m$ (d) $150\,m$

3. The angle of elevation of the top of a tower at point on the ground is 30°. If on walking 20 metres toward the tower, the angle of elevation become 60°, then the height of the tower is

(a) 10 metre (b) $\frac{10}{\sqrt{3}}$ metre

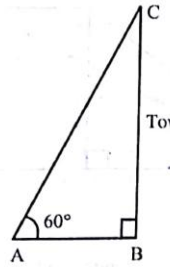
(c) $10\sqrt{3}$ metre (d) None of these

4. A vertical pole consists of two parts, the lower part being one third of the whole. At a point in the horizontal plane through the base of the pole and distance 20 meters from it, the upper part of the pole subtends an angle whose tangent is $\frac{1}{2}$. The possible heights of the pole are

(a) 20 m and $20\sqrt{3}\,m$ (b) 20 m and 60 m

(c) 16 m and 48 m (d) None of these

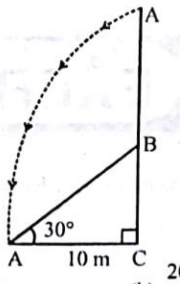
5. In the adjoining figure, if the angle of elevation is 60° and the distance $AB = 10\sqrt{3}\,m$, then the height of the tower is



(a) $20\sqrt{3}$ cm (b) 10 m

(c) 30 m (d) $30\sqrt{3}\,m$

6. The top of a broken tree has its top touching the ground (shown in the adjoining figure) at a distance of 10 m from the bottom. If the angle made by the broken part with ground is 30°, then the length of the broken part is



(a) $10\sqrt{3}$ cm (b) $\frac{20}{\sqrt{3}}\,m$

(c) 20 cm (d) $20\sqrt{3}\,m$

7. An aeroplane flying horizontally 1 km. above the ground is observed at an elevation of 60° and after 10 seconds the elevation is observed to be 30°. The uniform speed of the aeroplane in km/h is

(a) 240 (b) $240\sqrt{3}$

(c) $60\sqrt{3}$ (d) None of these

8. A 25 m ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4m, then the foot of the ladder will slide

(a) 5 m (b) 8 m

(c) 9 m (d) 15 m

9. If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is

(a) 15° (b) 30°

(c) 45° (d) 60°

10. The angles of elevation of the top of a tower from two points at distances m and n metres are complementary. If the two points and the base of the tower are on the same straight line, then the height of the tower is

(a) $\sqrt{mn}$ (b) mn

(c) $\frac{m}{n}$ (d) None of these

11. The Qutab Minar casts a shadow 150 m long at the same time when the Vikas Minar casts a shadow of 120m long on the ground. If the height of the Vikas Minar is 80m, find the height of the Qutab Minar.

(a) 180 m (b) 100 m

(c) 150 m (d) 120 m

12. From the bottom of a pole of height h, the angle of elevation of the top of a tower is α. The pole subtends an angle β at the top of a tower. The height of the tower is

(a) $\frac{h\sin\alpha\cos(\alpha+\beta)}{\cos\beta}$ (b) $\frac{h\sin\alpha\cos(\alpha-\beta)}{\sin\beta}$

(c) $\frac{h\sin\alpha\sin(\alpha+\beta)}{\cos\beta}$ (d) $\frac{h\sin\alpha\sin(\alpha-\beta)}{\sin\beta}$

13. An aeroplane at a height of 600 m passes vertically above another aeroplane at an instant when their angles of elevation at the same observing point are 60° and 45° respectively. How many metres higher is the one from the other ?

(a) 286.53 m (b) 274.53 m

(c) 253.58 m (d) 263.83 m

MTOC More than One Correct :

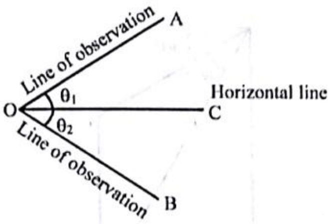
DIRECTIONS : *This section contains 3 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. A tree breaks due to storm and the broken part bends to that the top of the tree first touches the ground, making an angle of 30° with the horizontal. The distance from the foot of the tree to the point where the top touches the ground is 10 m. The height of the tree is

(a) $10(\sqrt{3}+1)\,m$ (b) $10\sqrt{3}\,m$

(c) $10(\sqrt{3}-1)\,m$ (d) $\frac{30}{\sqrt{3}}\,m$

2. Which of the following is/are correct?



(a) θ_1 is the angle of elevation.
(b) θ_2 is the angle of depression.
(c) The angle of elevation or depression is always measured from horizontal line through the point of observation.
(d) θ_1 and θ_2 are always equal.

3. I. The angle of elevation of the top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, then height of the hill is 150 m.

II. An aeroplane flying horizontally 1 km above the ground is observed at an angle of 60°. After 10 seconds, its elevation changes to 30°. Then the speed of the aeroplane is 527.04 km/h.

III. A man in a boat rowing away from light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Then the speed of the boat is 40 m/minute

Which is true?

(a) I (b) II
(c) III (d) None of these

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

From the top of a tower, the angles of depression of two objects on the same side of the tower are found to be α and β where $\alpha > \beta$.

1. If the distance between the objects is 'p' metres, then the height 'h' of the tower is

(a) $\frac{p\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$ (b) $\frac{\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

(c) $\frac{p(\tan\alpha-\tan\beta)}{\tan\alpha\tan\beta}$ (d) none of these

2. The height of the tower if $p = 50$m, $\alpha = 60°$ and $\beta = 30°$, is

(a) 120 m (b) 130 m
(c) 140 m (d) none of these

3. The distance of the extreme object from the top of the tower is

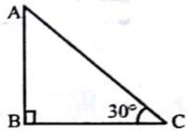
(a) 65 m (b) 130 m
(c) 260 m (d) none of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason are correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but **Reason is not the correct explanation** of Assertion.
(c) If **Assertion is correct** but **Reason is incorrect.**
(d) If **Assertion is incorrect** but **Reason is correct.**

1. Assertion :
If the above figure, if $BC = 20$ m, then height AB is 11.56 m.



Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$

where θ is the angle $\angle ACB$.

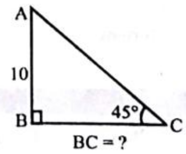
2. Assertion : If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.

Reason : According to pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base

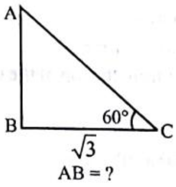
MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

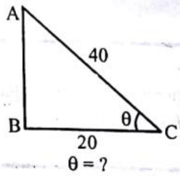
1. **Column-I** **Column-II**

(A) 

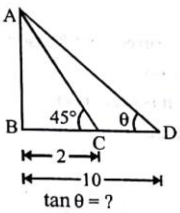
(p) 60°

(B) 

(q) 10

(C) 

(r) $\frac{1}{5}$

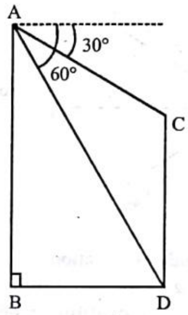
(D) 

(s) 3

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. An aircraft is flying along a horizontal line AB directly towards an observer P on the ground and maintaining an altitude of $5000m$. The angles of depression at A and B are 30° and 60° respectively. Find AB.
2. In the adjoining figure, from the top of a building AB, 60 metres high, the angles of depression of the top and the bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find
   (i) the horizontal distance between AB and CD.
   (ii) the height of the lamp post CD.



3. The angle of elevation of a cloud from a point 200 m above the lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud.
4. (i) The angle of elevation of a bird from a point 50 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the bird above the lake.
   (ii) The angle of elevation of a cloud from a point 200 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.
5. The angle of elevation of a cliff from a fixed point A is 45°. After going up at a distance of 60 metres towards the top of the cliff at an inclination of 30°, it is found that the angle of elevation is 60°. Find the height of the cliff.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. line of sight
2. angle of elevation
3. angle of depression
4. 30°
5. trigonometric ratios.
6. Alternate
7. $100\sqrt{3}$

TRUE / FALSE

1. True 2. False 3. True 4. False

MATCH THE FOLLOWING :

1. (A) → s; (B) → r; (C) → p; (D) → q
2. (A) → q; (B) → p; (C) → r; (D) → s

VERY SHORT ANSWER QUESTIONS :

1. Let K be the position of the kit at a height h above the ground OA.
   The length of the string = OK = 250m such that

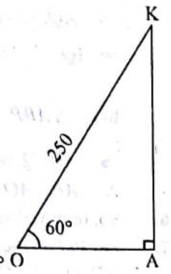
$\angle KOA = 60°$

In ΔKOA, we have

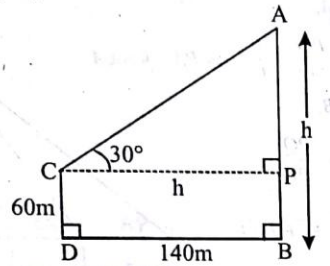
$\frac{KA}{KO} = \sin 60°$

$\Rightarrow KA = KO \sin 60°$

$\Rightarrow KA = 250 \times \frac{\sqrt{3}}{2} = 125\sqrt{3}\ m$



2. The angle of elevation of the sun = 45°
3. Let the height of the first tower be h (figure).



i.e., $AB = h$ and $AP = h - 60$

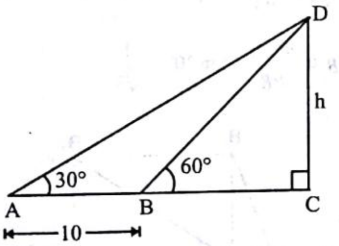
In ΔACP, $\tan 30° = \frac{AP}{CP} \Rightarrow h = 60 + \frac{140}{\sqrt{3}} = 140.83\ m$

4. $10\sqrt{3}\ m$
5. $100\sqrt{2}\ m$

Hint. Let the breadth of the river be b metres, then

$\Rightarrow \sin 45° = \frac{b}{200\ m}$

6. Let, A, B, C be three collinear points. Let CD be the vertical tower, with C as foot and D as top. From description of problem, $AB = 10$, $\angle DAC = 30°$ & $\angle DBC = 60°$



Let the tower height be h, i.e. $CD = h$. In ΔADC,

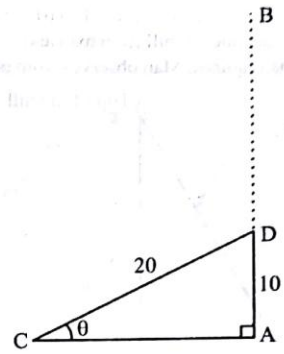
$\tan 30° = \frac{h}{AC}$ and in ΔBDC, $\tan 60° = \frac{h}{BC}$

$\Rightarrow h = \frac{10\sqrt{3}}{2} \Rightarrow h = 5\sqrt{3}m$.

Therefore, height of the tower $= 5\sqrt{3}m$.

7. Let the vertical tree $AB = 30\ m$, be broken at D.

So, $AD + DB = 30. \Rightarrow AD + DC = 30, AD = 10,$ hence $DC = 20\ m$. If $\angle ACD = \theta$, then



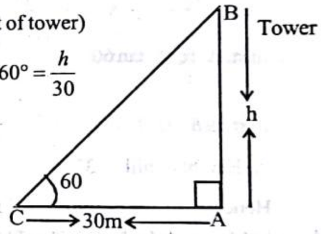
$\sin\theta = \frac{AD}{DC} = \frac{10}{20} = \frac{1}{2} \Rightarrow \theta = 30°$

Hence required angle = 30°

8. 60°
9. $40\sqrt{3}\ m$
10. Let $AB = h\ m$ (i.e height of tower)

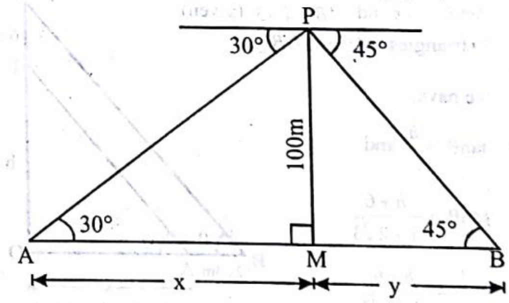
Now in rt ΔABC, $\tan 60° = \frac{h}{30}$

$\Rightarrow h = 30\sqrt{3}m$



SHORT ANSWER QUESTIONS :

1. Let A and B be the position of two boats (figure)



Let PM be the lighthouse such that $PM = 100m$.

Let $AM = x$ and $BM = y$

In ΔAPM, $\frac{AM}{PM} = \cot 30° \Rightarrow \frac{x}{100} = \sqrt{3}$

$\Rightarrow x = 100\sqrt{3}\ m$

Similarly in ΔBPM,

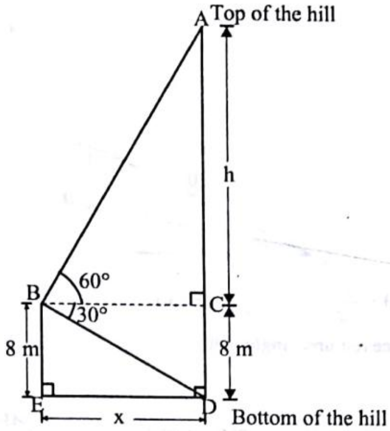
$\frac{BM}{PM} = \cot 45° \Rightarrow \frac{y}{100} = 1 \Rightarrow y = 100\ m$

$\therefore$ Required distance,

$AB = x + y = 100\sqrt{3} + 100 = 100(\sqrt{3} + 1)\ m = 273.2\ m$

2. The required height $= 1.732 \times 136.61 = 236.60$ m
The required distance $AC = 100 + 136.61 = 236.61$ (approx.)

3. Let x be the distance of hill from man and $h + 8$ be height of hill which is required. Man observes from point B.



In rt. ΔACB, $\tan 60° = \frac{AC}{BC} = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$

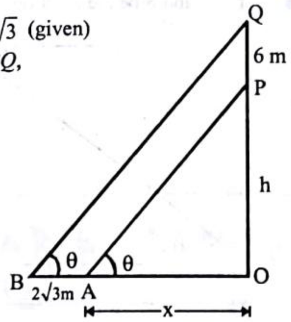
In rt. ΔBCD, $\tan 30° = \frac{CD}{BC} = \frac{8}{x}$

$\therefore$ Height of hill $= 32\,m$

Hence, distance of ship from hill $= x = 8\sqrt{3}\,m$

4. Let OA and AB be the shadows of tower OP and flag-staff PQ respectively on the ground. Suppose the sun makes an angle θ with the ground.

Let $OA = x$ and $AB = 2\sqrt{3}$ (given)
In triangles OAP and OBQ,



we have,

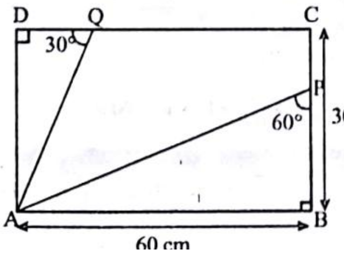
$\tan\theta = \frac{h}{x}$ and

$\tan\theta = \frac{h+6}{x+2\sqrt{3}}$

$\therefore \frac{h}{x} = \frac{h+6}{x+2\sqrt{3}}$

Hence, the angle that the sun makes with ground = 60°

5. In rt. ΔADQ, $\frac{AD}{AQ} = \sin 30° \Rightarrow \frac{AD}{AQ} = \frac{1}{2}$



$\Rightarrow AQ = 2AD$

$\Rightarrow AQ = 2 \times 30 = 60$ cm. $(\because AD = BC = 30)$ ...(1)

In $rt.\ \Delta ABP$, $\frac{AB}{AP} = \cos 60° \Rightarrow \frac{AB}{AP} = \frac{1}{2}$

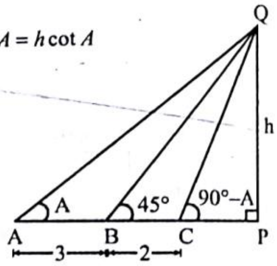
$\Rightarrow AP = 2AB \Rightarrow AP = 2 \times 60 = 120$ cm. ...(2)

$\therefore AP + AQ = 120 + 60 = 180$ cm.

6. So, required sum $= 45° + 45° = 90°$

7. Let PQ be the tower of height h.
$\angle PAQ = A$, $\angle PBQ = 45°$ and $\angle PCQ = 90° - A$.
Again $AB = 3$ and $BC = 2$. (see figure)
In ΔPQA,

$\tan A = \frac{PQ}{PA} = \frac{h}{PA} \Rightarrow PA = h\cot A$



In ΔPQB,

$\tan 45° = \frac{PQ}{PB}$

$1 = \frac{h}{PB} \Rightarrow PB = h$

$\frac{h-2}{h} = \frac{h}{h+3} \Rightarrow h = 6m.$

Therefor, height of the tower $= 6m$

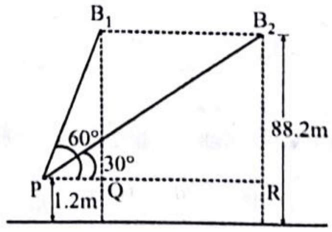
8. So length of the remaining part of the tree $= h - 20 = 10\,m$.

9. Height of the balloon from the ground $= 88.2m$. Height from the eye level of girl of $1.2\,m$ height $= 88.2 - 1.2 = 87.00\,m$
So, $B_1Q = B_2R = 87\,m$.

In ΔPB_1Q, $\frac{B_1Q}{PQ} = \tan 60° = \sqrt{3}$

$PQ = \frac{B_1Q}{\sqrt{3}} = \frac{87}{\sqrt{3}} = \frac{87\sqrt{3}}{3} = 29\sqrt{3}$

In ΔPB_2R, $\frac{B_2R}{PR} = \tan 30° = \frac{1}{\sqrt{3}}$



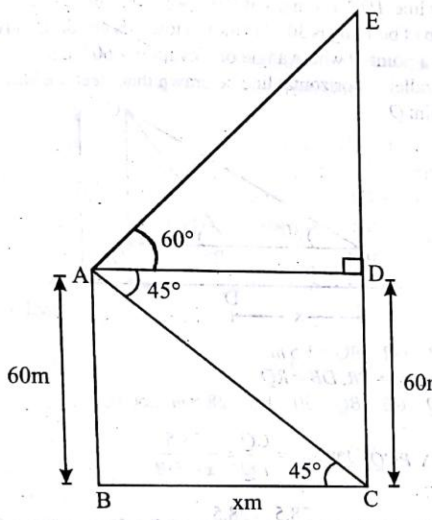
$\Rightarrow PR = \sqrt{3}\,B_2R = \sqrt{3} \times 87 = 87\sqrt{3}$

Distance travelled by the balloon $= QR = PR - PQ$

$= 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}\,m.$

10. Height of tower is $17.32\,m$

11. Let A be the window and CE be the opposite house



$CD = AB = 60m$ [opposite sides of rectangle] ....(1)

In $rt.\ \Delta$, $\tan 45° = \dfrac{AB}{BC} \Rightarrow 1 = \dfrac{60}{BC}$

$\Rightarrow\ BC = 60m$ ....(2)

$AD = BC$ [opposite sides of rectangle]

$\therefore\ AD = 60\,m$ [From (ii)] ....(3)

In $rt.\ \Delta ADE$, $\tan 60° = \dfrac{DE}{AD}$

$\Rightarrow\ \sqrt{3} = \dfrac{DE}{AD} \Rightarrow DE = 60\sqrt{3}$

$\therefore$ Height of the opposite house

$CE = CD + DE = 60 + 60\sqrt{3} = 60\,(1+\sqrt{3})m$

12. Distance of hill $= 17.3\ m$.

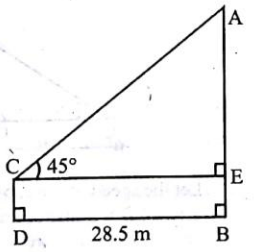
13. 30m

[**Hint:** $\tan 45° = \dfrac{AE}{CE}$

$\Rightarrow\ 1 = \dfrac{AE}{CE}$

$\Rightarrow\ AE = CE$

$AB = BE + AE$
$= (1.5 + 28.5)\,m$
$= 30\,m.$



14. $20(\sqrt{3}-1)m$

Hint: Let AB be the building of height 20m and BC be the transmission tower of height h metres.

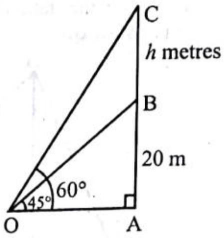
$\tan 45° = \dfrac{AB}{OA} \Rightarrow 1 = \dfrac{20m}{OA}$

$\Rightarrow OA = 20\,m$

$\tan 60° = \dfrac{h+20}{20}$

$\Rightarrow\ \sqrt{3} = \dfrac{h+20}{20}$

$\Rightarrow\ h = 20(\sqrt{3}-1)$



15. $3(\sqrt{3}+1)m$

Hint: A is a point on the bridge and C, D are points on the opposite sides of the bank of the river.

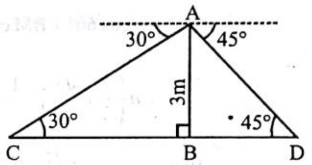
$\tan 30° = \dfrac{AB}{CB} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{AB}{CB}$

$\Rightarrow\ CB = \sqrt{3}\ AB$

$\tan 45° = \dfrac{AB}{BD}$

$\Rightarrow\ 1 = \dfrac{AB}{BD}$

$\Rightarrow\ BD = AB$



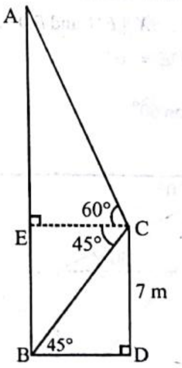
$\therefore$ Width of river $= CB + BD = \sqrt{3}\,AB + AB$

$= (\sqrt{3}+1)\,AB = 3(\sqrt{3}+1)\,m.$

16. Height of each pillar $= 20\sqrt{3}$ m and 20 m from the pillar whose angle of elevation is 60°.

17. $7(\sqrt{3}+1)\,m$;

Hint: $\tan 45° = \dfrac{CD}{BD}$



LONG ANSWER QUESTIONS :

1. Let B be the top of the building F_1, F_2 be the positions of the two fire stations. (figure)

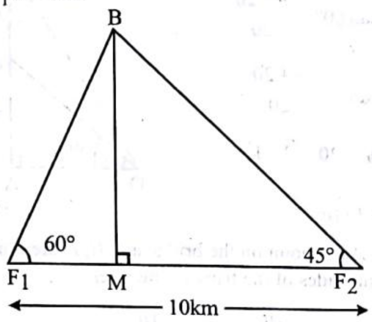
Since $BM = F_1B \sin 60° = F_2 B \sin 45°$

and $\sin 60° > \sin 45°$

$\Rightarrow F_1B < F_2B$

Therefore, the station F_1 should send its team.

Let $F_1B = a$ km



Then, $F_1F_2 = F_1M + MF_2$

$$= a\cos 60° + BM\cot 45° = a \times \frac{1}{2} + a\sin 60°$$

$$= a\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) = \frac{1+\sqrt{3}}{2}a \Rightarrow a = \frac{2F_1F_2}{(1+\sqrt{3})}$$

$$\therefore a = \frac{10\times 2}{\sqrt{3}+1} = 10\times 2\frac{(\sqrt{3}-1)}{2} = 10(\sqrt{3}-1)\,km = 7.32\,km$$

Thus the team from F_1 has to travel 7.32 km (approx.)

2. Let AB be the first church, having height 30 m. i.e. $AB = 30\ m$. There is another church CD, situated at the other bank. AC is the width of the river.

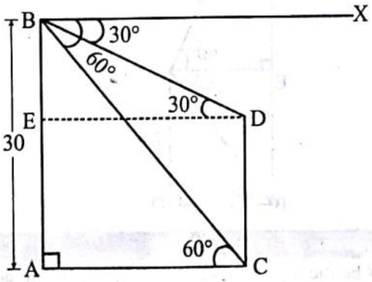
B is the top of the first church from where angle of depression of point D (top of the second church) is 30° and angle of depression of point C, (foot of the second church) is 60°.

We draw a line BX from B parallel to AC, which is the line joining foot of the two churches. We draw a line parallel to AC from D which meet AB at E.

So, $ED \parallel AC \parallel BX$. $BX \parallel ED$ and BD meet them.

So, $\angle DBX = \angle BDE = 30°$.

In ΔABC, $\frac{AB}{AC} = \tan 60°$

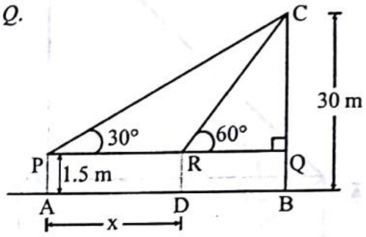


Hence, width of the river $= 10\sqrt{3}m$

$CD = EA = AB - BE = 30 - 10 = 20$ m.

Hence, height of the other church = 20 m.

3. Let eye of the boy be at P, 1.5 meter from ground line shown by line ADB. From point P the boy finds angle of elevatin of top of building is 30°. He moves towards building and comes to a point D where angle of elevation = 60°. Let a line PRQ parallel to horizontal line be drawn that meet the building at point Q.



$AP = DR = BQ = 1.5\ m$.

So, $AD = PR$, $DB = RQ$,

$CQ = CB - BQ = 30 - 1.5 = 28.5\ m$. Let $AD = x$.

In Δ PCQ, $\tan 30° = \frac{CQ}{PQ} = \frac{28.5}{x+DB}$

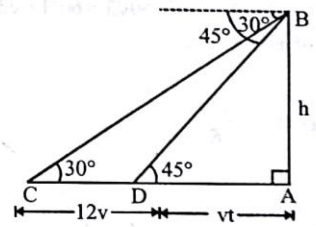
In ΔRQC, $\tan 60° = \frac{28.5}{RQ} = \frac{28.5}{DB} \Rightarrow DB = 28.5\cot 60° = \frac{28.5}{\sqrt{3}}$

So, $x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3} \Rightarrow x = (28.5)\sqrt{3} - (28.5)\frac{1}{\sqrt{3}}$

Hence, the distance walked by the boy towards the building $= 19\sqrt{3}m$.

4. $XQ = \frac{94.64\times 2\times\sqrt{3}}{3} = 109.3$metres , $PQ = 94.64$ m

5. Let AB be the tower of height h metres. Let C be the initial position of the car with angle of depression is 30° and let after 12 minutes the angle of depression at D is 45°.



Let the speed of the car be v meter per minute.

Then, CD = Distance travelled by the car in 12 minutes.

$\Rightarrow$ $CD = 12$ v metres

[$\because$ Distance = speed × time]

Suppose the car takes t minutes to reach the tower AB from D. Then, DA = vt metres.

In ΔABD, we have

$\tan 45° = \frac{AB}{AD} = \frac{h}{vt} \Rightarrow h = vt$ ...(i)

$\sqrt{3}h = vt + 12v$ ...(ii)

Substituting the value of h from equation (i) in equation (ii), we get

$\sqrt{3}vt = vt + 12v \Rightarrow t = 16$ minutes 23 seconds

[$\because$ 0.39 minutes = 0.39×60 seconds]

Thus, the car will reach the tower from D in 16 minutes and 23 seconds.

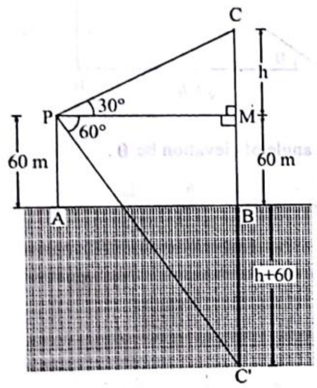
6. Let AB be the surface of the lake and P be the point of observation such that $AP = 60$ metres. Let C be the position of the cloud and C' be its reflection in the lake.

Then $CB = C'B$. Let PM be perpendicular from P on CB. Then, $\angle CPM = 30°$ and $\angle C'PM = 60°$. Let $CM = h$. Then, $CB = h + 60$. Consequently, $C'B = h + 60$. In ΔCMP, we have

$$\tan 30° = \frac{CM}{PM} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM} \Rightarrow PM = \sqrt{3}h \quad \text{...(i)}$$

In $\Delta PMC'$, we have

$$\tan 60° = \frac{C'M}{PM} \Rightarrow \quad \tan 60° = \frac{C'B + BM}{PM}$$



$$PM = \frac{h+120}{\sqrt{3}} \quad \text{... (ii)}$$

From equations (i) and (ii), we get

$$\sqrt{3}h = \frac{h+120}{\sqrt{3}}$$

Hence, the height of the cloud from the surface of the lake is 120 metres.

7. Let CD be the cliff and A and B the two points of observation (see figure).

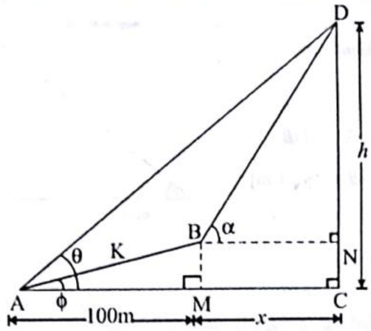
$AB = K$, $\angle BAM = \phi$ and $\angle DAC = \theta$

and $\angle DBN = \alpha$

Let $CD = h$

$$\text{In } \Delta ABM, \ \frac{AM}{AB} = \cos\phi \Rightarrow AM = K\cos\phi \quad \text{......... (1)}$$

$$\text{and } \frac{BM}{AB} = \sin\phi \Rightarrow BM = K\sin\phi \quad \text{......... (2)}$$



In ΔADC, $\frac{AC}{CD} = \cot\theta$

$\Rightarrow AC = h\cot\theta$ and $MC = AC - AM$ .... (3)

$MC = h\cot\theta - K\cos\phi$ [From eq. (1) and (3)]

In ΔBND, $\frac{DN}{BN} = \tan\alpha$

$DN = (h \cot\theta - K\cos\phi)\tan\alpha$.

Since, $h = DN + NC$ .... (4)

$h = (h\cot\theta\tan\alpha - K\cos\phi\tan\alpha) + K\sin\phi$

[From eq. (2) and (4), as NC = BM]

$$h = \frac{K(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$$

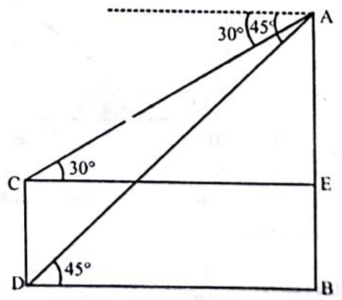
8. 97.6 m/sec.

Hint: Let the speed of the helicopter be x meters per sec.

9. $8\sqrt{3}$ m

10. $4(3+\sqrt{3})$ m; $4(3+\sqrt{3})$ m

Hint: Here AB is a multi-storeyed building and CD is other building. $CD = 8$ m,

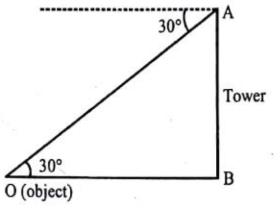


Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (d) **Hint:** $\sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{6\text{ cm}} \Rightarrow BC = 3$ cm.

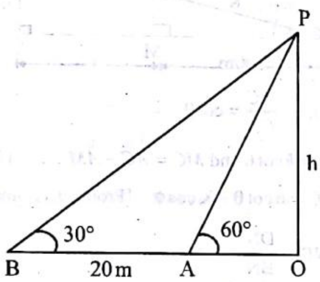
2. **(c)** **Hint:** $\tan 30° = \frac{AB}{OB}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75\text{ m}}{OB}$

$\Rightarrow OB = 75\sqrt{3}$ m]

3. **(c)**



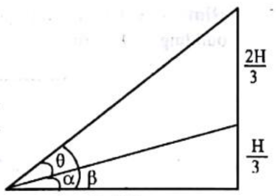
$OA = h\cot 60°,\ OB = h\cot 30°$

$OB - OA = 20 = h(\cot 30° - \cot 60°)$

$$\Rightarrow h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

4. **(b)** $\frac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$

or $\frac{H}{3d} = \tan\alpha$ and $\frac{H}{d} = \tan\beta$



$$\tan(\beta - \alpha) = \frac{1}{2} = \frac{\frac{H}{d} - \frac{H}{3d}}{1 + \frac{H^2}{3d^2}}$$

$$\Rightarrow 1 + \frac{H^2}{3d^2} = \frac{4H}{3d}$$

$\Rightarrow H^2 - 4dH + 3d^2 = 0 \Rightarrow H^2 - 80H + 3(400) = 0$

$\Rightarrow H = 20$ or 60 m

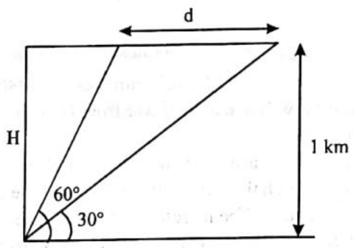
5. **(c)** **Hint:** $\tan 60° = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{BC}{10\sqrt{3}\text{ m}} \Rightarrow BC = 30$ m.

6. **(b)** **Hint:** $\cos 30° = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10\text{ m}}{AB} \Rightarrow AB = \frac{20}{\sqrt{3}}$ m.

7. **(b)** $d = H\cot 30° - H\cot 60°$

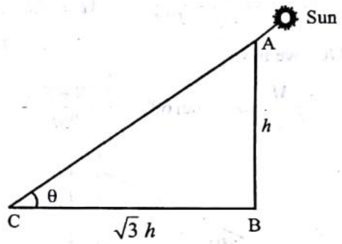
Time taken = 10 second

$$\text{speed} = \frac{\cot 30° - \cot 60°}{10} \times 60 \times 60 = 240\sqrt{3}$$



8. **(b)**

9. **(b)** **Hint:** Let height of tower (AB) be h metres, then length of its shadow $(BC) = \sqrt{3}\,h$ metres.



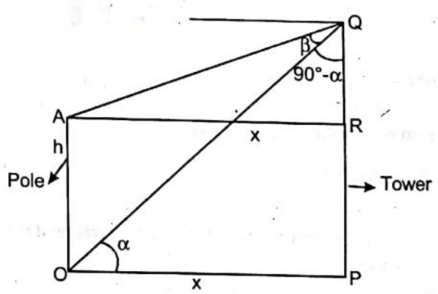
Let angle of elevation be θ,

then $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

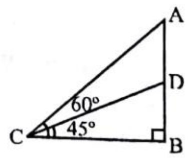
$\Rightarrow \theta = 30°$

10. **(a)** **11.** **(b)**

12. **(b)**



13. **(c)** Let the aeroplanes are at point A and D respectively. Aeroplane A is flying 600m above the ground.



So, $AB = 600$,

$\angle ACB = 60°, \angle DCB = 45°$

From ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow BC = \frac{600}{\sqrt{3}} = 200\sqrt{3}$.

From ΔDCB, $\frac{DB}{BC} = \tan 45° \Rightarrow DB = 200\sqrt{3}$.

So, the distance $AD = AB - BD = 600 - 200\sqrt{3}$

$= 200(3 - \sqrt{3}) = 200(3 - 1.7321) = 253.58$m.

MORE THAN ONE CORRECT :

1. (b, d)
2. (a, b & c)
3. (a,b)

PASSAGE BASED QUESTIONS :

1. (a) Height of the tower $(AB) = h$ m

distance $(CD) = p$ m

Let distance $(BC) = x$ m

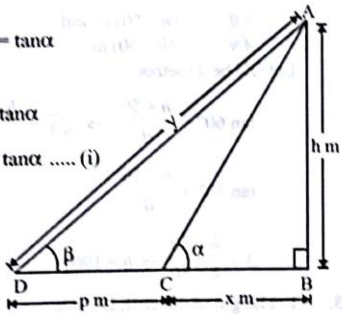
$\angle ACB = \alpha$ and $\angle ADB = \beta$

In right ΔABD,

$\frac{AB}{BC} = \tan\alpha$

$\Rightarrow \frac{h}{x} = \tan\alpha$

$\Rightarrow h = x\tan\alpha$ ..... (i)



In right ΔABD,

$\frac{AB}{BD} = \tan\beta$

$\Rightarrow \frac{h}{BC + CD} = \tan\beta$

$\Rightarrow h = (x + p)\tan\beta$ ..... (ii)

From (1), we get

$x = \frac{h}{\tan\alpha}$

Hence, $h = \frac{p \cdot \tan\alpha \cdot \tan\beta}{\tan\alpha - \tan\beta}$ proved.

2. (b) Putting $p = 150$ m, $\alpha = 60°$ and $\beta = 30°$, we get

$$h = \frac{150 \times \tan 30° \times \tan 60°}{\tan 60° - \tan 30°} \text{ m} = 129.9 \text{ m}$$

Hence, the height of the tower $= 129.9$ m ≈ 130 *m*.

3. (c) $\sin\beta = \frac{h}{y}, \Rightarrow y = \frac{h}{\sin\beta} = \frac{130}{\sin 30°} = 260m$.

ASSERTION & REASON :

1. (a) Both the assertion and reason are correct, reason is the correct explanation of the assertion.

$\tan 30° = \frac{AB}{BC} = \frac{AB}{20}$

$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56$ m.

2. (b) Both assertion and reason are correct, but reason is not the correct explanation of the assertion.

MULTIPLE MATCHING :

1. (A) → q ; (B) → s ; (C) → p ; (D) → r

(A) $\tan 45° = \frac{AB}{BC}$

$BC = 10$

(B) $\tan 60° = \frac{AB}{BC} = \frac{AB}{\sqrt{3}}$

$AB = \sqrt{3} \times \sqrt{3} = 3$

(C) $\cos\theta = \frac{20}{40} = \frac{1}{2} = \cos 60°$

$\theta = 60°$

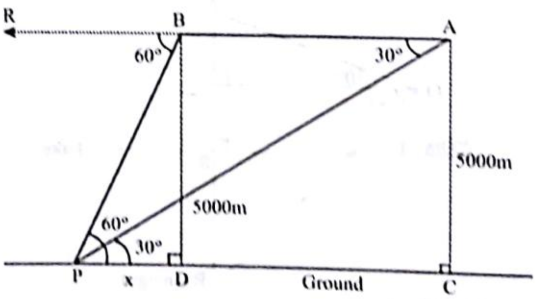
(D) $\tan 45° = \frac{AB}{BC}$

AB = 2 m

$\tan\theta = \frac{AB}{BD} = \frac{2}{10} = \frac{1}{5}$

HOTS SUBJECTIVE QUESTIONS :

1. Let P be the position of the observer. AB represent flying path of an aircraft at a height of 5000 m above the ground. As PB || PC



$\therefore \angle BAP = \angle APC = 30°$ and

$\angle RBP = \angle BPC = 60°$ (Alt $\angle$s)

Let PD = x

Now, in right-angled ΔACP

$$\tan 30° = \frac{AC}{PC} = \frac{5000}{x+DC}$$

$$\Rightarrow x + DC = 5000\sqrt{3} \quad \text{.........(1)}$$

Also, in right angled ΔBDP

$$\tan 60° = \frac{BD}{x} = \frac{5000}{x}$$

$$\Rightarrow x = \frac{5000}{\sqrt{3}} \quad \text{.........(2)}$$

From (1) and (2),

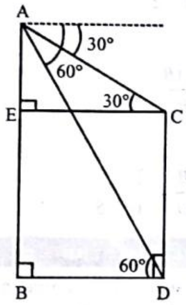
$$DC = 5000\sqrt{3} - \frac{5000}{\sqrt{3}} = \frac{17320}{3} \quad [\because \sqrt{3} \approx 1.732]$$

$$AB = 5773\frac{1}{3}\text{m} \quad (\because AB = DC)$$

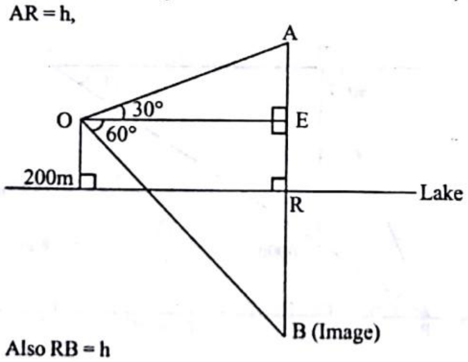
Hence, the distance $AB = 5773.33$ m

2. (i) $BD = \frac{AB}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ m

(ii) $CD = EB = 40$ m.



3. O is the point of observation. A is the cloud and B its reflection. Let height of cloud above the lake be h m; i.e., AR = h,



Also RB = h

$\therefore$ AE = h − 200 and EB = h + 200.

In rt. ΔABC,

$$\Rightarrow OE = (h-200).\sqrt{3} \quad \text{.........(1)}$$

In rt. ΔOEB,

$$OE = (h+200).\frac{1}{\sqrt{3}} \quad \text{.........(2)}$$

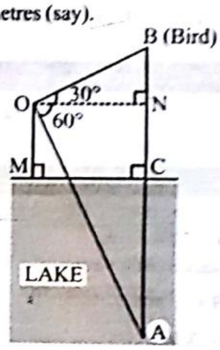
From (1) and (2), we get

$$(h-200)\sqrt{3} = \frac{h+200}{\sqrt{3}} \Rightarrow h = 400$$

$\therefore$ height of the cloud above the lake is 400m.

4. (i) 100 m (ii) 400m

[**Hint.** (i) Let A be the reflection of the bird B in the lake, then $CB = CA = h$ metres (say).



$\therefore$ $NB = (h-50)$ m and $AN = (h+50)$ m

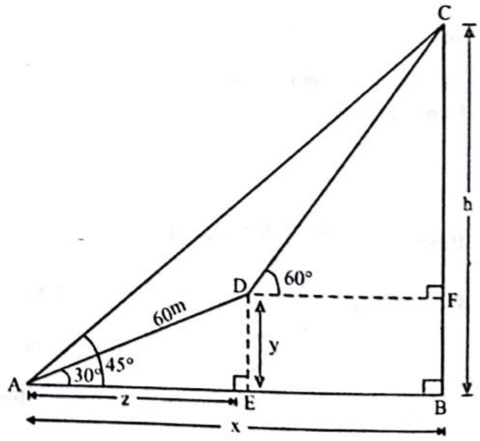
Let ON be d metres.

$$\tan 60° = \frac{h+50}{d} \Rightarrow \sqrt{3} = \frac{h+50}{d}$$

$$\tan 30° = \frac{h-50}{d}$$

$$3 = \frac{h+50}{h-50} \Rightarrow h = 100]$$

5. Let height of the cliff, BC = h,



height of cliff is 81.96 m.